

Building and Solving Macroeconomic Models using WinSolve: Stochastic simulation and Control*

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1 Introduction

This paper illustrates stochastic simulation and optimal control of a model using *WinSolve* (Pierse 2000). The tutorial and exercises make use of a small model of the Philippines developed by the *Asian Development Bank* (*ADB*) and described in Cagas *et al.* (2006). Section 2 presents a general discussion of the uses of stochastic simulation and the facilities available in *WinSolve* while Section 3 is a tutorial, illustrating the application of some of the techniques to the *ADB* model of the Philippines. Similarly, Section 4 presents a general discussion of optimal control while Section 5 illustrates the techniques on the *ADB* model.

2 Stochastic Simulation

Economic models are intrinsically stochastic (random). Empirical models with econometrically estimated parameter values will have uncertainty associated with the parameter estimates, in addition to the econometric disturbances assumed in the original model. Most modern theoretical models, such as *real business cycle* (*RBC*) models and *dynamic stochastic general equilibrium* (*DSGE*) models are also stochastic by construction because they involve

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agents forming *rational expectations* of future variables and predetermined variables being driven by exogenous random shocks.

Despite the intrinsic stochastic nature of economic models, most models are solved deterministically, with all the shocks set to their expected value of zero. When models involve agents who are forming rational expectations of the future, these are generally replaced by *model consistent expectations* in which the expectational shocks are dropped, which is equivalent to assuming that the agents have perfect foresight.

2.1 Certainty equivalence

Certainty equivalence is a property of a stochastic model which means that its deterministic solution is an unbiased estimator of the expected values of its model variables. When the conditions for certainty equivalence hold, then a deterministic solution is sufficient, provided that only the expected value and not the higher order moments are of interest.

Certainty equivalence holds in the case where a model is completely linear in both variables and shocks and the stochastic shocks have zero mean. (It also holds in a few other narrowly defined situations). However, in practice, most economic models are non-linear and do not satisfy the conditions for certainty equivalence. In the *DGSE* literature, it is common to construct a linear approximation to a model, generally around the deterministic steady-state, and then solve the approximate model deterministically, making use of the property of certainty equivalence. This approximation will be adequate for the expected values of the variables in a neighbourhood of the approximation point but, if the model is very non-linear, then this neighbourhood may be very small and still, higher order moments such as variance may be of interest.

2.2 What is stochastic simulation?

Stochastic simulation of a model is a technique whereby a model is solved repeatedly, each solution or *replication* adding pseudo-random shocks drawn from some probability distribution. The resulting set of replications can then be used to compute estimates of the empirical distribution of the solution values for the endogenous variables including moments of the distribution such as the expected value, variance etc. As the number of replications increases, these approximations will converge to the true (generally unknown) population values of the moments.

2.3 When to use stochastic simulation

When the conditions of certainty equivalence hold, then stochastic simulation is not necessary to find the expected value of the model variables. Even then, if higher order moments are of interest, then stochastic simulation provides a simple and consistent way to estimate them. In all other situations, then stochastic simulation should be employed.

2.4 Ways of generating shocks

In order to do stochastic simulation, some way is needed to determine how the shocks should be generated. In a theoretical model, some assumption may be made about which equations have shocks and what the form of the distribution should be (generally univariate normal with some specified variance). In an empirical model, the econometric estimation residuals provide a good basis for generating shocks, either by the Cholesky method or by bootstrapping.

2.4.1 Cholesky method

The Cholesky method uses the empirical data moment

$$\hat{\Sigma} = \frac{\mathbf{E}'\mathbf{E}}{T - k}$$

as a consistent estimator of the covariance matrix of shocks, where \mathbf{E} is the $T \times n$ matrix of single equation residuals from the n model equations with T observations where k is an optional degrees-of-freedom correction. The shocks are assumed to be normally distributed with covariance matrix $\hat{\Sigma}$. This estimator will only be positive-definite when $T > n$ and if this condition is not satisfied, then the method is not feasible. (Note that since some equations such as identities will have zero residuals, a slight relaxation of the condition of positive-definiteness is required to allow for some zero rows and columns in $\hat{\Sigma}$).

In the case where $T < n$, the McCarthy method is available. This can be considered as an approximation to the Cholesky method that can be applied even when there are insufficient time observations to apply the Cholesky method directly.

2.4.2 Bootstrap method

In this method, shocks are drawn from repeated re-sampling of rows of the matrix of single equation residuals of the model equations \mathbf{E} . The distribu-

tion of these shocks is a consistent estimate of the empirical distribution of the single equation residuals. In particular, there is no guarantee that the resulting shock distribution will be symmetric or even mean zero, although the method of course does rely on an implicit assumption that the distribution is time-invariant. .

2.4.3 User Covariance

This method is useful for theoretical models where the shock covariance matrix can be assumed *a priori*. Shocks are assumed to be multivariate normal with zero mean. The user specifies the non-zero elements of (one triangle of) the covariance matrix. A check is made to ensure that the specified covariance matrix is (semi-)positive definite.

2.4.4 General distributions

In principle it is possible to generate shocks using any user-specified distribution, allowing the shocks to be non-symmetric and to have any other desired properties including non-invariance over time. In practice it is quite difficult to generate sets of correlated pseudo-random shocks except from the normal distribution, but independent shocks from arbitrary distributions are certainly possible. *WinSolve* has a facility where the user can specify a file of shocks to be used in stochastic simulation. Each row of the file needs to define the shocks for all model variables in a time period/replication. Because of the possibility that the model may fail to solve in a certain time period for a particular drawing of random shocks (in which case the replication will be abandoned and a new drawing will be taken), the number of rows in the file should exceed $T * R$ where R is the number of replications. In view of this problem of non-convergence, it is impossible to guarantee that the shocks will be applied in the desired time periods, which is a potential problem for non-time-invariant distributions.

