Persistence, cointegration, and aggregation: a disaggregated analysis of output fluctuations in the U.S. economy*

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A framework is developed for measuring the persistence of shocks to aggregate output in the context of a multisectoral model. It is argued that persistence coefficients can be estimated more precisely using a disaggregated model of output growths rather than univariate representations. The effect of cointegration among sectoral output series on the persistence measure is also analysed, and a decomposition of the persistent effect of output innovations into 'monetary' and 'other' shocks provided. The framework is applied to U.S. data, and although 'money' shocks are shown to be statistically significant, their contribution to the total persistence of output fluctuations is found to be relatively unimportant.

1 Introduction

Whether the effect of supply or demand shocks on output is temporary or long lasting is an issue of utmost importance in macroeconomics which has attracted a great deal of attention over the past few years. The traditional view of business cycle decomposes the variations in aggregate output into a deterministic trend and a stationary cyclic component, so that the effect of innovations in output are transitory, having no influence on output levels in the long run. But following the influential work of Nelson and Plosser (1982), many economists have come to view the variations in aggregate output in terms of first-difference stationary processes, thus arguing that shocks have a permanent effect on the level of output. One important issue in this literature is the sire of the long-run response of aggregate output to a unit shock, commonly referred to as the *persistence* of shocks to output.

Several studies have provided estimates of the persistence measure for the real gross national product (GNP) in the United States. These estimates vary considerably depending on the data set used and the estimation procedure adopted. On the basis of low-order ARIMA models estimated on the quarterly U.S. data over the period 1947-85. Campbell and Mankiw (1987a) conclude that 'a 1 percent innovation to real GNP should change one's forecast of GNP over a long horizon by over 1 percent'. Harvey (1985) obtains a similar result using an unobserved component model applied to annual data over the period 1948-70. However, Clark (1987a) and Watson (1986) have obtained substantially lower estimates of persistence using an unobserved component model estimated on a quarterly data set comparable to that employed by Campbell and Mankiw. In these studies a 1 percent shock would lead to around a 0.6 percent change in output in the long run, Cochrane (1988), using a nonparametric procedure also finds little evidence of persistence in GNP. The evidence on the persistence of aggregate output fluctuations in the U.S. is mixed and inconclusive, and as argued in Christiano and Eichenbaum (1989) the issue of

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whether real GNP is trend or difference-stationary may be very difficult to resolve on the basis of the available post-war quarterly data.

The situation is not, however, as hopeless as it may appear at first. All the studies cited above and reviewed in Christiano and Eichenbaum (1989) base the estimation of the persistence measure on *univariate* representations of real GNP, and therefore ignore the information contained in other variables such as aggregate consumption and investment in the estimation of the persistence of output fluctuations. A number of recent studies have followed this line of argument and have used multivariate models, containing variables in addition to the real GNP, to obtain a more reliable estimate of the size of the random walk component in GNP. These include the studies by Campbell and Mankiw (1987b), Clark (1987b), Shapiro and Watson (1988), Evans (1989) and Blanchard and Quah (1989). Evans (1989), for example, argues that due to the presence of feedback between output growth and unemployment and the strong negative contemporaneous correlation between output growth and unemployment innovations, the unemployment data contain important information for the analysis of output persistence. A similar argument is also made in King et al. (1987) with respect to consumption and investment.

In this paper we consider using a different type of additional information, namely sectoral output growth rates, in the analysis of output persistence at the aggregate level. We develop a suitable framework for the measurement of persistence of shocks to aggregate output in the context of a multisectoral model of output growths. We will be arguing that the information contained in the relations between sectoral growth rates, and the correlations that exist between innovations in output growth of different sectors, enables us to obtain a more precise estimate of the persistence of shocks to aggregate output via the disaggregated model than can be obtained through a direct analysis of the aggregate series.

The disaggregated framework also allows us to decompose the persistence effect of output innovations into 'macro' and 'other' possibly sector-specific shocks, under the identifying assumption that the two types of shocks are contemporaneously uncorrelated. The same assumption is also made by Blanchard and Quah (1989). However, unlike Blanchard and Quah's analysis which imposes the additional restriction of a zero long-run impact of 'demand' shocks on output, our approach does not require any further identifying restrictions.

The plan of the paper is as follows: Section 2 presents a brief overview of the literature on measurement of persistence in univariate models. This material prepares the ground for our multisectoral generalization in section 3, where we propose a general measure of persistence based on the spectra1 density of first differences, evaluated at zero frequency. We investigate the effect of cointegration among the sectoral outputs on the sectoral and cross-sectoral persistence measures (section 3.1) and consider the effect that aggregation may have on persistence (section 3.2). Section 3.3 of the paper is concerned with the measurement of persistence in multisectoral models with macroeconomic shocks. Finally, section 4 applies the disaggregated framework developed in the paper to the analysis of output growth in the U.S. economy, using data disaggregated by ten industrial sectors, and provides estimates of persistence for each sector and for the economy as a whole. We also present separate estimates of the persistence measures for the 'monetary' and 'other' shocks, under the identifying restriction that these two types of shocks are uncorrelated. The results show that the estimate of the aggregate persistence measure based on the multisectoral model is appreciably below that obtained from the aggregate series directly, thus providing further evidence on the upward bias of the estimates of persistence measures obtained on the basis of low order univariate ARIMA specifications. We also present results on the statistical significance of the short-term and the long-term effect of unanticipated monetary growth of sectoral outputs, and show that in five out of the ten sectors studied 'money' shocks are statistically significant and their effects do not die out in the long term. Despite this, due to the relatively unimportant nature of 'money' shocks as compared to the other shocks affecting the economy over the period 1955–87, the contribution of 'money' shocks to the total persistence of output fluctuations in the economy turned out to be rather small.

2 Persistence measures in univariate models

In this section we give a brief overview of the different approaches taken to analyze the problem of persistence in univariate models. The aim here is to prepare the ground for our multisectoral generalization of the persistence concept and its measurement in the next section.[1]

[1] Useful surveys of the persistence literature are already available in Diebold. and Nerlove (1989), Stock and Watson (1988) and Christiano and Eichenbaum (1989).

Suppose that y_t follows the genera1 first-difference linear stationary process:

$$\Delta y_t = \mu + a(L)\epsilon_t. \tag{1}$$

where Δ is the first difference operator,

$$a(L) = a_0 + a_1 L + a_2 L^2 + \cdots \tag{2}$$

is a polynomial in the lag operator L, and μ is a scalar constant. The ϵ_t are mean zero, serially uncorrelated shocks with common variance σ^2_ϵ. The trend-stationary process

$$y_t = \gamma t + b(L)\epsilon_t \tag{3}$$

is a limiting case of (1) and arises if $\mu = \gamma$, and the lag polynomial in (1) has a unit root, namely if $a(1) = 0$. In general, the extent to which the first-difference process (1) deviates from the trend-stationary process (3) is clearly related to the magnitude of $a(1)$. A natural measure of the size of the random walk, or the unit root component of (1), is therefore given by $a(1)$, and this is in fact that measure proposed by Campbell and Mankiw (1987a).

The importance of the stochastic trend can also be measured directly in terms of the size of the stochastic variability of the trend component of y_t. Using the Beveridge and Nelson (1981) decomposition, $y_t = \tau_t + z_t$, whereτ_t is the stochastic trend and z_t is the cyclical component, the magnitude of the random walk component can be measured by

$$\mathrm{V}(\tau_t|\Omega_{t-1}) = \sigma^2_\epsilon a^2(1). \tag{4}$$

Alternatively, Cochrane (1988) has proposed using the variance of the long differences of y_t, $\mathrm{V} = \lim_{s\to\infty}(\mathrm{V}_s)$, as a measure of persistence, where

$$\mathrm{V}_s = \mathrm{V}(y_t - y_{t-s})/s\,\mathrm{V}(\Delta y_t),$$

$\mathrm{V}(\cdot)$ denotes the variance operator, and $\mathrm{V}(y_t - y_{t-s})$ is the variance of s-differences of y_t. The relationships between $a(1)$, $\mathrm{V}(\tau|\Omega_{t-1})$, V as measures of persistence can be motivated by noting that all the three measures represent different methods of scaling the spectral density of Δy_t at zero frequency. Let $f_{\Delta y}(\omega)$ be the spectral density function of Δy_t. Then under (1),

$$2\pi f_{\Delta y}(\omega) = \sigma^2_\epsilon a(e^{i\omega})a(e^{-i\omega}), \quad -\pi \le \omega < \pi,$$

and it is easily seen that

$$a^2(1) = 2\pi f_{\Delta y}(0)/\sigma^2_\epsilon, \tag{5}$$

$$\mathrm{V} = 2\pi f_{\Delta y}(0)/\sigma^2_{\Delta y}, \tag{6}$$

where $\sigma^2_\epsilon = \mathrm{V}(y_t|\Omega_{t-1})$ is the conditional variance of Δy_t and $\sigma^2_{\Delta y} = \mathrm{V}(\Delta y_t)$ is the unconditional variance of Δy_t. Therefore, the problem of measurement and estimation of persistence of fluctuations in y_t reduces to the problem of estimating the spectral density of Δy_t at zero frequency.[2] This estimate can then be deflated by the conditional or the unconditional variance of Δy_t to obtain scale-free measures of persistence.

