# Stability of a U. K. Money Demand Equation: a Bayesian Approach to Testing Exogeneity*

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**Abstract**

The paper analyses an M3 demand for money equation for the United Kingdom. Attention is paid to the policy change that occurred in 1971 with the introduction of the measure known as Competition and Credit Control. Classical and Bayesian single equation instrumental variables procedures are developed to investigate the exogeneity of the short-term interest rate and the constancy of the parameters of the underlying relationships. The parameters of the short-term equation have changed as well as the exogeneity status of the interest rate variable but the parameters of the long-run equation appear to be less affected by the policy change.

## 1 Introduction

A large number of demand for money equations in the United Kingdom exhibit parameter instability across the major policy change that occurred in 1971 with the introduction of the measures known as Competition and Credit Control (CCC).[1] Their instability has been attributed to a structural break on the (implicit) justification that the CCC changes were specifically directed to changing the competitive structure of the banking system. We note, however, that these equations are often estimated by Ordinary Least Squares (OLS) though it has occasionally been argued that the interest rate should be treated as an endogenous variable— see e.g. Artis and Lewis (1976). Furthermore, as we shall see below, the interest rate setting process was indeed fundamentally modified with the introduction of CCC. Following Engle et al. (1983) these are precisely the circumstances under which an invalid exogeneity assumption entails instability of OLS estimators. This issue, which is central to our paper, is developed further in Section 3.1 below.

The main object of our paper is, therefore, to develop operational classical and Bayesian Instrumental Variables (IV) procedures for analysing the exogeneity of a variable in a single structural equation. These procedures atre then used to investigate whether the instability of a demand for money equation has been induced by the invalid assumption that the interest rate is exogenous or whether it corresponds to a

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[1] A noticeable exception is the $M_1$ demand for money equation estimated by Hendry (1980) and updated in Hendry and Richard (1983), whose coefficients are stable over the period 1963(i)–1980(ii).

genuine structural break. These alternativesd whose policy implications are quite different are formalised in Section 3.1 below.

The paper is organised as follows. In Section 2 we discuss the specification of an M3 demand for money equation that is first estimated by OLS under the working assumption that the interest rate is exogenous; multiplicative dummies are then introduced and the equation is reestimated by Weighted Least Squares (WLS) in order to obtain a parsimonious description of parameter insdtability; in Section 3 we develop calssical and Bayesian IV procedures for investigating the exogeneity of a variable within a bivariate linear model; these procedures are described in general terms in Section 3.1 while Sections 3.2 to 3.5 regroup the more technical material; in Section 4 our money demand equation is imbedded with an interest rate equation in a two-equation model and the exogeneity of the interest variable is then formally analysed; conclusions are drawn in Section 5 and the technical details are presented in an appendix.

Sections 2 and 3 are largely autonomous with respect to each other and the reader may consider skipping Sections 3.2 to 3.5 that contain the more technical material. Those whose interest lies in the algebra of an exogeneity analysis may wish to read first Section 3 that provides the theoretical background for the empirical analysis.

# 2 Single Equation Analysis of the Demand for Money

## 2.1 The institution background: competition and credit control

The introduction of the measures known as Competition and Control (CCC) in October 1971 was an attempt by the monetary authorities to move from a regime where the primary objective was to restrain movements in short term interest rates, to a regime in which control over the monetary aggregates could be achieved through the free operation of market forces. To this end, restrictive practices in the banking sector were swept away, the clearing bank cartel (which had previously linked the rates on Advances and time deposits to the administered "Bank Rate") was abolished and the banks were encouraged to compete for funds by offering competitive rates. In 1972 the Bank Rate was replaced by a "Minimum Lending Rate" that was market determined, being related to the Treasury bill rate.

Part of the rationale for the policy change was evidence from published studies of the demand for monet in the U.K. (Fisher (1968), Laidler and Parkin (1970), Goodhart and Crockett (1970)) of a stable behavioural relationship that could be exploited to achieve the objectives of monetary control. However, the immediate result of the switch to the new regime weas an upsurge in holdings of interest earning deposits that was unpredicted by these demand functions and Hacche (1974) reported a complete breakdown of the Bank of England's own forecasting equations for M3 after 1971. The operation of the CCC regime proved difficult as Goodhart (1980) describes and some of the direct control mechanisms abolished in 1971 were later reintroduced before the regime finally came to an end on 20th August 1981.