2.4.5 Antithetic shocks

This is not a method of generating shocks as such but is a way of reducing the simulation variance involved in stochastic simulation and will improve the resulting estimates provided that the assumption that the shock distribution is symmetric is appropriate. The idea is that, for every vector of random shocks drawn in replication r , ε_t^r , a corresponding antithetic vector of shocks $-\varepsilon_t^r$ is used for replication $r + 1$. Thus shocks are generated in pairs that cancel out exactly and the shock distribution is forced to be exactly symmet-

ric. In a linear model, the average of the replications (the expected value of model variables) will be identical to the deterministic model solution.

2.4.6 Unanticipated shocks

In models including forward rational expectations, there is an issue in stochastic simulation of whether or not agents anticipate future shocks. Certainly, expectational shocks should be unobservable by agents (unless they are assumed to have perfect foresight). In *WinSolve*, it is possible to specify either that all future shocks are anticipated or no future shocks are anticipated. In the latter case, the computational problem to be solved is considerably increased and the cost of stochastic simulation increases *pro rata*.

3 Stochastic Simulation with the Philippines model

In this section we use stochastic simulation to explore the properties of the small model of the Philippines, built at the *Asian Development Bank* and described in *Cagas et al.* (2006). Since this is an empirically estimated model with historical data available from 1997:4 to 2004:4, an appropriate shock generation method is the bootstrap method. This method has the advantage of not assuming a normal distribution for shocks. However, it will also not guarantee a symmetric distribution unless the antithetic shock option is also checked.

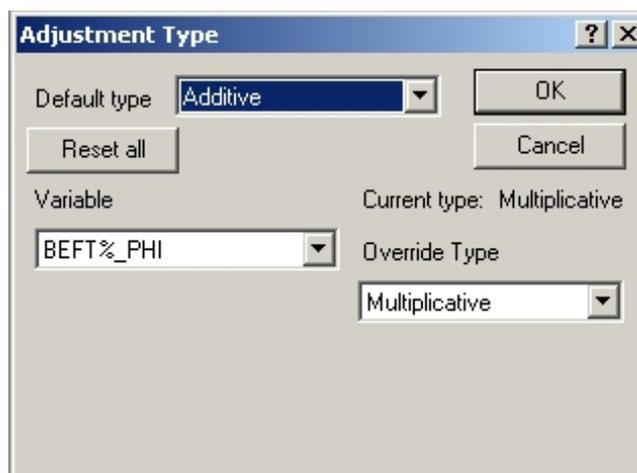


Figure 1: Adjustment type dialog box

The model to be opened is the file *phi model-Nov_2005.txt* and the data file is the base forecast simulation data base saved in the previous tutorial. (If this is no longer available, then read in the forecast data base, *phi_data_Nov2005.xls*, do a dynamic simulation to recreate the base forecast and set the run you have just done as the simulation base using the option *Set last solution as base* on the *Solve* menu).

Before starting a stochastic simulation with the *ADB* model it is important to ensure that all model adjustments are set as additive. This is because, in stochastic simulation, some large shocks may be generated (with low probability), which if applied multiplicatively could cause the model to fail to converge. All the adjustments can be set to be additive in the *Change adjustment type* option on the *Assumptions* menu. In the dialog box (Figure 1) select *Default type* as *Additive* and click on the *Reset all* button. Then click *OK* to accept the changes and exit.

3.1 The stochastic simulation dialog box

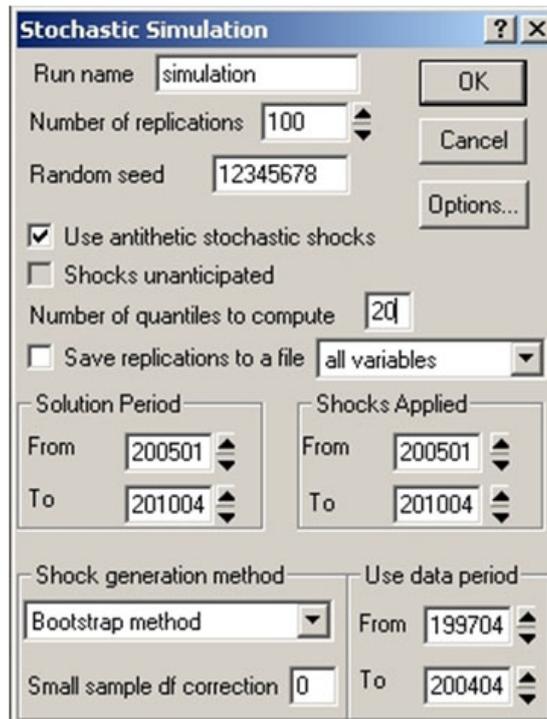


Figure 2: Stochastic simulation dialog box

To start stochastic simulation, click on the *Stochastic simulation* icon 

or select the *Stochastic simulation* option from the *Solve* menu. This opens the stochastic simulation dialog box (Figure 2). This dialog box is quite complicated and is worthy of some detailed explanation. The key selection is the *shock generation method*, which here should be specified as *Bootstrap method*. For the bootstrap method (and other empirical shock generation methods) it is necessary to specify the historical data period to use for generating the shocks, usually corresponding to the econometric estimation period. Here it is set to the period 1997:4 to 2004:4. The solution period for the stochastic simulation is set to the forecast period 2005:1 to 2010:4.