3 Measurement of persistence in a multisectoral model

Consider the following multivariate generalization of (1):

$$\Delta \mathbf{y}_t = \boldsymbol{\mu} + \mathbf{A}(L)\boldsymbol{\epsilon}_t, \tag{7}$$

where $\Delta \mathbf{y}_t$ denotes ther $m \times 1$ vector of output growths $\{\Delta y_{it}$, $\boldsymbol{\mu}$ is an $m \times 1$ vector of constants representing sector-specific mean growth rates, and $\boldsymbol{\epsilon}_t$ is an $m \times 1$ vector of white noise innovations with mean zero and covariance matrix $\boldsymbol{\Sigma} = \{\sigma_{ij}\}$. The matrix polynomial

$$\mathbf{A}(L) = \sum_{i=0}^{\infty} \mathbf{A}_i L^i$$

[2] Some authors, notably Durlauf (1989, 1990) have argued that it is more appropriate to base the analysis of persistence or testing for unit roots on the spectral density (or the spectral distribution function) of first differences at *all* frequencies rather than only at the zero frequency. This seems a useful extension of the spectral density approach to the analysis of persistence, but will not be followed in this paper.

is assumed to be absolutely summable. The $\mathbf{A}_i$'s are $m \times m$ matrices of fixed parameters and $\mathbf{A}_0 = \mathbf{I}_m$ (the $m \times m$ identity matrix). We denote the (i, j) element of $\mathbf{A}(L)$ by the lag polynomials $a_{ij}(L)$.

Almost all the multivariate models analysed in the persistence literature are nested within (7). The bivariate models of Evans (1989) and Blanchard and Quah (1989) and the equilibrium real-business-cycle model discussed in Long and Plosser (1987) can be readily cast in the form of (7). Although the model is specified in the first-difference-stationary form, it allows for one or more of the variables in $\mathbf{y}_t$ to be trend-stationary.[3]

In the above multisectoral model, shocks originating from sector j can influence the long-run level of output in sector i both directly through the lag filter $a_{ij}(L)\epsilon_{jt}$ and indirectly through their correlations with shocks in the other sectors. In computing the persistence measures, it is therefore important that the effect of output fluctuations through both of these channels are taken into account. With this in mind, consider the blind application of the Campbell and Mankiw procedure to (7). This yields

$$\lim_{s\to\infty} \{\partial\, \mathrm{E}(\mathbf{y}_{t+s}|\Omega_t)/\partial \boldsymbol{\epsilon}_t\} = \mathbf{A}(1),$$

which clearly ignores the effects of cross-correlations that may exist between shocks in different sectors and is appropriate only in the orthogonal case where $\sigma_{ij} = 0$ for $i \neq j$. In an attempt to render the Campbell and Mankiw approach generally applicable to multivariate systems, one can make use of the Choleski decomposition $\mathbf{\Sigma} = \mathbf{T}^{-1}\mathbf{D}\mathbf{T}'^{-1}$, where $\mathbf{T}$ is lower triangular with unit diagonal elements and $\mathbf{D}$ is a diagonal matrix. Then (7) can also be written as

$$\Delta \mathbf{y}_t = \mu + [\mathbf{A}(L)\mathbf{T}^{-1}]\mathbf{u}_t, \tag{7'}$$

where $\mathbf{u}_t = \mathbf{T}\boldsymbol{\epsilon}_t$. In this representation the u_{it}'s (the elements of $\mathbf{u}_t$) are contemporaneously uncorrelated, and it may therefore seem appropriate to measure the persistence of shocks by the following multivariate generalization of the Campbell and Mankiw measure.[4]

$$\lim_{s\to\infty} \{\partial\, \mathrm{E}(\mathbf{y}_{t+s}|\Omega_t)/\partial \mathbf{u}_t\} = \mathbf{A}(1)\mathbf{T}_{-1}. \tag{8}$$

This measure is, however, subject to two main criticisms. Firstly, the Choleski decomposition is not unique and there exist many other orthogonal transformations of $\boldsymbol{\epsilon}_t$. Secondly, the (i, j) element of $\mathbf{A}(1)\mathbf{T}^{-1}$ refers to the long-run response of y_{it} to changes in u_{jt}, which is composed of a linear combination of all the sectoral innovations, ϵ_{ij}, $i = 1, 2, \ldots, m$, and does not necessarily correspond to the persistence effect of a shock originating in a *particular* sector.[5]

The spectral density approach to the measurement of persistence is not, however, subject to the above-mentioned shortcomings, and can be easily adapted to derive persistence measures both at the level of individual sectors and at the aggregate level. The (unscaled) sectoral measures of persistence are given by the spectral density of matrix $\Delta\mathbf{y}_t$ evaluated at zero frequency; namely,

$$2\pi f_{\Delta y}(0) = \mathbf{A}(1)\mathbf{\Sigma}\mathbf{A}(1)'. \tag{9}$$

As in the univariate case, this result can also be rationalised directly as measuring the size of the random walk components of the $\mathbf{y}_t$ process. To see this, consider the following multivariate version of the Beveridge-Nelson decomposition:

$$\mathbf{y}_t = \boldsymbol{\tau}_t + \mathbf{z}_t,$$

$$\boldsymbol{\tau}_t = \boldsymbol{\mu} + \boldsymbol{\tau}_{t-1} + \mathbf{A}(1)\boldsymbol{\epsilon}_t,$$

$$\mathbf{z}_t = \sum_{i=0}^{\infty} \mathbf{C}_i\boldsymbol{\epsilon}_{t-i}, \quad \mathbf{C}_i = -\sum_{j=i+1}^{\infty} \mathbf{A}_j,$$

where $\boldsymbol{\tau}_t$ is the $m \times 1$ vector of the (stochastic) trend components and $\mathbf{z}_t$ is the $m \times 1$ vector of the transitory components. Therefore,

$$\mathrm{V}(\boldsymbol{\tau}_t|\Omega_{t-1}) = \mathbf{A}(1)\mathbf{\Sigma}\mathbf{A}(1)',$$

[3] The conditions for y_{it} to be trend-stationary are given by $a_{ij}(1) = 0$, for $j = 1, 2, \ldots, m$.

[4] This is in fact the persistence measure used by (Evans, 1989, p. 220) in his bivariate model of output and unemployment.

[5] In their bivariate model, Blanchard and Quah (1989) manage to avoid these difficulties by first identifying 'supply' and 'demand' shocks with the composite disturbances such as those in $\mathbf{u}_t$, and secondly by imposing the identifying restriction that only 'supply' shocks have a long-run impact on output. This restriction allows them to uniquely determine the Choleski factor of $\mathbf{\Sigma}$, defined by $\mathbf{S} = \mathbf{T}^{-1}\mathbf{D}^{1/2}$.

which is identical to the expression in (9). The (i, j) element in this matrix can now be scaled either by the conditional variance of Δy_{jt}, $\mathrm{V}(\Delta y_{jt}|\Omega_{t-1}) = \sigma_{jj}$, or by its unconditional variance, $\mathrm{V}(\Delta_{jt})$, to obtain scale-free measures of persistence of output fluctuations in sector i caused by a unit shock in sector j. The former method of scaling yields a multisectoral generalisation of Campbell and Mankiw's univariate measure $(a(1)|)$, and the latter gives a generalisation of the Cochrane measure (V). While in principle there is little to choose between the two scaling methods, in practice most researchers have focussed on the Campbell and Mankiw type measure which in the univariate case can be interpreted as the long-run response of y_t to shocks. In what follows we shall also contine our analysis to persistence measures scaled by the conditional variance of first differences.

Let $\mathbf{e}_t$ be a selection vector which has unity on its ith element and zeros elsewhere. Then the cross-sectoral persistence measures, being the long-term effects of shocks in sector j on the level of output in sector i, can be written as

$$P_{ij} = \mathbf{e}_i'\mathbf{A}(1)\mathbf{\Sigma}\mathbf{A}(1)'\mathbf{e}_j/\mathbf{e}_i\mathbf{\Sigma}\mathbf{e}_j, \quad i, j = 1, 2, \ldots, m. \tag{10}$$

The sector-specific measures of persistence, which we denote by P_i (> 0), can be obtained from (10) and are given by $P_i = \sqrt{P_{ii}}$. It is clear that in a univariate model P_i reduces to $A(1)$, which is the familiar Campbell and Mankiw measure. Also, in the case where y_{it} is trend-stationary, it easily follows that $P_i = 0$, as it should.[6]

3.1 Cointegration and persistence

In this section we briefly consider the effect that cointegration among the elements of y_t may have for the cross-sectoral measures of persistence. Here we assume that all components of $\mathbf{y}_t$ are first-difference-stationary, and that there exists an $m \times r$ matrix $\mathbf{a}$ of rank $r(< m)$, such that $\mathbf{a}'\mathbf{y}_t$ is stationary.[7] The matrix $\boldsymbol{\alpha}$ is called the cointegrating matrix and its columns the cointegrating vectors of $\mathbf{y}_t$. The necessary and sufficient conditions for cointegration are given by [see, for example, Engle and Granger (1987)] :[8]

$$\boldsymbol{\alpha}'\mathbf{A}(1) = 0 \text{ and } \boldsymbol{\alpha}'\boldsymbol{\mu} = 0. \tag{11}$$

The condition $\boldsymbol{\alpha}'\mathbf{A}(1) = 0$, which plays a central role in the analysis of cointegrated systems, also implies that $\mathbf{A}(1)\mathbf{\Sigma}\mathbf{A}(1)'$ is singular and there will therefore be some exact linear relationships between the cross-sectoral persistence measures, P_{ij} defined in (10). These relationships are given by $\boldsymbol{\alpha}'\mathbf{P} = 0$, where $\mathbf{P} = \{P_{ij}\}$ is the matrix of cross-sectoral persistence measures. The conditions $\boldsymbol{\alpha}'\mathbf{P} = 0$ and $\boldsymbol{\alpha}'\mathbf{A}(1) = 0$ are in fact mathematically equivalent. Also, $\boldsymbol{\alpha}'\mathbf{P} = 0$ implies that the matrix of persistence measures, $\mathbf{P}$, has rank $m - r$, and there are, therefore, only $m - r$ independent sources of random variations that can have persistence effects on the level of sectoral outputs. This is in line with the Stock and Watson (1988) characterisation of cointegrated systems in terms of common trends and represents an alternative formalization of the cointegration property in terms of independent sources of random variations that have persistence effects.