The breakdown of M3 equations with CCC have several alternative explanations. One possibility is that the equations were misspecified because of omitted variables and the obvious candidate here is the own-rate of interest on money. We discuss this further below. However, money demand equations estimated by OLS implicitly assume that it is legitimate to treat the rate of interest as an exogenous variable. Since the introduction of CCC has moved the economy from a regime of administered interest rates to a regime where interest rates are market determined, we have to consider the possibility that the exogeneity status of the interest rate has changed. Finally, we must also consider the possibility that a demand for money function may not be invariant to a change in the process generating interest rates even if, within each regime, the interest rate is a valid exogenous variable. The Lucas (1976) critique would suggest that agents might modify their behaviour in response to an attempt by the authorities to control it in this way.

## 2.2 Specification search

Our discussion of the institutional background suggests that we should, ideally, conduct a joint search for the money demand and for the interest rate equations. Such a search would, however, prove computationally ery demanding and, anyway, hard to conduct given the difficulties encountered in the specification of the interest rate equation. Therefore, on grounds of tractability, we have adopted the following sequential procedure: in this section, we conduct a single equation specification search for the money demand equatioon equation by means of OLS and WLS estimation under the working assumption that interest rates, as well as the other current dated regressors are weakly exogenous in the terminology of Engle

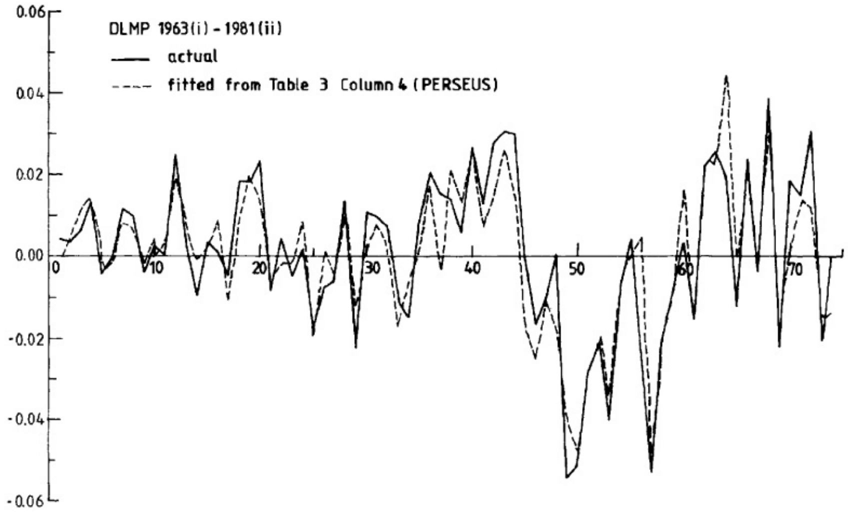


Figure 1: Actual and fitted (Table III Col 4) values of $\Delta \ln(M/P)$, 1963(i)–1981(ii)

et al. (1983); the specification that emerges from this analysis is then used as such in Section 4, where it is embedded within a bivariate model for the purpose of investigating the exogeneity of the interest rate.

As discussed further in Section 5, the empirical evidence relative to the present application seems to suggest that our final conclusions about the exogeneity of the interest rate are unlikely to be severely biased by the adoption of this operational stepwise seach procedure.

## 2.3 The data

The data consists of 79 seasonally adjusted quarterly observations (1961(iv)–1981(ii)), for which the first five are used for the initialisation of the lagged variables and the last four for predictive tests. The remaining 70 observations are eventually divided into the subperiods $A\{$1963(i)–1971(iii)$\}$ and $B$ $\{$1971(iv)–1980(ii)$\}$. The variables relevant to our analysis are:

- $M$: the M3 personal sector monetary aggregate;
- $Y$: real personal disposable income;
- $P$: the deflator of $Y$
- $R$: the local authorities short-term interest rate

The sources are described in Appendix A. Graphs of $\Delta \ln(M/P)$, $\Delta \ln(1 + R_t)$ and $\ln(M/(PY))$ are reproduced in Figures 1–3. Our choice of variables calls for a number of comments.