It is possible to restrict the period over which stochastic shocks are applied. This is particularly important for models with rational expectations where the terminal condition is a long run steady state condition. If shocks are applied right up to the terminal date, it is difficult for the model to satisfy the terminal condition and convergence will be a problem. In the case of the Philippine model under consideration, this is not an issue and so the period to apply shocks can be set to the solution period 2005:1 to 2010:4.

The number of replications has been kept at the default value of 100. This is quite a small number of replications and, in serious simulations, should probably be increased for higher precision at the cost of greater execution time. The *use antithetic variables* box has been checked. This forces the distribution of generated shocks to be (exactly) symmetric. However, in a non-linear model, it does not guarantee that the distribution of the model variables will be symmetric.

The random number seed initialises the random number generator used for generating pseudo-random shocks. Two stochastic simulations, over the same period and using the same random number seed, will generate the same results. This is useful when a simulation needs to be repeated at a later date. Changing the seed will result in a new set of random drawings. It is useful to make a note of the random number seed used in important simulations.

The stochastic simulation box has the option to specify that simulation quantiles are to be computed. These allow a complete probability distribution to be estimated (and graphed in a fan chart). This option is not set by default since it can be expensive, because it necessitates that all replications are written to a temporary file and then read back to compute the necessary quantiles. The input box specifies the number of quantiles to be computed. The value of 20 allows quantiles in 5% intervals from 5% to 95%. The *save replications to a file* option may be used in conjunction with the quantiles option to save replications permanently for use outside *WinSolve*. Note that if only a single variable is required, it is considerably cheaper to specify the variable name than accept the default of all variables.

Once all the desired options have been set in the dialog box, click the

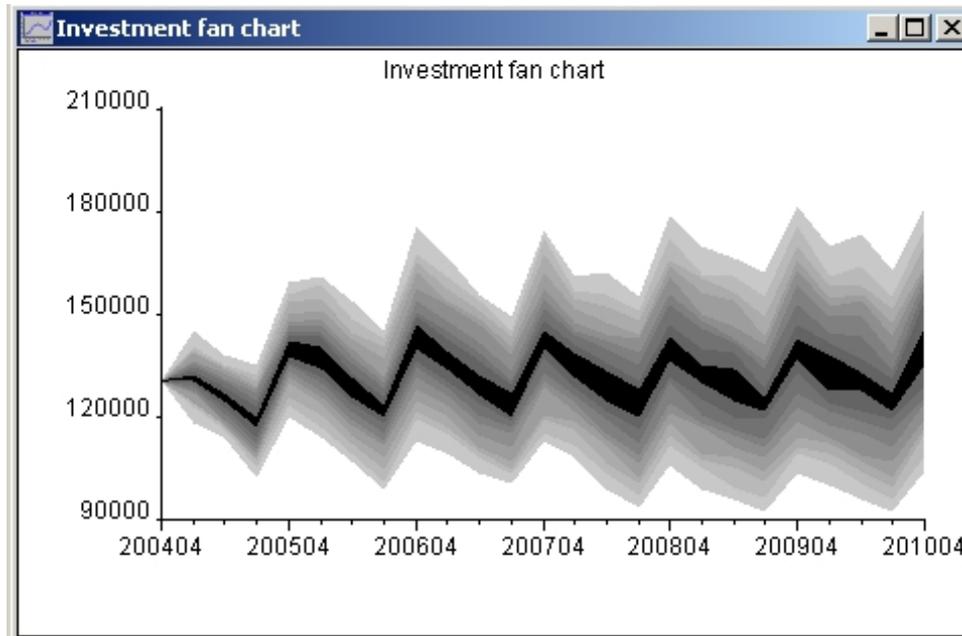


Figure 3: Fan chart for real investment

OK button to exit which opens the status box and starts the simulation. *WinSolve* will allow a maximum number of failed replications before giving up on a stochastic simulation. By default this number is set to 10 but it can be increased if necessary using the *maximum errors* control in the *solution options* dialog box. This dialog is accessible from the stochastic simulation dialog box through the *Options* button.

When the stochastic simulation has finished, the results are available to view in the usual way through the *New table/graph* box on the *Results* menu. After a stochastic simulation, various additional statistics are available to view: the simulation mean, standard deviation, skewness, mode and the $\pm 5\%$ simulation bands. In addition to these, if quantiles were requested, these are also available (the median of course is the 50% quantile) as well as two fan chart options. The *quantile fanchart* is a standard fanchart of quantiles, centred on the median of the distribution. The *modal fanchart* is an alternative fanchart, centred on the estimated mode of the distribution (the sort of fanchart reported by the Bank of England). Figure 3 presents a quantile fanchart for real investment. The bands represent 5% quantile intervals with the darkest central band showing the 45% to 55% interval including the median and the outermost bands the 5% and 95% bands. Notice that although the bands tend to fan out from the start of the simulation in

2005:1, they eventually tend to stabilise. This is as expected since the bands should asymptotically approach the unconditional error bands.

4 Optimal Control

In optimal control, a policy-maker (for example the government) aims to achieve a set of objectives, defined by the maximisation of a welfare function (or the minimisation of a loss function), subject to the equations that define the economic model. The welfare or loss function depends on certain endogenous variables in the economic model that are known as the *targets*. The policy-maker is assumed to be able to set certain exogenous variables (the *controls*) in order to achieve its objectives.

There may be more than one policy-maker (for example two countries or government and consumers), each trying to achieve its own objectives. In this case, the optimisers play a game with each other and may choose either to co-operate or to act independently. When the optimisers co-operate, a joint welfare or loss function is negotiated that reflects the negotiating power of the agents. When the agents act independently, there may be a dominant agent that takes the lead (Stackleberg leader) or the agents may play a Nash game, each taking the moves of all other agents as given.