3.2 Aggregation and persistence

Suppose now we are interested in measuring the persistence effect of shocks at the level of aggregate output, Y_t, defined by

$$Y_t = \sum_{i=1}^{m} w_i y_{it} = \mathbf{w}'\mathbf{y}_t, \tag{12}$$

where $\mathbf{w}' = (w_1, w|_2, \ldots, w_m)$, is an $m \times 1$ vector of positive fixed weights. Under the multisectoral model (7) we have

$$\Delta Y_t = \mathbf{w}'\boldsymbol{\mu} + \mathbf{w}'\mathbf{A}(L)\boldsymbol{\epsilon}_t. \tag{13}$$

This specification is directly comparable to the *univariate* ARIMA models used in the literature for the measurement of persistence at the aggregate level. The main advantage of using (13) over the univariate

[6] When y_{it} is trend-stationary, all the elements in the ith row and in the ith column of matrix $\mathbf{A}(1)\mathbf{\Sigma}\mathbf{A}(1)'$ will be identically equal to zero (see footnote 3).

[7] To ensure that $\mathbf{a}'\mathbf{y}_t$ has bounded variance we also assume that $\mathbf{A}(L)$ is I-summable, namely that $\sum_{i=1}^{\infty} i|\mathbf{A}_i| < \infty$. This condition is satisfied when $\mathbf{A}(L)$ is the lag polynomial matrix in the Wold representation of a vector ARMA specification of $\Delta\mathbf{y}_t$.

[8] We are assuming that the variance matrix of the innovations, $\mathbf{\Sigma}$, is nonsingular.

aggregate models lies in the fact that by exploiting even very simple univariate time series specifications at the disaggregate level, we are still able to arrive at very-high-order ARIMA specification for the aggregate output, Y_t. For example, it is possible to obtain an ARIMA(m,$m-1$) specification for Y_t, even if the sectoral output growths are specified to follow independent AR(1) processes.[9] Thus, given the difficulties involved in obtaining accurate estimates of high-order ARIMA processes using available aggregate time series, one possible way out would be to base the estimation of the aggregate persistence measure on (13) instead of relying on low-order ARIMA specifications of the aggregate output directly.[10]

Let P_y be the persistence measure of aggregate output obtained on the basis of the disaggregate specification (7). Applying the spectral density approach to (13) and using the conditional variance of ΔY_t as the scaling factor, we have

$$P_y^2 = \mathbf{w}'\mathbf{A}(1)\mathbf{\Sigma}\mathbf{A}(1)'\mathbf{w}/\mathbf{w}'\mathbf{\Sigma}\mathbf{w}. \tag{14}$$

This measure is directly comparable to the Campbell and Mankiw measure $a(1)$, derived using a univariate time series specification of aggregate output.[11]

It is important to note that the 'aggregate persistence measure', P_y, is valid irrespective of whether one or more of the sectoral outputs in y_t are trend- stationary. For example, in the case where output of all sectors except the output of sector i are trend-stationary we have

$$P_y = \{\sigma_{ii} w_i^2/\mathbf{w}'\mathbf{\Sigma}\mathbf{w}\}^{1/2} P_i,$$

where P_i is the persistence measure of sector i.

Clearly, $P_y = 0$ if $P_i = 0$, $i = 1, 2, \ldots, n$. The reverse is not, however, true. In principle we could have $P_i = 0$, even if none of the sectoral persistence measures is equal to zero. This arises when $\mathbf{y}_t$ is cointegrated and the aggregating $\mathbf{w}$ is proportional to one of the cointegrating vectors in $\boldsymbol{\alpha}$.[12] In general, however, the effect of cointegration among the sectoral outputs on P_y is complex and involves all the cross-sectoral persistence measures, P_{ij}. Using (10) and (14), we have

$$(\mathbf{w}'\mathbf{\Sigma}\mathbf{w})P_y^2 = \sum_{i,j=1}^{m} w_i w_j \sigma_j j P_{ij}, \tag{15}$$

where P_{ij} are related through $\boldsymbol{\alpha}'\mathbf{P} = 0$ where $\mathbf{y}_t$ is cointegrated with the cointegrating matrix $\boldsymbol{\alpha}$.

An interesting specialisation of (15) arises when sectoral outputs are pairwise cointegrated. Consider first the simple case where $m = 2$, and let $\boldsymbol{\alpha} = (\alpha_1, \alpha_2)'$ be the cointegrating vector. Then using the result in Proposition 1 and noting that $\sigma_{22}P_{12} = \sigma_{11}P_{21}$, we have

$$P_{21} = (-\alpha_1/\alpha_2)P_{11},$$

$$P_{11}/P_{22} = (\alpha_2/\alpha_1)^2(\sigma_2 2/\sigma_{11}).$$

Substituting these results in (15), it is now easily seen that[13]

$$\begin{aligned}(\mathbf{w}'\mathbf{\Sigma}\mathbf{w})^{1/2}P_y &= |w_1\sigma_{11}^{1/2}P_1 - w_2\sigma_{22}^{1/2}| \text{ if } \alpha_1/\alpha_2 > 0,\\ &= w_1\sigma_{11}^{1/2}P_1 + w_2\sigma_{22}^{1/2}P_2, \text{ if } \alpha_1/\alpha_2 < 0,\end{aligned}$$

where $P_i = \sqrt{P_{ii}} > 0$. Therefore, the value of the aggregate persistence measure crucially depends on whether in large samples the sectoral outputs are correlated positively (i.e., $\alpha_1/\alpha_2 < 0$) or negatively (i.e., $\alpha_1/\alpha_2 > 0$). Under the more likely case where $\alpha_1/\alpha_2 < 0$, P_y can be written as the weighted sum of the two sectoral persistence measures, namely,

$$P_y = \lambda_1 P_1 + \lambda_2 P2, \tag{16}$$

where $\lambda_i = (w_i^2\sigma_{ii}/\mathbf{w}'\mathbf{\Sigma}\mathbf{w})^{1/2}, i = 1, 2$. This result readily extends to the m-sector case.

[9] See, for example, Granger and Morris (1976) and the review of small scale aggregation in Granger (1990).
[10] The use of low-order ARIMA processes in the estimation of persistence at the aggregate levels has been crittcized by Cochrane (1988) and Christiano and Eichenbaum (1989).
[11] The counterpart of Cochrane's measure V, based on the disaggregated model, is given by

$$V_d = \{\mathbf{w}'\mathbf{A}(1)\mathbf{\Sigma}\mathbf{A}(1)'\mathbf{w}/\,\mathrm{V}(\Delta Y_t)\}.$$

[12] This follows immediately from (11) and (14).
[13] Notice that in the present case $\mathbf{w} = (w_1, w_2)'$, and $\mathbf{\Sigma}$ is the variance matrix of $(\epsilon_{1t}, \epsilon_{2t})'$.

Proposition 1. *Let P_i be the sectoral persistence measures, P_y the aggregate persistence measure, and suppose thut the sectoral output levels $y_{it}, i = 1, 2, \ldots, m$, are pairwise cointegrated. Then, assuming that sectoral outputs are correlated positively, we have*

$$P_y = \sum_{i=1}^{m} \lambda_i P_i, \tag{17}$$

where $\lambda_i = (w_i^2 \sigma_{ii} / \mathbf{w}'\mathbf{\Sigma}\mathbf{w})^{1/2}, i = 1, 2, \ldots, m$. (See appendix A for a proof.)

The above result can also be written as

$$f_{\Delta y}^{1/2}(0) = \sum_{i=1}^{m} w_i f_{\Delta y_i}^{1/2}(0), \tag{18}$$

which provides a decomposition of the spectral density of the aggregate growth rate at zero frequency, $f_{\Delta y}(0)$, in terms of the spectral densities of the sectoral growth rates also evaluated at the zero frequency, $f_{\Delta y_i}(0)$.

3.3 Measurement of persistence in models with macroeconomic shocks

Consider now the model

$$\Delta \mathbf{y}_t = \boldsymbol{\mu} + (L)v_t + \mathbf{A}(L)\boldsymbol{\epsilon}_t, \tag{19}$$

where v_t represents a *scalar* white-noise process with mean zero and constant variance σ_t^2, and (L) is an $m \times 1$ vector of lag polynomials,

$$\mathbf{d}(L) = \mathbf{d}_0 + \mathbf{d}_1 L + \mathbf{d}_2 L^2 + \cdots, \quad \sum_{i=1}^{\infty} i|\mathbf{d}_i| < \infty.$$

This model provides a generalisation of (7) and allows for the effect of a common shock, v_t, on sectoral output growths in addition to the sector-specific shocks, ϵ_{it}. We refer to v_t as the 'macro' shock and in this paper identify it with the unexpected growth of money supply.[14] We use the following specification for the money growth equation:

$$\Delta m_t = \boldsymbol{\beta}'\mathbf{z}_t + v_t, \tag{20}$$

where $\mathbf{z}_t$ is a vector of predetermined variables to be specified more fully in the next section. To ensure that the parameters of the equation systems (19) and (20) are identified, we assume that v_t and $\boldsymbol{\epsilon}_t$ are uncorrelated.