The choice of personal sector M3 as the appropoiate money aggregate follows from our discussion of the institutional background. Some initial work was done using total M3 but proved unsatisfactory in many respects probably reflecting different behaviour by individuals and companies. That led us to look at the sectoral disaggregation of M3 following, thereby, the Bank of England practice. A complete study of M3 would then require the specification of separate personal and company sectors demand equations typically depending on different interest rates. An exogeneity analysis within this joint context would prove computationally very demanding, espaecially within a Bayesian framework and goes beyond the objectives of the present paper. Therefore, we restricted ourselves to looking at the personal sector only.

The choice of the interest rate variables raises a number of issues. On theoretical grounds our equation should include the opportunity cost of holding M3, a substantial part of which is non-interest bearing. It should, therefore, include the differential between an outside interest arte and the own-rate on the interedst bearing component of M3 as well as that outside rate itself. Before 1971, since all short-term interest rates were closely liked to the Bank Rate, the own-rate differential is essentially constant so that its impact is only estimible in the second subperiod. The Local Authority rtae was chosen as a representative outside rate. WE included both rates in initial empirical work over the whole period but found the own rate wholly insignificant.[2] This seems to indicate that the motivation for holding bank

[2]An $F$-test of the specification corresponding to column 1 in Table I against a specification that included in addition lags of the own interest rate up to 5th gave an $F$ value of 0.24 with degrees of freedom 6 and 37.

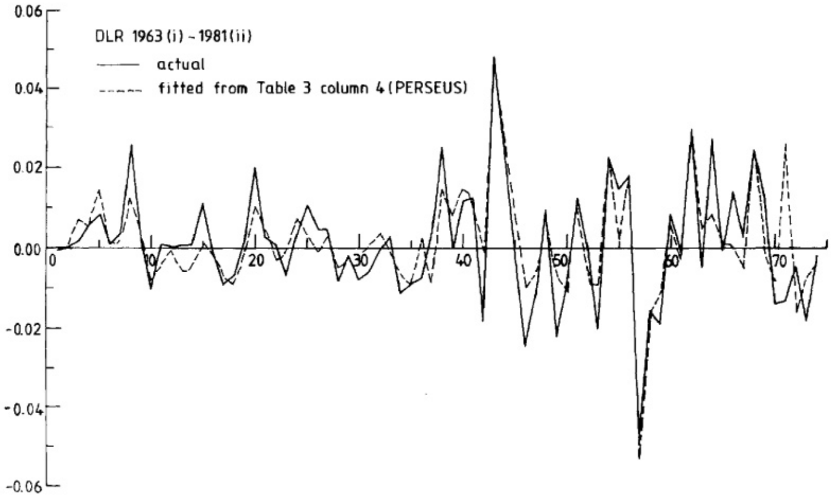


Figure 2: Actual and fitted (Table III Col 4) values of $\Delta \ln(1 + R_t)$, 1963(i)–1981(ii)