Apart from these formal considerations, optimal control can also be thought of as an interesting way to explore the properties of a model. It can answer hypothetical questions such as: suppose the government wanted to achieve a particular set of targets, what would it need to do in setting the variables under its control?

4.1 The Optimal Regulator problem

The *optimal regulator* problem solves

$$\min_{\mathbf{u}_t} \sum_{t=1}^T \beta^t (\mathbf{y}_t - \mathbf{y}_t^*)' \mathbf{W}_t (\mathbf{y}_t - \mathbf{y}_t^*) \quad (4.1)$$

subject to the (non-linear) model equations

$$\mathbf{f}(\mathbf{y}, \mathbf{x}) = \mathbf{0}$$

where \mathbf{y}_t is a vector of *endogenous* model variables and \mathbf{x}_t is a vector of *exogenous* variables, both at time t , \mathbf{y}_t^* is a vector of target values for \mathbf{y}_t , \mathbf{W}_t is a matrix of weights, β is a (scalar) time discount factor and the minimisation is with respect to a set of controls \mathbf{u}_t that is a subset of \mathbf{x}_t . When the model is

linear and \mathbf{W}_t is time invariant, this is known as the *optimal linear regulator* problem and the solution is a linear rule for the control variables.

In the loss function (4.1), there is nothing to prevent the control variables \mathbf{u}_t from fluctuating wildly from period to period. Clearly, this may be considered an undesirable property, in which case it is possible to incorporate an extra term into the loss function to penalise changes in the controls as in

$$\min_{\mathbf{u}_t} \sum_{t=1}^T \beta^t [(\mathbf{y}_t - \mathbf{y}_t^*)' \mathbf{W}_t (\mathbf{y}_t - \mathbf{y}_t^*) + (\mathbf{u}_t - \mathbf{u}_{t-1})' \mathbf{P}_t (\mathbf{u}_t - \mathbf{u}_{t-1})]. \quad (4.2)$$

The relative scale of the weighting matrices \mathbf{W}_t and \mathbf{P}_t in (4.2) determines the importance of the secondary objective of desiring smooth movements in the control variables compared to the primary objective of achieving its targets.

In many cases, the optimiser will not want to change the values of the control variables in every period, but only at discrete intervals. For example, the Monetary Policy Committee of a Central Bank, targetting inflation through control of interest rates, will typically only meet once a month to discuss whether rates need to be changed. Thus the interest rate control variable will never change more frequently than once a month, and will remain fixed in all intervening periods. This discrete response can be incorporated into the control problem. In the extreme case, when the interval between control adjustments is equal to or greater than T , the control is set once and for all over all time periods and becomes a scalar value rather than a time path. As will be discussed below, this can be used as a way to estimate *optimal simple rules* where the parameters of the rule are treated as scalar control variables.

4.2 The Linear Quadratic Control Problem

The *linear quadratic* (*LQ*) control problem solves

$$\min_{\mathbf{u}_t} \sum_{t=0}^{\infty} \beta^t \frac{1}{2} (\mathbf{y}_t' \mathbf{Q} \mathbf{y}_t + 2\mathbf{y}_t' \mathbf{S} \mathbf{u}_t + \mathbf{u}_t' \mathbf{R} \mathbf{u}_t) \quad (4.3)$$

subject to the *linear* model equations

$$\mathbf{A} \mathbf{y}_t + \mathbf{B} \mathbf{y}_{t-1} + \mathbf{C} \mathbf{x}_t = \mathbf{0}$$

where \mathbf{y}_t is a vector of *endogenous* model variables and \mathbf{x}_t is a vector of *exogenous* variables, both at time t , \mathbf{Q} , \mathbf{S} and \mathbf{R} are matrices of weights, β is a (scalar) time discount factor and the minimisation is with respect to a

set of controls \mathbf{u}_t that is a subset of \mathbf{x}_t . The solution to this problem is a linear control rule

$$\mathbf{u}_t = \mathbf{F}\mathbf{y}_t.$$

Note that the loss function (4.3) includes a quadratic term $\mathbf{u}_t' \mathbf{R} \mathbf{u}_t$ in the controls, \mathbf{u}_t . This makes it possible to incorporate a penalty on changes in the controls if some degree of smoothness is desired.

4.3 Time consistent control

In models in which agents have rational expectations, there is an issue that the optimal control solution is not always *time consistent*. The forward-looking agents in the economy will be aware of the optimal policy being pursued by the optimiser (the government) from the first period, and will make their own plans accordingly. However, in subsequent periods, there will often be an incentive for the government to deviate from its original policy, since what it did in previous periods is no longer of interest and, from the point of view of the present period, it can often do better than the original announced optimal policy. The problem is that, if the government does choose to renege on its original announced policy, then rational agents will see this and the government policy will lack all credibility.

There are two possible solutions to this problem. It may be possible for the government to be made to *commit* to its original policy, for example by entering into a legal contract whereby it will be penalised for any deviation from its original policy. However, a commitment made by the current government will not necessarily be binding on future governments. The alternative solution is to design a policy from which there will never be an incentive to renege. Such a policy is known as a *time consistent* policy. The time consistent policy will be credible but will not be as good as the optimal policy under commitment.

In the linear quadratic control problem, several methods have been proposed for finding time consistent control solutions. Some, such as the dynamic programming method of Oudiz and Sachs (1985), give closed-loop solutions in the form of a decision rule for the control variables and have the desirable property of also being *sub-game perfect*. Other solutions only give open-loop paths for the control variables and are not sub-game perfect. In non-linear control problems, time consistent solutions are less easy to find and only open-loop solutions will be available.