The identifying nature of the restriction $\mathrm{Cov}(\boldsymbol{\epsilon}_t, v_t) = \mathbf{\Sigma}_{\epsilon v} = 0$ can be easily shown in the case of normally distributed shocks. Suppose $\mathbf{\Sigma}_{\epsilon v} \neq 0$. Then, under the normality assumption we may write

$$\boldsymbol{\epsilon}_t = (\sigma_v^{-2}\mathbf{\Sigma}_{\epsilon v})v_t + \mathbf{u}_t,$$

where v_t and $\mathbf{u}_t$ are now uncorrelated. Using this result in (19) gives

$$\Delta y_t = \boldsymbol{\mu} + [\mathbf{d}(L) + \sigma_v^{-2}\mathbf{A}(L)\mathbf{\Sigma}_{\epsilon v}]v_t + \mathbf{A}(L)\mathbf{u}_t. \tag{21}$$

The joint maximum likelihood (ML) estimation of (21) and (20) now yields consistent estimates of $\mathbf{A}(L)$ and $\mathbf{d}^*(L) = \mathbf{d}(L) + \sigma_v^{-2}\mathbf{A}(L)\mathbf{\Sigma}_{\epsilon v}$. However, it is not possible to recover a consistent estimate of $\mathbf{d}(L)$, unless $\Sigma_{\epsilon v} = 0$.[15]

Under the above assumptions, it is now possible to decompose the persistence of output fluctuations into the components due to 'money' and due to 'other' shocks. This can be done both at the level of individual sectors and for the economy as a whole. Here we focus on the decomposition of the aggregate persistence measure, P_y. Using (19), the spectral density of $\Delta Y_t = \mathbf{w}'\Delta y_t$ at zero frequency is now given by

$$2\pi f_{\Delta y}(0) = \sigma_v^2[\mathbf{w}'\mathbf{d}(1)]^2 + \mathbf{w}'\mathbf{A}(1)\mathbf{\Sigma}\mathbf{A}(1)'\mathbf{w}.$$

[14] Other types of macroeconomic shocks, such as unexpected changes in oil prices or exchange rates, may also be considered. Indeed, the inclusion of more than one type of shock can be easily accomodated within this framework, subject to identifying restrictions similar to that discussed below [see Lee et al. (1992)].

[15] This results also highlights the general difficulty involved with the decomposition of output innovations into 'supply' and 'demand' shocks, and shows that such a decomposition is only meaningful if one is prepared to assume that the two types of shocks are contemporaneously uncorrelated. [See Blanchard and Quah (1989).]

Scaling this by the conditional variance of ΔY_t,

$$\mathrm{V}(\Delta Y_t|\Omega_{t-1}) = \sigma_v^2(\mathbf{w}'\mathbf{d}_0)^2 + \mathbf{w}'\mathbf{\Sigma}\mathbf{w},$$

yields the following decomposition of P_y:

$$P_y^2 = \lambda P_m^2 + (1-\lambda)P_O^2, \tag{22}$$

where P_m is the component of the persistence measure due to 'monetary' shocks, P_O is the component due to 'other' shocks:

$$P_m = \mathbf{w}\mathbf{d}(1)/\mathbf{w}'\mathbf{d}_O, \tag{23}$$

$$P_O = \{\mathbf{w}'\mathbf{A}(1)\mathbf{\Sigma}\mathbf{A}(1)'\mathbf{w}/\mathbf{w}'\mathbf{\Sigma}\mathbf{w}\}^{1/2}, \tag{24}$$

and λ is the mixture coefficient defined by

$$\lambda = \sigma_v^2(\mathbf{w}'\mathbf{d}_O)^2/(\sigma_v^2(\mathbf{w}'\mathbf{d}_O)^2 + \mathbf{w}'\mathbf{\Sigma}\mathbf{w}). \tag{25}$$

Similar decompositions can also be obtained at the sectoral levels. The formulae for the decomposition of P_i, the sectoral persistence measures, can be obtained from (22)–(25) by replacing $\mathbf{w}$ in these expressions with $\mathbf{e}_i$. Note that care must be taken in interpreting the persistence measure due to 'monetary' shocks since misspecification in (19) can result in biased estimates for $\mathbf{d}(L)$. In particular, the omission of other important macroeconomic shocks which are negatively corre- lated to 'money' shocks (through, for example, a feedback rule in which monetary policy aims to offset the effects of the omitted shock) will result in downward bias in the measure of persistence due to 'money' shocks.

4 Empirical results: Measures of sectoral and aggregate persistence for the U.S. economy

The persistence effect of shocks to the U.S. real GNP has been extensively investigated at the aggregate level. [See, for example, Campbell and Mankiw (1987a,b), Clark (1987a), Cochrane (1988), Watson (1986), Haubrich and Lo (1989), and Durlauf (1989).] Here we apply the methods discussed in the previous section and provide a disaggregated analysis based on a multisectoral model composed of ten sectors[16] The available data set covers the 1947–87 period.

4.1 Testing for unit roots at the sectoral levels

The first stage in the analysis is to test for unit roots in sectoral outputs (measured in logarithms).[17] Table 1 gives the Augmented Dickey-Fuller (ADF) statistics for four different lag lengths computed over the sample period 1952–87.[18] [See Fuller (1976) and Dickey and Fuller (1981).] All the underlying ADF regressions are estimated on the same data set and include simple linear trends. The relevant 5% and 10% critical values for the ADF statistics are equal to −3.54 and −3.20. respectively.[19] None of the tests come even close to rejecting the unit root hypothesis, and this is true of all the sectors. With the exception of the durable manufacturing, this finding is in accordance with the results reported in (Durlauf, 1989, table 6). The difference in the two results in the case of the durable manufacturing seems to be primarily due to the different procedure used by Durlauf to correct the simple DF statistic for the residual serial correlation.[20] To check the robustness of the unit root tests to the specification of the trend, following Perron (1988, 1989), we also computed ADF statistics assuming a different growth

[16] The sources of the data and the detals of the sectoral classifications are given m appendix C.

[17] Although the aggregate persistence measure, P_Y, proposed in this paper is applicable even if (log) outputs in one or more sectors arc trend-stationary (see section 3.2), nevertheless at the estimation stage where finite-order AR or ARMA processes are fitted to the data, it is more appropriate to exclude sectors with trend-stationary output processes from the analysis. Ambiguities, however, arise when we are not sure whether the output of a particular sector is trend- stationary or not.

[18] The choice of the sample period in the computation of the ADF statistics is governed by the available data and the highest order chosen for the ADF text, namely 4.

[19] These critical values are obtained using the simulated response surfaces gven in (MacKinnon, 1990, table I).

[20] Durlauf (1989) employs the Phillips-Perron type correction instead of the augmentation of the simple DF regression by lagged values of Δy_{it}. [See Phillips (1987) and Phillips and Perron (1988).] But recent Monte Carlo evidence by Schwert (1989) suggests that the ADF procedure tends to have better small sample properties than the Phillips-Perron method.

Table 1: Augmented Dickey-Fuller statistics for tests of a unit root[a] in U.S. sectoral outputs (in logs); 1952–1987.

	Dickey-Fuller statistics[b]			
Sectors	ADF(1)	ADF(2)	ADF(3)	ADF(4)
1. Agriculture	−1.58	−0.35	0.06	0.14
2. Mining	−0.17	−0.10	−0.30	−0.52
3. Construction	−2.14	−2.14	−2.15	−2.17
4. Dur. manuf.	−2.99	−2.96	−2.67	−2.78
5. Nondur. manuf.	−1.82	−0.93	−0.52	−0.38
6. Transport	−2.95	−2.73	−2.65	−2.72
7. Utilities	−0.51	−0.52	−0.50	−0.53
8. Trade	−2.65	−2.31	−2.04	−1.54
9. Services	−1.46	−1.22	−0.78	−0.62
10. Government	−1.99	−1.11	−0.70	−0.60

[a]The underlying augmented Dickey-Fuller regressions contain a simple linear time trend and are based on the same number of observations.

[b]The (asymptotic) 5% and 10% critical values are −3.54 and −3.20 respectively.

path before and after the first oil shock in 1973 for each sector. This is the 'changing growth' model in Perron (1989). The results are summarized in table 2. Only in the case of the 'government' sector is there a clear cut case against the unit root hypothesis. For all the other sectors the hypothesis is either not rejected, or if rejected, the rejection has been confined to one out of the four ADF statistics that were computed for each sector. With the exception of the 'government' sector, Perron's alternative trend specification does not significantly alter the conclusion reached earlier, namely that the hypothesis of a unit root in sectoral outputs cannot be rejected.

Table 2: ADF statistics for tests of a unit root in sectoral outputs under Perron's 'changing growth' model;[a] 1952–1987.

	Dickey-Fuller statistics[b]			
Sectors	ADF(1)	ADF(2)	ADF(3)	ADF(4)
1. Agriculture	−4.21[b]	−3.05	−2.73	−2.59
2. Mining	−2.55	−2.39	−3.10	−4.74[b]
3. Construction	−2.10	−1.61	−1.79	−1.75
4. Dur. manuf.	−3.15	−3.24	−3.01	−3.21
5. Nondur. manuf.	−4.39[b]	−3.24	−2.77	−2.61
6. Transport	−2.22	−1.99	−1.94	−2.19
7. Utilities	−1.82	−1.75	−1.97	−1.94
8. Trade	−3.45	−3.21	−3.01	−2.38
9. Services	−3.40	−3.52	−3.06	−2.81
10. Government	−6.62[b]	−5.26[b]	−4.65[b]	−4.80[b]

[a]The Augmented Dickey-Fuller (ADF) statistics are based on the OLS residuals computed over the period 1947–87 from the regressions of sectoral outputs (in logs) on an intercept, a simple linear trend, t, and the broken trend line defined as $DT_t^* = t - T_\beta$ if $t > T_\beta$ and 0 otherwise, where $T_\beta = 27$ is the time break in 1973 and $t = 1$ in 1947. See model B in Perron (1989).

[b]Statistical significance at the 5% level. For relevant critical values, see table V.B in Perron (1989), with $\lambda = T_\beta / T = 0.65$.

Finally, before we can utilize the multisectoral model (7), it is important to show that output growths, Δy_{it}, are in fact stationary. Table 3 gives the ADF statistics for the test of a unit root in sectoral growth rates. For most sectors (six out of ten) there is a clear-cut rejection of the unit root hypothesis, and even for the sectors where the evidence is mixed the hypothesis is still rejected on the basis of the simple DF and the ADF(1) statistics. We therefore proceed with the presumption that sectoral growth rates are stationary and that the multisectoral model is a suitable framework for the analysis of persistence in the U.S. post-war economy.

Table 3: Augmented Dickey-Fuller statistics for tests of a unit root in U.S. sectoral output growths,[a] 1952–1987.