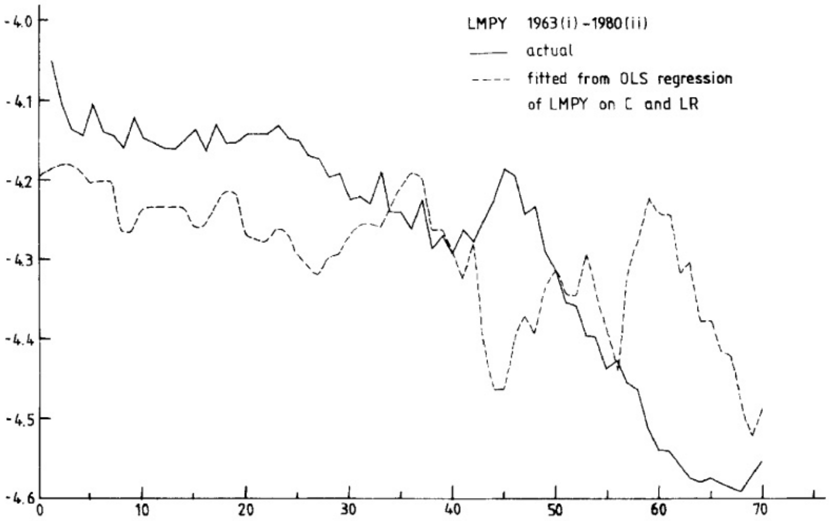


Figure 3: Actual and fitted (Table III Col 4) values of $\ln(M/(PY)$, 1963(i)–1981(ii)

time-deposit accounts instead of such substitutes as Building Socity accounts, etc., lies elsewhere than in the interest rate differential. We therefore kept only the Local Authority rate, while wishing to stress that the issue of the substitution effect of interest rates is not thereby closed.

## 2.4 Notation

The following mnemonics are used throughout the paper: $MP$ for $M_t/P_t$, $MPY$ for $M_t/(P_tY_t)$, $D$ for the difference operator ($\Delta$ when conventional notation is used), $i$ for the $i$-th lag operator, $D_i$ for the $i$-th difference operator ($\Delta_i$), $DD$ for the squared difference operator ($\Delta^2$). $L$ for natural logarithms (ln) and $R$ for $1 + R_t$. For example , $DDLP2$ reads as $\Delta^2 \ln P_{t-2}$, $D4LY$ as $\Delta_4 \ln Y_t$, $LR5$ as $\ln(1 + R_{t-5})$, and so on. Also $C$ stands for the constant term and $C_i$ for the $i$-th quarter seasonal dummy ($i = 1, 2, 3$).

Other notations are: SSR for the sum of squared residuals (these sums are instrumental in the computation of several $F$-test statistics), SDR for the unbiased standard deviation of the regression error, $R^2$ for the unadjusted squared multiple correlation coefficient and DW for the conventional Durbin-Watson statistic. Following Kiviet (1985) no formal significance is attached to the DW statistic. However, values well within the $d_u$ critical interval are at least not worrying.

The following statistics are reported when available:
$\eta_1(4)$ is a forecast test (see Hendry (1979)) asymptotically distributed as $\chi^2_4$ on the null of no predictive failure;
$\eta_2(4,k)$ is the Chow (1970) test for parameter constancy for four periods approximately distributed as $F_{4,k}$ on the null of parameter constancy;
$\eta_3(1)$ is the squared $h$-test for first-order autocorrelation, asymptotically distributed as $\chi^2_1$ on the null of serial independence;
$\eta_4(8)$ is the Box and Pierce (1970) test statistic for 8th-order residual autocorrelation, asymptotically distributed as $\chi^2_8$ on the null of serial independence;
$\eta_5(4)$ is a Lagrange multiplier test for 4th-order autocorrelation and
$\eta_5(4,j)$ is an $F$-version thereof (see Godfrey (1978)), approximately distributed as $\chi^2_4$ and $F_{4,j}$ on the null of serial independence;
$\eta_7(1)$ is the test for first-order ARCH (see Engle (1982a)) asymptotically distributed as $\chi^2_1$ on the null of no ARCH effect.

### 2.5 Results

The specification search has been conducted along the principles described in Hendry and Richard (1982, 1983) and consists of three main steps.

*Step 1.* Independent specification searches over the pre- and post-1971 periods, each of which consists of 35 observations only, cannot be envisaged because of lack of degrees of freedom. Therefore, the starting point of our analysis is an unrestricted OLS regression over the 70 observations of $DLMP$—i.e. $\Delta\ln(M_t/P_t)$—on 27 regressors consisting of a constant term, three seasonal dummies, current and lagged values up to the fifth-order of $LP$, $LY$, $LR$ and lagged values of $LM$. THis equation serves essentially to calibrate the error standard deviation, which equals here 0.0095 and to ensure that the error process is a mean innovation process (MIP) relative to our data base. The individual coefficient values are of little interest and are not reported here.