Time consistent control for the linear quadratic control problem will be available in the next release (version 4) of *WinSolve*.

4.4 Optimal simple rules

Sometimes, rather than pursuing a full optimal control policy, the policy-maker is assumed instead to set the instruments under its control by following a simple rule, such as the Taylor rule, Taylor (1993),

$$r_t = \alpha + \beta(\pi_t - \pi^*) + \gamma(y_t - y_t^*) \quad (4.4)$$

where r_t is the interest rate, π_t is inflation with target rate π^* and y_t is output with target (potential output) y_t^* . The Taylor rule has three parameters α , β , and γ and, in Taylor's original formulation (in which the inflation target was assumed to be 2%), these were set to the values 4%, 1.5 and 0.5 respectively. These values, however are just arbitrary, and may not reflect the true weights in the policy-maker's loss function.

Instead of just imposing parameter values, it is possible to use optimal control to find *optimal* values of the parameters of a simple rule like (4.4) that minimise the policy-maker's loss function (4.1). In this control problem, the control variables are simply the scalar parameters α , β , and γ of the simple rule, so these control variables are assumed to be set once in the first solution period and thereafter to remain constant.

5 Optimal Control with the Philippine model

In this exercise, optimal control will be used to revisit the fiscal problem considered in the previous tutorial to explore how the Philippine government could tackle its budget deficit by reducing government expenditure or increasing tax collection.

It will be assumed that the objective of the government is to achieve a particular target path for the debt to *GDP* ratio of the previous year (*TDEBTr_PHI*) and that, in order to achieve this objective, it is able to effect cuts in government non-interest payment expenditure *GPAY_PHI* and/or increases in government tax collection *GTAX_PHI*. Since these variables are endogenous in the model, new alternative equations will need to be specified, based on the original model equations but including the additional new exogenous variables *GPAYEX* and *GTAXEX* which will be the control variables of the optimal control problem. When these control variables are zero, the equation will revert to the standard model equation but when non-zero, *GPAY_PHI* or *GTAX_PHI* will be multiplied by a factor determined by the control variable. The original model equations for *GPAY_PHI* and *GTAX_PHI* are given in Figure (4) and the new equations in Figure (5). Description lines (starting with the code '*C') have been included before each

```

*M d4log(GPAY_PHI) = 0.0825225 + 0.616565 * d4log(GREV_PHI)
  + 0.0227026 * (UEMP%_PHI(-3) - UEMP%_PHI(-5)) - 0.908833 *
  ( log(GPAY_PHI(-4)) - log(GREV_PHI(-4))
  - 0.04 * UEMP%_PHI(-5) + 0.24 * (DDEBT_PHI(-8)
  + FDEBT_PHI(-8))/GNP_PHI(-8) );

*M dlog(GTAX_PHI) = -0.271031*(log(GTAX_PHI(-1)) - log(GTAX_PHI(-3)) )
  -0.210960*seas(1) + 0.175272*seas(2) + 0.644831*dlog(GNP_PHI)
  + 0.833144*dlog(GNP_PHI(-1)) + 0.447027*dlog(GNP_PHI(-2))
  - 0.576901*( log(GTAX_PHI(-1)) - log(GNP_PHI(-1))
  + 0.095*TAX%_PHI(-1) +5.17);

```

Figure 4: Original equations for $GPAY_PHI$ and $GTAX_PHI$

new equation so that, when these alternative equations are switched in, they can be easily identified.

Note that, because of the logarithmic nature of the original equations, the level of $GPAY_PHI$ and $GTAX_PHI$ in the new equations will actually be multiplied by the *anti-logarithm* of the control variable $GPAYEX$ and $GTAXEX$ respectively. Of course, it would have been possible to specify the new equations using the logarithm of the control variable instead of the level. However, this would not have been a good idea since, in the search for a solution to the the optimal control problem, it is quite possible that the control variables may take on negative values. In this case, had the equations been specified using the logarithms of the control variables, the solution would have broken down. By specifying the equations using the control variable in levels, this problem is avoided since the anti-logarithm of course is guaranteed to be positive.

However, since it is the anti-logarithm in which we are really interested, it is a good idea to include extra equations explicitly defining the anti-logarithm of the controls such as

$$\begin{aligned}
 EXPGPAYEX &= \exp(GPAYEX); \\
 EXPGTAXEX &= \exp(GTAXEX);
 \end{aligned}$$

so that these variables will be available to view, once the solution has been found.

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*C control equation
*M d4log(GPAY_PHI) = GPAYEX + 0.0825225 + 0.616565 * d4log(GREV_PHI)
    + 0.0227026 * (UEMP%_PHI(-3) - UEMP%_PHI(-5)) - 0.908833 *
    ( log(GPAY_PHI(-4)) - log(GREV_PHI(-4))
    - 0.04 * UEMP%_PHI(-5) + 0.24 * (DDEBT_PHI(-8)
    + FDEBT_PHI(-8))/GNP_PHI(-8) );