	Dickey-Fuller statistics[b]			
Sectors	DF	ADF(1)	ADF(2)	ADF(3)
1. Agriculture	-6.73^b	-6.34^b	-4.56^b	-3.25^b
2. Mining	-5.15^b	-3.81^b	-2.11	-1.42
3. Construction	-3.78^b	-4.43^b	-3.05^b	-2.70
4. Dur. manuf.	-6.21^b	-4.95^b	-4.52^b	-3.67^b
5. Nondur. manuf.	-6.23^b	-6.26^b	-4.91^b	-3.76^b
6. Transport	-5.60^b	-4.69^b	-3.79^b	-2.52
7. Utilities	-5.39^b	-3.25^b	-2.28	-2.16
8. Trade	-5.18^b	-4.68^b	-4.22^b	-4.22^b
9. Services	-4.68^b	-3.91^b	-3.70^b	-3.00^b
10. Government	-10.46^b	-11.10^b	-10.43^b	-8.65^b

[a]The ADF regressions contain an intercept term but not a time trend.

[b]Statistical significance at the 5% level.

5 Estimates of the persistence measures

To obtain estimates of the persistence measures we need consistent estimates of $\mathbf{A}(L)$ and $\mathbf{\Sigma}$, the parameters of the multisectoral model (7). For this purpose we initially considered two different versions of a second-order vector autoregressive, VAR(2), version of (7): a fully unrestricted version.[21]

$$M_1: \quad (\mathbf{I}_m - \mathbf{C}_1 L - \mathbf{C}_2 L^2)\Delta \mathbf{y}_t = \mathbf{C}(L)\Delta \mathbf{y}_t = \mathbf{a} + \boldsymbol{\epsilon}_t \tag{26}$$

where $\mathbf{C}_1 = \{c_{1,ij}\}$ and $\mathbf{C}_2 = \{c_{2,ij}\}$ are $m \times m$ matrices and $\mathbf{a}$ is a vector of fixed constants: and a version that restricts the coefficients of $\Delta y_{j,t-1}$ (and $\Delta y_{j,t-2}$ in the ith output growth equation to be the same for all $j (j \neq i)$. That is,

$$M_2: \quad \Delta y_{it} = a_i + c_{1,ii}\Delta y_{i,t-1} + c_{2,ii}\Delta y_{i,t-2} + b_{1i}\Delta \mathbf{y}_{-i,t-1} + b_{2i}\Delta_{-i,t-2} + \epsilon_{it}, \quad i = 1, 2, \ldots, m,$$

where

$$\Delta y_{-i,t} = \sum_{j=1, j\neq i}^{m} \Delta y_{jt}.$$

Model M_2 imposes $2m(m-2)$ parametric restrictions on M_1, and has the interpretation that output growth in sector i is related to a simple aggregation of output growths in the rest of the economy. In terms of the parameters of M_1 these restrictions are

$$c_{1,ij} = b_{1i}, \quad c_{2,ij} = b_{2i}, \quad i, j = 1, 2, \ldots, m, j \neq i. \tag{27}$$

With $m = 10$, M_2 imposes 160 restrictions on M_1. Despite these restrictions, this model still allows for feedbacks from the rest of the economy to the ith sector. In addition to M_1 and M_2 we also estimated a third model, M_3, which imposed further parameter restrictions on M_2 by dropping regressors in M_2 whose coefficients had a t-ratio (in absolute value) less than unity. All the three models were estimated by the Full Information Maximum Likelihood (FIML) method over the period 1955–87.[22] Table 4 gives

[21] Notice that given the available sample size, the highest order VAR that we can fit to the data is a second-order one.

[22] Clearly, in the case of the unrestricted VAR(2) model, M_1, the FIML and the OLS estimates coincide. In the case of the restricted VAR models, M_2 and M_3, the FIML estimates are computed by iterating on the Seemingly Unrelated Regression Equations (SURE) estimates, described in Zellner (1962).

Table 4: Maximised log-likelihood values[a]

Models	*LLF*	*N*
M_1	897.99	210
M_2	825.62	50
M_3	819.87	31

[a] *LLF* is the maximised log-likelihood values and N is the number of estimated regression coefficients.

the maximised log-likelihood values (LLF) and the number of regression coefficients estimated under each of the three models. The log-likelihood ratio statistic for the test of M_2 against M_1 is equal to 144.74 and for the test of M_3 against M_2 it is equal to 11.50. Both these statistics are well below the 95 percent critical values of the chi-squared distribution with 160 and 19 degrees of freedom, respectively.[23] The nonrejection of M_2 against M_1 is particularly noteworthy, and has important consequences for the precision with which persistence measures are estimated. The use of an overparameterised model such as M_1 will, in general, lead to poorly determined persistence measures, and this can be seen clearly in the estimates of the aggregate persistence measures obtained for the U.S. economy on the basis of models M_1 to M_3 (see the last row of table 5).[24] The highly overparameterised model M_1 yields a large but extremely unreliable estimate of the aggregate persistence measure, $\widehat{P}_y$. Under M_1, the (asymptotic) standard error of $\widehat{P}_y$ is estimated to be equal to 2.67, which is well in excess of the value estimated for P_y itself![25] The situation, however, is very different when we consider the estimates of P_y based on the more parsimonious models M_2 and M_3. The best estimate of P_y (in the sense of having the least variance) is obtained under model M_3.[26] It is equal to 0.83 with an (asymptotic) standard error of 0.0849. This estimate is well below unity and is more in line with the estimates obtained by Watson (1986) and Clark (1987a) than the ones obtained by Campbell and Mankiw (1987a).[27]

For a more direct comparison of the above estimate of the aggregate persistence measure with an estimate based on a univariate model, we fitted a number of ARMA specifications directly to the aggregate output growth defined by $\Delta Y_t = \sum_{i=1}^{m} \Delta y_{it}$.[28] The results for ARMA processes of orders (i, j), $i, j = 1, 2, 3, 4$, are summarised in tables 6a and 6b. The maximised values of the log-likelihood function given in table 6a are all close to one another and the only model which come close to rejecting the ARMA (1,1) specification at the 5% level is ARMA(1,3). Overall, there is very little to choose between the different ARMA processes. In view of this, in table 6b we give the estimates of the aggregate persistence measures for all the ARMA specifications. These estimates are all above unity and fall in the range 1.00–1.85, which are clearly compatible with Campbell and Mankiw's estimates, but not with our estimate of P_y based on the multisectoral model, M_3. Turning now to the persistence of output fluctuations at the sectoral levels, using (16) we also estimated sector-specific persistence measures, P_i, under models M_1 to M_3. The results are summarised in table 5. In view of the highly overparameterised nature of M_1, we shall confine our attention to the estimates obtained under the restricted models M_2 and M_3. These two sets of estimates give a similar pattern of persistence across the sectors, with high values estim- ated for 'construction', 'services', and 'government' sectors, and low values estimated for 'agriculture', 'trade', and 'nondurable manufacturing' sectors.

[23] Notice that for degrees of freedom $v > 100$, we have

$$\sqrt{2\chi_v^2} - \sqrt{(2v-1)} \overset{a}{\sim} \mathrm{N}(0, 1).$$

[24] The aggregate persistence measure, P_y, relates to $Y_t = \sum_{i=1}^{m} y_{it}$, and is computed according to (14) with $\mathbf{w}' = (1, 1, \ldots, 1)$, $\mathbf{A}(1) = \mathbf{I}_m - \mathbf{C}_1 - \mathbf{C}_2$, and $m = 10$.

[25] A derivation of the asymptotic variance of $\widehat{P}_y$, together with the variance of other persistence measures, can be found in appendix B.

[26] Also, recall that M_3 could not be rejected against either M_2 or M_3 (see table 4).

[27] Notice that our measure of aggregate output is based on the sum of the logarithms of sectoral outputs, while the measure of aggregate output used in the literature is the logarithm of the sum of sectoral outputs. However, the two measures are very closely related. The correlation between the output growth rates calculated using the two measures is equal to 0.97.

[28] To estimate the ARMA models we used the exact ML algorithm proposed in Pesaran (1988a), which allows for the possibility of obtaining ML estimates on the unit circle. This is an important consideration, especially when the aim is the estimation of persistence measure. On this see also Campbell and Mankiw (1987a).

Table 5: Sectoral and aggregate persistence measures.[a]

	Models[b]		
Sectors	M_1	M_2	M_3
1. Agriculture	2.75	0.84	0.89
	(3.73)	(0.01)	(0.004)
2. Mining	2.61	1.77	1.38
	(3.53)	(0.51)	(0.11)
3. Construction	4.01	3.45	3.64
	(5.02)	(1.46)	(1.54)
4. Dur. manuf.	1.42	0.91	1.00
	(0.72)	(0.04)	(0.02)
5. Nondur. manuf.	1.82	0.58	0.64
	(2.31)	(0.01)	(0.003)
6. Transport	1.63	1.08	1.17
	(0.93)	(0.04)	(0.01)
7. Utilities	5.22	1.77	1.76
	(7.06)	(0.28)	(0.24)
8. Trade	1.43	0.66	0.73
	(1.37)	(0.01)	(0.01)
9. Services	4.44	2.77	3.75
	(6.00)	(2.73)	(6.93)
10. Government	4.69	2.45	2.65
	(5.07)	(0.20)	(0.18)
Aggregate output	2.09	0.76	0.83
	(2.67)	(0.1473)	(0.0849)

[a]The sectoral persistence measures, $P_i = \sqrt{P_{ii}}$, are estimated using (10). The aggregate persistence measure P_y, is estimated using (14) with $\mathbf{w}' = (1, 1, \ldots, 1)$. The figures in parentheses are asymptotic standard errors, computed according to the formulae given in appendix B.

[b]M_1 is an unrestricted VAR(2) specification of the multisectoral model, and M_2 and M_3 are restricted versions of M_1. See the text for further details.

Table 6a: Maximised log-likelihood values for different ARMA models fitted to ΔY_t: 1955–1987

	Order of AR			
Order of MA	1	2	3	4
1	80.69	80.71	82.43	83.10
2	80.69	82.63	82.79	83.11
3	83.59	83.84	83.86	83.89
4	83.85	83.87	83.88	83.90

The relationship between the estimates of sectoral persistence measures, $\widehat{P}_y$, given in table 5 is complex and, as shown in section 3.1, depends on the pattern of cointegration among the sectoral outputs. We first tested for the presence of cointegration among the sectoral output series using Johansen (1988, 1989) maximum likelihood procedure[29] and found evidence of between five and eight cointegrating vectors, depending on whether we use the trace or the maximum eigenvalue test criterion. These results indicate that there are substantially fewer than ten independent sources of random variation that affect sectoral outputs, although they should be treated with caution, given the paucity of degrees of freedom. We

[29] We computed both of Johansen's proposed test statlstlcs, namely the 'trace' and the 'maximal eigenvalue' statistlcs, over the period 1955–87 on the basis of a VAR(2) model allowing for an intercept term, a linear time trend, and a time trend in the underlyng data generatIon process. The computations were carried out on *Microfit 3.30* [Pesaran and Pesaran (1991)].

Table 6b: Aggregate persistence measures estimated on the basis of ARMA models fitted to ΔY_t: 1955–1987

	Order of AR			
Order of MA	1	2	3	4
1	1.22	1.17	0.93	1.15
	(0.0093)	(0.228)	(0.185)	(0.352)
2	1.20	1.53	1.03	1.16
	(0.393)	(0.197)	(0.238)	(0.376)
3	1.00	1.07	1.06	1.07
	(0.307)	(0.316)	(0.329)	(0.382)
4	1.10	1.09	1.08	1.09
	(0.357)	(0.351)	(0.361)	(0.405)

then tested the hypothesis of pairwise cointegration of sectoral outputs. Surprisingly, we found very little statistically significant evidence of pairwise cointegration. The hypothesis of pairwise cointegration was easily rejected for the 'agriculture', 'mining', and 'utilities' sectors. None of these sectors showed significant evidence (at the 5% level) of cointegration with the other sectors in the economy. We only found significant evidence of pairwise cointeg- ration in the case of sectors (3,4), (3,5), (3,8), (5,10), and (8,9).[30] We therefore do not expect to find a simple relationship between the sectoral and the aggregate persistence measures in the case of the U.S. economy.

5.1 Persistent effects of 'monetary' and 'other' shocks

In this section we provide evidence on the relative importance of 'monetary' and 'other' shocks for the long-run evolution of the U.S. output. In view of the highly overparameterised nature of M_1, and since M_2 could not be rejected against M_1, we base our analysis on model M_2 and augment it with the current and the one-period-lagged values of the unanticipated growth of money supply, v_t. This augmented model, which we denote by $\widetilde{M}_2$, may be written as

$$\widetilde{M}_2 : \quad \mathbf{C}(L)\Delta \mathbf{y}_t = \mathbf{a} + (\gamma_0 + \gamma_1 L)v_t + \boldsymbol{\epsilon}_t \tag{28}$$

where $\gamma_j' = (\gamma_{j1}, \gamma_{j2}, \ldots, \gamma_{jm}), j = 0, 1$. Recall that under M_2 there are $2m(m-2)$ restrictions on the coefficients of $\mathbf{C}(L)$ and these are given by (27). For the money supply growth equation we adopt the following specification:

$$\Delta m_t = \beta_0 + \beta_1 \Delta m_{t-1} + \beta_2 \Delta m_{t-2} + \beta_3 \Delta G_{t-1} + \beta_4 UN_{t-1} + v_t \tag{29}$$

where ΔG_t is the rate of change of real federal government expenditure, $UN_t = \log[RU_t/(1 - RU_t)]$, and RU_t is the unemployment rate.[31] This specification is a simplified version of the money supply growth equation used in Pesaran (1991) and avoids the complications associated with the use of the Barro (1977) type money growth equation which includes the contemporaneous effect of real federal government expenditure.[32] The above output-money equations can also be viewed as a multisectoral, stochastic trend version of the 'New Classical' model where money shocks can affect output levels in the short run. but not in the long run. Under this interpretation $\widetilde{M}_2$ will also be subject to the following further restrictions:

$$H_{NC} : \quad \gamma_0 + \gamma_1 = 0.$$

These restrictions impose zero persistence for money shocks in all sectors.

The consistent and efficient estimation of the parameters of the multivariate model, (28) and (29), is discussed in detail in Pesaran (1991).[33] In order to obtain efficient estimators and avoid some of the

[30] Notice that in this application Johansen's test procedure does not satisfy the transitivity property of pairwise cointegration. Given the evidence of cointegratton between sectors 4 and 3 and between sectors 3 and 5, we would expect, *a priori*, to find evidence of cointegration between sectors 4 and 5, but we do not.

[31] See appendix C for data sources.

[32] On this, see Pesaran (1982) for further details.

[33] The problem of estimatlng univariate 'surprise' models is discussed in Pagan (1984, 1986) and reviewed in (Pesaran, 1987, ch 7).

difficulties associated with the use of two-step estimators we estimated the parameters of the output and money equations jointly by the FIML method over the period 1955–87.[34] Here we focus on the estimates of the money supply shock coefficients and report these estimates together with their asymptotic standard errors in table 7.[35] In this table we also give Wald statistics for testing the joint hypothesis of zero restrictions on the money supply shocks, $\gamma_{0i} = \gamma_{1i} = 0$, and for testing the new classical hypothesis H_{NC}, i.e., $\gamma_{0i} + \gamma_{1i} = 0$, for $i = 1, 2, \ldots, m$.

Table 7: FIML estimates of the coefficients of the money supply shocks.

	Coefficient estimates[a]		Test statistics[b]	
Sectors	$\widehat{\gamma}_{0i}$	$\widehat{\gamma}_{1i}$	$\gamma_{0i} = \gamma_{1i} = 0$	$\gamma_{0i} + \gamma_{1i} = 0$
1. Agriculture	−1.10	−0.50	4.35	3.97
	(−1.77)	(−0.81)		
2. Mining	−0.16	1.22	6.56[c]	2.62
	(−0.30)	(2.56)		
3. Construction	1.59	−0.06	14.05[c]	7.49[c]
	(3.74)	(−0.14)		
4. Dur. manuf.	1.04	−0.00	1.34	0.81
	(1.14)	(−0.00)	1.34	0.81
5. Nondur. manuf.	1.19	0.26	12.11[c]	9.23[c]
	(3.30)	(0.74)		
6. Transport	1.22	0.52	14.63[c]	13.06[c]
7. Utilities	−0.32	0.89	7.01[c]	1.58
	(−0.87)	(2.61)		
8. Trade	0.97	0.09	9.39[c]	6.24[c]
	(2.99)	(0.27)		
9. Services	0.38	0.12	9.23[c]	7.65[c]
	(2.76)	(0.83)		
10. Government	0.16	−0.01	1.39	0.77
	(1.17)	(−0.10)		

[a]The estimates $\widehat{\gamma}_{01}$ and $\widehat{\gamma}_{11}$ respectively refer to the FIML estimates of the coefficients of the current and the one-period-lagged unanticipated money growth variable, v_t, in sector i. (The figures in parenthesees are asymptotic t-ratios.)

[b]The test statistics are the Wald statistics for tests of the hypotheses $\gamma_{0i} = \gamma_{1i} = 0$ and $\gamma_{0i} + \gamma_{1i} = 0$, respectively.

[c]Statistical significance at the 5% level.

The results clearly show that the effect of money supply shocks on output is not uniform across the sectors. Money shocks have no statistically significant long-term, or even short-term, effects on outputs in the 'agricultural', 'durable manufacturing', and the 'government' sectors. In the case of 'mining' and 'utilities', money shocks have significant short-term effects, but these effects tend to die out in the long run. For the five remaining sectors, however, money shocks have statistically significant effects on output levels both in the short run and in the long run. Overall, the evidence on the new classical hypothesis is mixed. It is upheld for half of the sectors studied and rejected for the rest. There is clearly a need for further empirical analysis of the possible short-run and long-run impact of monetary shocks on sectoral outputs.

In view of the above results we base our estimates of the persistence measures and their decomposition on a restricted version of $\widetilde{M}_2$, obtained in the following manner:

[34] The use of two-step estimators, whereby r_t is estimated first bv the application of the OLS method to (29) and then used as regressors in (18), besides being subject to the familiar 'generated regressor' problem, is further complicated in the present multivariate application due to the contemporaneous correlation across the output dlsturbances, ϵ_{it}.

[35] The computation of the FIML estimators were carried out on GAUSS by iterating on a multivariate generalisation of the double-length regression proposed in Pagan (1986). Also see (Pesaran, 1987, pp. I77–179). The details of the algorithm and the associated computer codes can be obtained from the authors on request.

(i) In the case of sectors 1, 4, and 10, we imposed the zero restrictions $\gamma_{0i} = \gamma_{1i} = 0$.

(ii) In the case of sectors 2 and 7, we imposed the new classical restriction, $\gamma_{0i} + \gamma_{1i} = 0$. As table 7 shows, none of these restrictions can be rejected at the 5% level. We also dropped regressors whose coefficients were less than unity (in absolute value). We refer to this further restricted model as $\widetilde{M}_3$.[36]

Table 8: Decomposition of sectoral and aggregate persistence measures by the type of shocks.[a] FIML estimates 1955–1987.