*C control equation
*M dlog(GTAX_PHI) = GTAXEX -0.271031*(log(GTAX_PHI(-1))- log(GTAX_PHI(-3)) )
    -0.210960*seas(1) + 0.175272*seas(2)+ 0.644831*dlog(GNP_PHI)
    + 0.833144*dlog(GNP_PHI(-1)) + 0.447027*dlog(GNP_PHI(-2))
    - 0.576901*( log(GTAX_PHI(-1)) - log(GNP_PHI(-1))
    - 0.095*TAX%_PHI(-1) +5.17);

```

Figure 5: New control equations for *GPAY_PHI* and *GTAX_PHI*



Figure 6: The model object

5.1 Defining the problem

As in the previous exercise in Section 3, the model to be opened is the file *phi model-Nov_2005.txt* and the data file is the base forecast simulation data base saved in the previous tutorial. (As before, if this is no longer available, then read in the forecast data base, *phi_data_Nov2005.xls*, do a dynamic simulation to recreate the base forecast and set the run you have just done as the simulation base using the option *Set last solution as base* on the *Solve* menu).

The new equations can be added to the text file by double clicking on the icon in the model object window (Figure 6) or selecting *Edit model* from the *File* menu, which opens the text editor. Add the new equations at the end of the file (so that they will not be the default equations) and then save the file. *WinSolve* will compile the new version of the model.

The new equations can now be switched in through the *switch alternate equations* option on the *Assumptions* menu as shown in Figure (7). Select



Figure 7: Alternative equations box

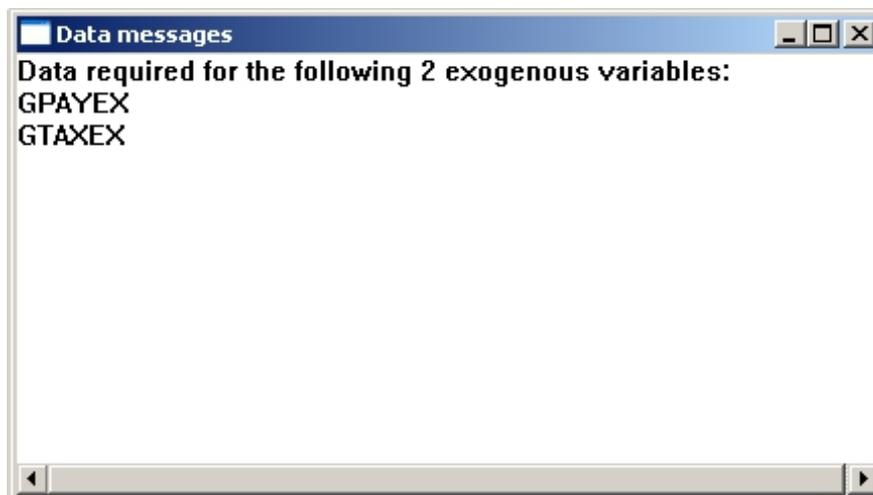


Figure 8: Data required warning message

each of the two variables $GPAY_PHI$ and $GTAX_PHI$ and choose the alternative equation labelled ‘*control equation*’. Then click *Done* to exit. The warning message in Figure (8) will appear. This is because no data values have yet been defined for the new exogenous variables $GPAYEX$ and $GTAXEX$. (Note that no data is required for $EXPGPAYEX$ and $EXPGTAXEX$ because these are working variables). The data on these variables can simply be set to zero. Use the *Edit data/adjustments* option on the *Data* menu or the icon  to edit the data and project zero values for these variables.

Before setting up the optimal control problem to be solved, it is necessary to solve the model once dynamically and set this solution as a new simulation base. Click on the Solve model icon  or select the *Solve model* option from the *Solve* menu. Select the solution period as 200501 to 201004 and make

sure that the *data has precedence over equations* box is unchecked. Select the *dynamic model solution* option and click *OK* to start solution. When the solution has finished, select the option *Set last solution as base* from the *Solve* menu.

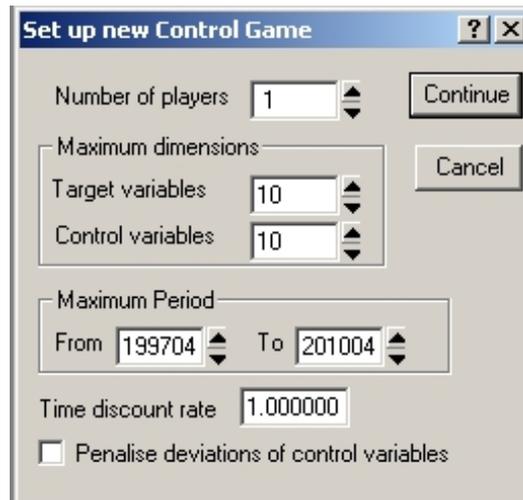


Figure 9: Optimal control setup dialog

The optimal control problem can now be defined. Select the *set up new control game* option on the *Control* menu. The dialog in Figure 9 is displayed. This defines maximum dimensions for the size of the problem and allows the rate of time discount β to be chosen. It also allows us to choose to penalise changes to the control variables as in the loss function (4.2). For the present though, we will leave the default settings unchanged.

Click on the *Continue* button to bring up the next dialog, Figure 10, which allows us to specify the target and control variables for the optimal control problem. Select the variable *TDEBTr_PHI* and add it to the list of target variables. Select the variable *GPAYEX* and add it to the list of control variables. Then click on the *Continue* button.

Finally, target values (and weights) need to be defined for all the target variables in the problem. In this case, since there is only one target variable (*TDEBTr_PHI*), the weights can be left at their default values of unity. The target value for the debt to *GDP* ratio, *TDEBTr_PHI* is assumed to be 0.75. Choose the *project level/growth rate* option and set a constant value of 0.75 in the projection dialog box (Figure 11). Click *OK* to return to the edit dialog and then *Update* to accept the changes. Finally, click *Close* to finish.

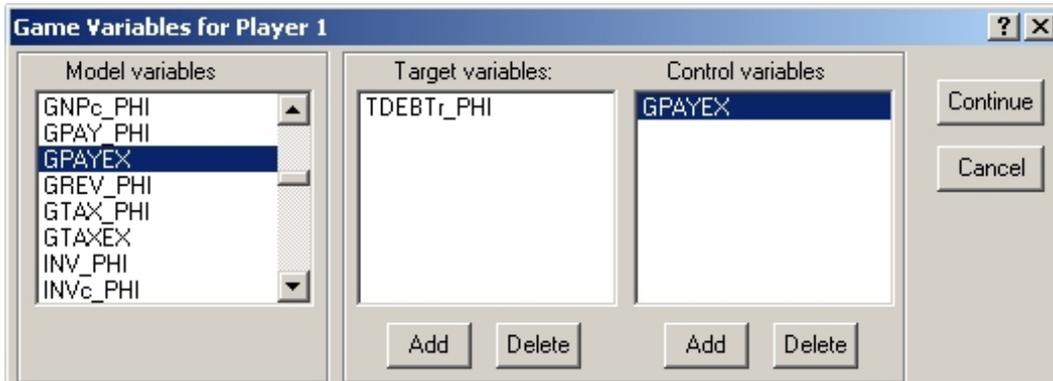


Figure 10: Defining targets and controls

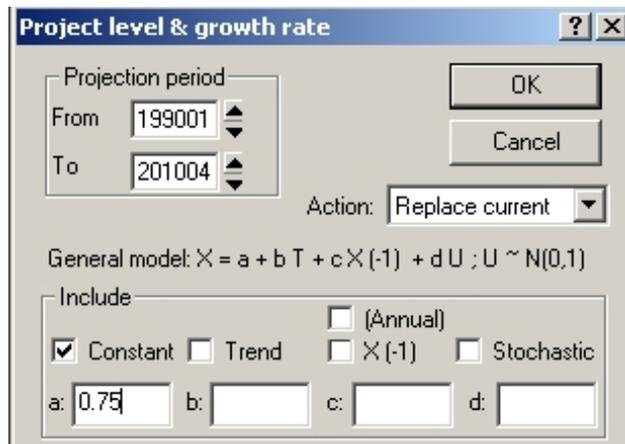


Figure 11: Control target projection box

The optimal control information box will appear (Figure 12) to summarise the properties of the control game that has been defined. Click *OK* to exit. At this stage, the control game can be saved to file from the *Control* menu.

5.2 Solving the problem

The optimal control problem has now been defined and so can now be solved. Select the *Optimal control* option from the *Control* menu or click on the optimal control icon, , which should now be ungreyed. The optimal control dialog box Figure 13 will be displayed. By default, *WinSolve* uses the



Figure 12: Control information box

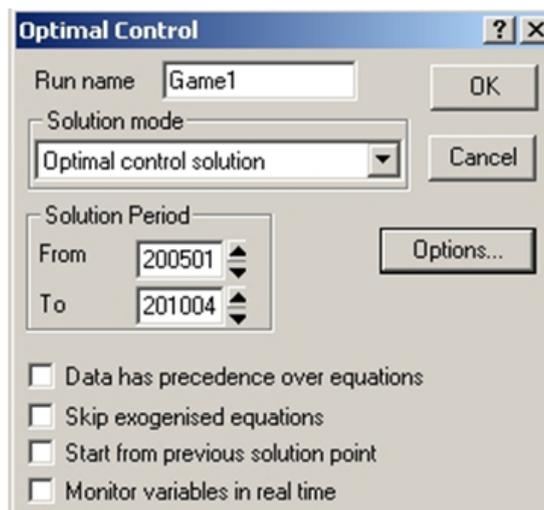


Figure 13: Optimal control dialog box

Powell conjugate directions algorithm for solving optimal control problems. Although this algorithm is very robust, it also tends to be very slow. It is generally better to switch to the *BFGS* algorithm, which is normally considerably faster. Click on the *Options* box to bring up the optimal control options dialog box (Figure 14) and select the *BFGS* algorithm from the list. Note that the options dialog box also has an option labelled *Time aggregation*. This controls the frequency with which control variables are updated. The default is 1, so that controls are updated every period. By increasing this value to a value m , it is possible to specify that control variables are only updated every m periods. In the limit, when the value is set to be equal to or greater than the number of solution periods, the control variables will be set once and for all, and so become scalar controls.