Sectors	'Money' shocks	'Other' shocks	Total
1. Agriculture	0.00	0.76	0.76
		(0.05)	(0.05)
2. Mining	0.00	1.31	1.25
		(0.25)	(0.23)
3. Construction	0.50	4.93	4.37
	(0.56)	(1.88)	(1.64)
4. Dur. manuf.	0.00	0.69	0.69
		(0.04)	(0.04)
5. Nondur. manuf.	0.55	0.58	0.58
	(0.22)	(0.04)	(0.04)
6. Transport	1.57	0.87	0.98
	(0.29)	(0.04)	(0.05)
7. Utilities	0.00	4.71	4.41
		(2.79)	(2.61)
8. Trade	0.67	0.58	0.60
	(0.15)	(0.05)	(0.05)
9. Services	3.36	2.23	2.38
	(1.57)	(0.99)	(1.03)
10. Government	0.00	2.00	2.00
		(0.24)	(0.24)
Aggregate output	1.83	0.62	0.67
	(0.55)	(0.0763)	(0.0720)

[a]The decomposition of aggregate and sectoral persistence measures are carried out using the formulae (22)–(25) and their counterparts at the sectoral levels. The figures are computed using the FIML estimates of model $\widetilde{M}_3$ defined in the text. The figures in parentheses and asymptotic standard errors, computed according to the formulae given in appendix B.

Using the parameter estimates obtained under the restricted model we com- puted the estimates reported in table 8 for the persistence measures decomposed into 'money' and 'other' shocks.[37] At the aggregate level, the persistence of 'money' shocks, P_m, is estimated to be 1.85 with an (asymptotic) standard error of 0.55. The persistence effects of 'money' shocks on aggregate output are, therefore, statistically significant, but the estimate of P_m is subject to a wide margin of uncertainty. However, it is important to note that despite the statistical significance of the long-term impact of money shocks on output, the contribution of P_m to the total persistence measure is rather small. This is true of all the sectors and can be seen clearly from a comparison of the last two columns of table 8. In the case of aggregate output the difference between the persistence measure due to 'other' shocks and the total persistence measure is only 0.05. The reason for this is primarily due to the fact that the size of money shocks (as measured by σ_v^2) compared to the size of the other shocks (as measured by $\mathbf{w}'\mathbf{\Sigma}\mathbf{w}$) has been very small over the sample period [see relations (22) and (25)].

[36] Model$\widetilde{M}_3$ imposes 32 restrictions on $\widetilde{M}_2$. To ensure that we have not inadvertently imposed an invalid restriction on $\widetilde{M}_2$ we also tested the overall validity of the 32 restrictions by the likelihood ratio procedure. The value of the log-likelihood ratio statistic turned out to be equal to 17.23, which is well below the 95% critlcal value of the chi-squared distribution with 32 degrees of freedom.

[37] The relevant formulae for the decomposition of persistence measures are given at the end of section 3.3. Also see appendix B for the derivation of their asymptotic variances of the persistence measures estimated by the FIML method.

A comparison of the results in tables 8 and 5 also shows that by including money shocks explicitly in the output growth equations it has in fact been possible to obtain a more precisely determined estimate of the aggregate persis- tence measure, as compared to the 'best' estimate obtained on the basis of model M_3, i.e., the model excluding the money shocks. The estimate of the aggregate persistence measure based on model $\widehat{M}_3$ is 0.67 (0.072) as compared to the estimate of 0.83 (0.085) based on model M_3. Figures in brackets are asymptotic standard errors.[38] In conclusion, while it is true that estimating long-run effects from finite data sets is, in general, a hazardous undertaking, the results reported in this paper show that by utilizing information on sectoral outputs or other relevant information on variables other than the past history of the variable under investigation, it is possible to reduce the margin of errors involved in the estimation of the aggregate persistence measure. In this paper we have only analyzed the effect of incorporating money shocks in the multisectoral model. Other possibilities would be to try oil price shocks and shocks to the capital and foreign exchange markets.

Appendix A Proof of Proposition 1

We present a proof by induction. Suppose that the proposition holds for $m = s$. That is,

$$P_y(s) = \sum_{i=1}^{s} \lambda_i(s) P_i, \tag{A.1}$$

where $P_y(s)$ represents the aggregate persistence measure for the s-sector aggregate, $Y_t(s) = \sum_{i=1}^{s} w_i y_{it}$, $\lambda_i(s) = \{w_i^2 \sigma_{ii} / \mathbf{w}_s' \boldsymbol{\Sigma}_s\}^{1/2}$, $i = 1, 2, \ldots, s$, $\mathbf{w}_s' = (w_1, w_2, \ldots, w_s)$, and $\boldsymbol{\Sigma}_s$ is the variance matrix of $(\epsilon_{1t}, \epsilon_{2t}, \ldots, \epsilon_{st})'$. Consider now adding a further sector to the aggregate, $Y_t(s)$. Namely,

$$Y_t(s+1) = Y_t(s) + w_{s+1} y_{t,s+1}. \tag{A.2}$$

Since the cointegration property is transitive it follows from the pairwise cointegration condition that $Y_t(s)$ and $y_{t,s+1}$ are also cointegrated. Moreover, since by assumption $y_{t,s+1}$ and $y_{1t}, y_{2t}, \ldots, y_{st}$ are all pairwise positively correlated, it follows that $Y_t(s)$ and $y_{t,s+1}$ will also be positively correlated. Hence, the result (16) in the text obtained for the case $m = 2$ is also applicable to the right-hand side components of (A.2) and we have

$$P_y(s+1) = \mu_1 P_y(s) + \mu_2 P_{s+1}, \tag{A.3}$$

where the weights in this case are given by

$$\mu_1 = \{\mathbf{w}' \boldsymbol{\Sigma}_s \mathbf{w}_s / \mathbf{w}_{s+1}' \boldsymbol{\Sigma}_{s+1} \mathbf{w}_{s+1}\}^{1/2},$$

$$\mu_2 = \{\mathbf{w}' \boldsymbol{\Sigma}_s \mathbf{w}_s / \mathbf{w}_{s+1}' \boldsymbol{\Sigma}_{s+1} \mathbf{w}_{s+1}\}^{1/2},$$

Substituting from (A.2) in (A.3) and after some algebraic simplifications, we have

$$P_y(s+1) = \sum_{i=1}^{s+1} \lambda_i(s+1) P_i,$$

which establishes that if the proposition holds for $m = s$ it will also hold for $m = s + 1$. But we have already established that the proposition holds for $m = 2$ so it should hold for any m. **Q.E.D.**

Appendix B Derivation of the variance of the persistence measures

This appendix gives a derivation of the variance of persistence measures estimated on the basis of the multisectoral model (28) that contains the money supply shocks v_t. Clearly the results are also applicable to the estimates of the persistence measures based on the VAR model (26) which does not include the money shocks. Here the derivations are given in terms of the aggregate persistence measures, but relevant variance expressions for the sectoral persistence measures can be obtained by replacing $\mathbf{w}$ in the expressions below by $w_i \mathbf{e}_i$, where $\mathbf{e}_i$ is the $m \times 1$ selection vector defined in the text.

[38] Given the asymptotic nature of our results, it may be worthwhile to consider the small properties of the persistence measures using Monte Carlo techniques. This is beyond the scope of the present paper however.

Consider the following general version of (28) and (29):

$$\mathbf{C}(L)\Delta\mathbf{y}_t = \mathbf{a} + \boldsymbol{\gamma}(L)v_t + \boldsymbol{\epsilon}_t, \tag{B.1}$$

$$x_t = \boldsymbol{\beta}'\mathbf{z}_t + v_t, \tag{B.2}$$

for $t = 1, 2, \ldots, T$, where $x_t = \Delta m_t$, $\boldsymbol{\beta}$ is a $k \times 1$ vector of unknown parameters

$$\mathbf{C}(L) = \mathbf{I}_m + \mathbf{C}_1 L + \cdots + \mathbf{C}_p L^p,$$

$$\gamma(L) = \gamma_0 + \gamma_1 L + \cdots + \gamma_q L^q \tag{B.3}$$

and $\mathbf{I}_m$ is an identity matrix of order m. Stacking all the observations using the notations

$$\Delta\mathbf{Y}' = [\Delta\mathbf{y}_1, \ldots, \Delta\mathbf{y}_T], \quad \mathbf{v}' = (v_1, \ldots, v_t),$$

$$\mathbf{E}' = [\boldsymbol{\epsilon}_1, \ldots, \boldsymbol{\epsilon}_T], \quad \mathbf{x}' = (x_1, \ldots x_T).$$

$$\mathbf{Z}' = [\mathbf{z}_1, \ldots, \mathbf{z}_T], \quad \boldsymbol{\tau}' = (1, \ldots, 1), \boldsymbol{\tau}' \text{ is } 1 \times T,$$

the model (B.1) and (B.2) can be written as

$$\mathbf{C}(L)\Delta\mathbf{Y}' = \mathbf{a}\boldsymbol{\tau}' + \gamma(L)\mathbf{v}' + \mathbf{E}', \tag{B.4}$$

$$\mathbf{x} = \mathbf{Z}\boldsymbol{\beta} + \mathbf{v}. \tag{B.5}$$

Using (B.3) and casting the above system in vector forms, (B.4) now becomes

$$\operatorname{vec}(\Delta\mathbf{Y}) = (\mathbf{I}_m \otimes \mathbf{W})\mathbf{a} + \operatorname{vec}(\mathbf{E}), \tag{B.6}$$

where $\mathbf{W} = [\boldsymbol{\tau}, \Delta\mathbf{Y}_0, \mathbf{V}_0]$, $\Delta\mathbf{Y}_0 = [\Delta\mathbf{Y}_{-1}, \ldots, \Delta\mathbf{Y}_{-p}]$, $\mathbf{V}_0 = [\mathbf{v}, \mathbf{v}_{-1}, \ldots, \mathbf{v}_{-q}]$,
and $\mathbf{a} = \operatorname{vec}\left([\mathbf{a}, -\mathbf{C}_1, \ldots, -\mathbf{C}_p, \gamma_0, \ldots, \gamma_q]'\right)$.

The parameters of the model (B.5) and (B.6) which we denote by $\boldsymbol{\theta}$ are the *unrestricted* elements of

$$\{\boldsymbol{\beta}, \mathbf{a}\} = \{\boldsymbol{\beta}, \boldsymbol{\mu}, \mathbf{c}^*, \boldsymbol{\gamma}^*\}. \tag{B.7}$$

If no restrictions are imposed on $\{\boldsymbol{\beta}, \mathbf{a}\}$ (as in the unrestricted VAR model), then the dimension of $\boldsymbol{\theta}$ is $k + m + pm2 + (q+1)m$. In general, $\mathbf{a}$ will be a function of $\boldsymbol{\theta}$.