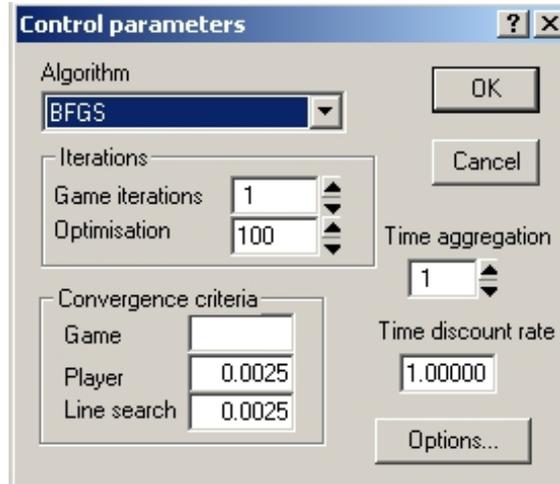


Figure 14: Optimal control options dialog

Click *OK* to return to the previous dialog box and select the solution period to be 200501 to 201004. Finally, click *OK* to start the control solution.

The results of the simulation are illustrated in Figures 15 and 16. The first shows optimal values of the target variable $TDEBTr_PHI$. Note that because this variable is the annual debt to GDP ratio for the previous year, the control variable is unable to affect it until the start of 2006. After this, the target ratio of 0.75 comes quite close to being achieved. The second figure shows the path of the (anti-logarithm of the) control variable, $\exp(GPAYEX)$. When this variable is below one, then government pay is being kept below what it would otherwise be. From 2009:4, $\exp(GPAYEX)$ is above one so that pay is being kept above what it would otherwise be. This illustrates a drawback to the control exercise since, in practice, the government should be happy to accept a debt to GDP ratio below its threshold target, but not above. This problem could be solved by changing the target variable to be

$$\max(TDEBTr_PHI, 0.75) \tag{5.1}$$

since this new target is automatically met whenever $TDEBTr_PHI$ is below the threshold of 0.75.

5.3 Exercises

Several other interesting optimal control exercises can be performed with this model. Most of these problems can be set up by simply editing the control

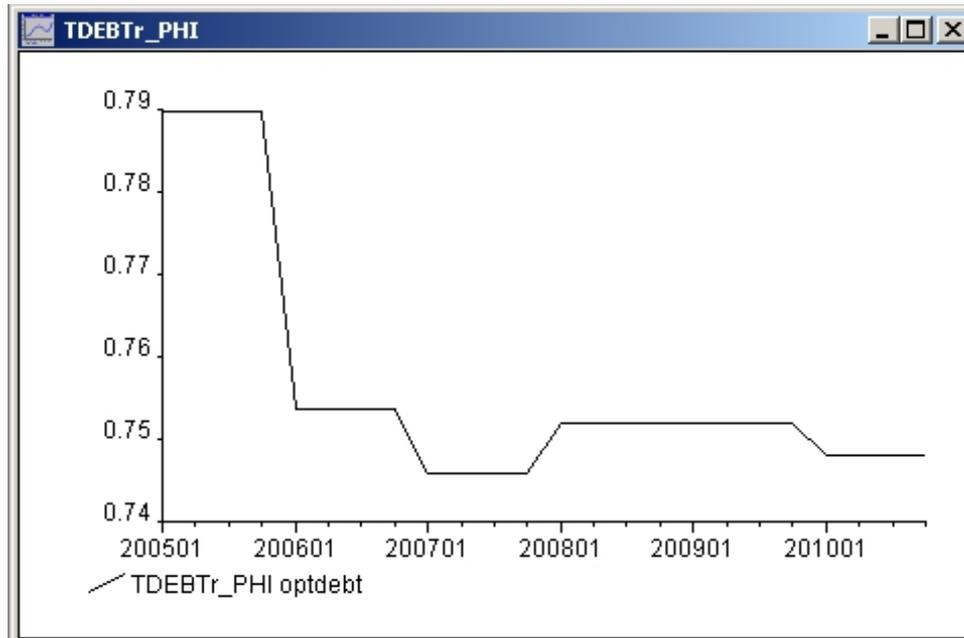


Figure 15: Optimal values of debt to *GDP* ratio

game defined above through the *edit control game* option on the *Control* menu.. The exception is control with penalties on changes to the controls which requires redefining a completely new control game.

1. Add a second target variable *GDEFr_PHI* with target value 0.04 and experiment with the weighting of the two targets
2. Add a second control variable *GTAX_EX*
3. Create new targets

$$\max(TDEBTr_PHI, 0.75)$$

and

$$\max(GDEFr_PHI, 0.04)$$

so as not to penalise being below the thresholds

4. Experiment with the time discount factor β in the control options dialog to change the relative weighting of present and future
5. Experiment with the aggregation parameter in the control options dialog to change how frequently controls get updated

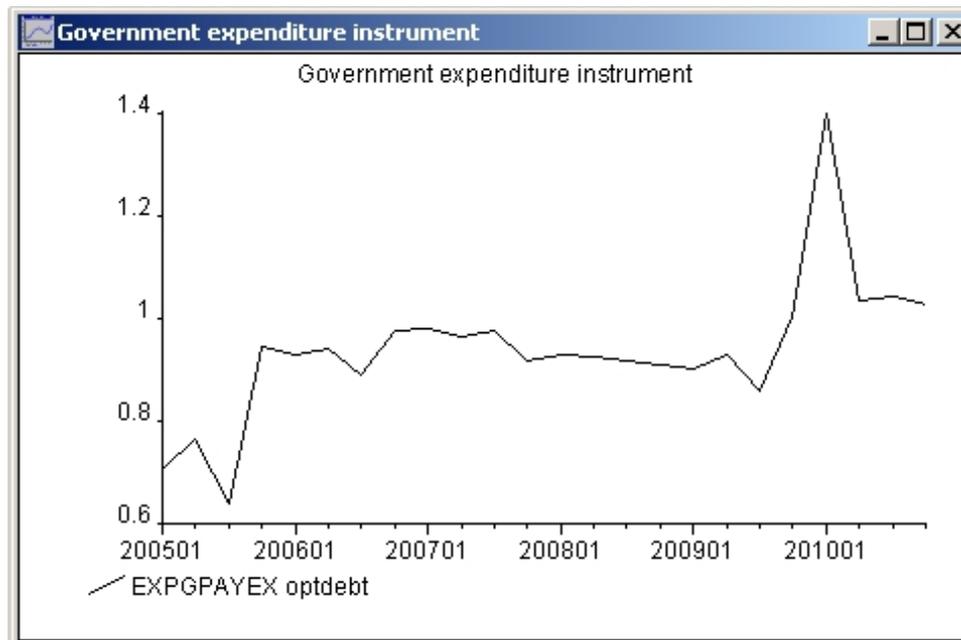


Figure 16: Optimal values of *GPAYEX* expenditure instrument

6. Impose a smooth path on the controls by penalising changes in the controls (this requires creating a new control game)

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