The various persistence measures discussed in the paper are all scalar functions of $\boldsymbol{\theta}$. We represent this functional relation by $P(\boldsymbol{\theta})$, and assume that $P(\boldsymbol{\theta})$ is evaluated at the Maximum Likelihood (ML) estimators of $\boldsymbol{\theta}$ which we denote by $\widehat{\boldsymbol{\theta}}$. The asymptotic variance of $P(\widehat{\boldsymbol{\theta}})$ is given by

$$\operatorname{Avar}[P(\widehat{\boldsymbol{\theta}})] = (\partial P/\partial\boldsymbol{\theta}')\operatorname{Avar}(\widehat{\boldsymbol{\theta}})(\partial P/\partial\boldsymbol{\theta}). \tag{B.8}$$

To derive $\operatorname{Avar}(\widehat{\boldsymbol{\theta}})$ we first note that the joint log-likelihood function of (B.5) and (B.6) is proportional to

$$L(\boldsymbol{\theta}) = \frac{T}{2}\log\sigma_v^2 - \frac{T}{2}\log|\boldsymbol{\Sigma}| - \frac{1}{2\sigma_v^2}\mathbf{v}'\mathbf{v} - \frac{1}{2}\operatorname{vec}(\mathbf{E})'\left(\boldsymbol{\Sigma}^{-1} \otimes \mathbf{I}_T\right)\operatorname{vec}(\mathbf{E}), \tag{B.9}$$

where $\mathbf{I}_T$ is an identity matrix of order T. Other notations are defined in the text. Using (B.9) and after some algebra, we have[39]

$$\operatorname{Avar}[\widehat{\boldsymbol{\theta}}]^{-1} = \begin{bmatrix} \begin{pmatrix} \frac{\partial\boldsymbol{\beta}}{\partial\boldsymbol{\theta}'} \\ \frac{\partial\mathbf{a}}{\partial\boldsymbol{\theta}'} \end{pmatrix} \end{bmatrix}' \begin{bmatrix} \mathbf{Z}_0'\left(\boldsymbol{\gamma}^*\boldsymbol{\Sigma}\boldsymbol{\gamma}^{*\prime} \otimes \mathbf{I}\right)\mathbf{Z}_0 + \frac{1}{\sigma_v^2}\mathbf{Z}'\mathbf{Z} & -\left(\boldsymbol{\Sigma}^{-1}\boldsymbol{\gamma}^{*\prime} \otimes \mathbf{W}'\right)\mathbf{Z}_0 \\ -\mathbf{Z}_0'\left(\boldsymbol{\gamma}^*\boldsymbol{\Sigma}^{-1} \otimes \mathbf{W}\right) & \left(\boldsymbol{\Sigma}^{-1} \otimes \mathbf{W}'\mathbf{W}\right) \end{bmatrix} \begin{bmatrix} \begin{pmatrix} \frac{\partial\boldsymbol{\beta}}{\partial\boldsymbol{\theta}'} \\ \frac{\partial\mathbf{a}}{\partial\boldsymbol{\theta}'} \end{pmatrix} \end{bmatrix}, \tag{B.10}$$

where $\mathbf{Z}_0' = [\mathbf{Z}', \mathbf{Z}_{-1}{}', \ldots, \mathbf{Z}_{-q}{}']'$.

[39] Also see Pesaran (1991), where similar derivations can be found for a related class of multivariate rational expectations models.

It only remains to derive the first derivatives $\partial P/\partial\boldsymbol{\theta}'$, for the various persistence measures of interest. First consider the persistence measure P_0 defined by (24) in the text. It is relatively easy to show that

$$\begin{aligned}\frac{\partial P_0}{\partial\boldsymbol{\theta}'} &= \frac{1}{P_0}\left[\frac{\mathbf{w}'\mathbf{A}(1)\boldsymbol{\Sigma}}{\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}}\right](\mathbf{w}'\mathbf{A}(1)\otimes\mathbf{A}(1)'\mathbf{S}_p)\left(\frac{\partial\mathbf{c}^*}{\partial\boldsymbol{\theta}'}\right)\\ &-\frac{1}{TP_0}\frac{(\mathbf{w}'\mathbf{A}(1)\otimes\mathbf{w}'\mathbf{A}(1)\mathbf{E}'\mathbf{W})}{\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}}\left(\frac{\partial\mathbf{a}}{\partial\boldsymbol{\theta}'}\right)\\ &+\frac{P_0}{T}\frac{(\mathbf{w}'\times\mathbf{w}'\mathbf{E}'\mathbf{W})}{\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}}\left(\frac{\partial\mathbf{a}}{\partial\boldsymbol{\theta}'}\right),\end{aligned}\tag{B.11}$$

where $\mathbf{S}_p = [\mathbf{I}_m, \mathbf{I}_m, \ldots, \mathbf{I}(m]$ is an $m\times mp$ matrix. Next for P_m defined by (23) in the text, we have[40]

$$\begin{aligned}\frac{\partial P_m}{\partial\boldsymbol{\theta}'} &= \boldsymbol{\gamma}(1)'\frac{(\mathbf{w}'\mathbf{A}(1)\otimes\mathbf{A}(1)'\mathbf{S}_p)}{\mathbf{w}'\mathbf{d}(0)}\left(\frac{\partial\mathbf{c}^*}{\partial\boldsymbol{\theta}'}\right)\\ &+\frac{(\mathbf{w}'\mathbf{A}(1)\otimes\mathbf{S}_q)}{\mathbf{w}'\mathbf{d}(0)}\left(\frac{\partial\boldsymbol{\gamma}^*}{\partial\boldsymbol{\theta}'}\right)-\frac{P_m}{\mathbf{w}'\mathbf{d}(0)}\mathbf{w}'\left(\frac{\partial\boldsymbol{\gamma}_0}{\partial\boldsymbol{\theta}'}\right),\end{aligned}\tag{B.12}$$

where $S_q = (1, 1, \ldots, 1)$ is an $1\times(q+1)$ vector.

Finally, for the measure P_y, defined by (22) in the text, we have

$$\frac{\partial P_y}{\partial\boldsymbol{\theta}'} = \lambda\frac{P_m}{P_y}\frac{\partial P_m}{\partial\boldsymbol{\theta}'}+\frac{P_m^2}{2P_y}\frac{\partial\lambda}{\partial\boldsymbol{\theta}'}+\frac{(1-\lambda)P_0}{P_y}\frac{\partial P_0}{\partial\boldsymbol{\theta}'}-\frac{P_0^2}{2P_y}\frac{\partial\lambda}{\partial\boldsymbol{\theta}'}.\tag{B.13}$$

λ is defined by (25) and has the following derivatives with respect to θ:

$$\begin{aligned}\frac{\partial\lambda}{\partial\boldsymbol{\theta}'} &= \frac{2(\lambda-\lambda^2)}{\mathbf{w}'\mathbf{d}(0)}\mathbf{w}'\left(\frac{\partial\boldsymbol{\gamma}_0}{\partial\boldsymbol{\theta}'}\right)\\ &+\left(\frac{2\lambda(\mathbf{w}'\otimes\mathbf{w}'\mathbf{E}'\mathbf{W})}{T[\sigma_v^2(\mathbf{w}'\mathbf{d}(0))^2+\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}]}\right)\left(\frac{\partial\mathbf{a}}{\partial\boldsymbol{\theta}'}\right)\frac{2(\lambda-\lambda^2)}{\sigma_v^2}\left(\frac{\mathbf{v}'\mathbf{Z}}{T}\right)\left(\frac{\partial\boldsymbol{\beta}}{\partial\boldsymbol{\theta}'}\right).\end{aligned}\tag{B.14}$$

Consistent estimates of (B.9) may now be computed for any of the persistence measures of interest using (B.11) and the relevant expressions for the derivatives of $\partial P/\partial\boldsymbol{\theta}'$ given above, all evaluated at the ML estimators.

Appendix C Data

Industrial output data series for the period 1947–87 were taken from the U.S. Department of Commerce publications *The National Income and Product Accounts of the United States, 1929–1982*, and the July 1986 and July 1988 issues of the *Survey of Current Business*. Figures were taken from table 6.2, which provides annual data on Gross National Product by Industry in constant prices (billions of 1982 dollars). The ten-sector classification used in the empirical work was obtained from the more disaggregated figures provided in these publications as described in table 9 below.

For the sample period up to 1985, the data used in the estimation of the money supply growth equation are the same as those employed by Rush and Waldo (1988) and Pesaran (1988b). Data for RU_t (the annual average unemployment rate in the total labor force, including military personnel) and for FED, (real Federal Government expenditure) were extended to 1987 using the Economic Report of the President (1989 edition), while M_t (annual average M1) was extended using the Federal Reserve Bulletin (various issues).

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[40] Notice that in (23) $\mathbf{d}(1) = \mathbf{A}(1)\gamma(1) = \mathbf{C}^{-1}\gamma(1)$ and $\mathbf{d}(0) = \mathbf{d}_0 = \gamma_0$.

Table 9

Survey of Current Business industry titles	Abbreviated industry titles	Line(s)
1. Agriculture, Forestry and Fisheries	Agriculture	4
2. Mining	Mining	$8+9+10+11$
3. Construction	Construction	12
4. Durable Manufacturing	Dur. manuf.	$15-25$
5. Nondurable Manufacturing	Nondur. manuf.	$27-36$
6. Transportation and Communications	Transport	$38+46$
7. Electric, Gas and Sanitary Services	Utilities	49
8. Wholesale and Retail Trade	Trade	$50+51$
9. Finance, Insurance, Real Estate and Services	Services	$52+60$
10. Government and Government Enterprises	Government	74

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