Successsive simplifications lead to equation (2.1)

$$\Delta\ln(M/P)_t = \beta_0 + \beta_1\Delta\ln(M/P)_{t-1} + \beta_2\Delta\ln P_t + \beta_3\Delta^2\ln P_{t-2} + \beta_4\ln(M/(PY))_{t-5} + \beta_5\Delta_4\ln Y_t + \beta_6\Delta\ln(1+R_t) + \beta_7\Delta^2\ln(1+R_{t-3}) + \beta_8\ln R_{t-5} + u_t \tag{2.1}$$

or, in our notation,

$$DLMP = C + \beta_1 DLMP1 + \beta_2 DLP + \beta_3 DDLP2 + \beta_4 LMPY5 + \beta_5 D4LY + \beta_6 DLR + \beta_7 DDLR3 + \beta_8 LR5$$

whose coefficients are reported in column 1 of Table I. In short, equation (2.1) takes the form of an error correction mechanism (ECM) for real money balance. Its steady-state equilibrium solution is characterised by a constant velocity of circulation of money. The disequilibrium feedback coefficient ($LMPY5$) exhibits an unusually long lag of 15 months though the time-lag is in fact poorly identified and equation (2.1) is only marginally better than those in which $LMPY5$ is replaced by any one of the other $LMPYi$variables.[3] The coefficient of the interest rate $DLR$ has the "wrong" sign according to conventional wisdom, an issue to which we shall pay further attention below.

*Step 2.* Equation (2.1) is then reestimated over the subperiods A and B separately. The results are found in columns 2A and 2B of Table I. Four salient features emerge from the comparison between those two regressions:

(i) The sample variance is substantially larger in period B than it is in period A with a variance ratio of about 3. While this could be a symptom of model misspecification arising from changes in omitted variables orthogonal to those included, we know of no such variables and have treated the problem as one of changes in the market structure;

(ii) the coefficient of $DLR$ changes sign with the introduction of CCC and the explanation for the overall positive coefficient of $DLR$ in equation (2.1) lies in the post-1971 period. Though at this stage of our analysis the difference is not yet statistically significant, it will prove critical for our purpose and has obvious policy implications, some of which are discussed in Section 5;

[3] In connection with this issue of time lag, note also the significance of the coefficient $LR5$.Throughout the simplification search we have run auxiliary regressions to test for the inclusion of additional lags but none has turned out significant.

Table I: OLS and WLS estimators for the UK money demand equation
Dependent variable: $DLMP$

| Case | 1 | 2A | 2B | 3 | 4 |
|---|---|---|---|---|---|
| estimator | OLS | OLS | OLS | WLS | WLS |
| periods | A+B | A | B | A+B | A+B |
| $DLMP1$ | 0.28(0.10) | 0.11(0.16) | 0.36(0.15) | 0.27(0.10) | 0.29(0.10) |
| $DLP$ | −0.36(0.13) | −0.48(0.24) | −0.44(0.22) | −0.45(0.15) | −0.38(0.13) |
| $DDLP2$ | −0.16(0.12) | −0.36(0.20) | −0.16(0.18) | −0.21(0.13) | −0.23(0.12) |
| $LMPY5$ | −0.04(0.01) | −0.09(0.06) | −0.03(0.02) | −0.03(0.02) | −0.04(0.01) |
| $D4LY$ | 0.08(0.05) | 0.11(0.06) | 0.01(0.08) | 0.07(0.05) | 0.08(0.05) |
| $DLR$ | 0.12(0.10) | −0.08(0.18) | 0.17(0.15) | { -0.17 (0.18)<br>0.16 (0.13) | { -0.13 (0.17)<br>0.16 (0.12) |
| $DDLR3$ | −0.06(0.07) | −0.28(0.15) | −0.04(0.10) | { -0.30 (0.15)<br>-0.01 (0.09) | { -0.34 (0.14)<br>— |
| $LR5$ | −0.17(0.08) | −0.13(0.11) | −0.12(0.13) | −0.14(0.07) | −0.16(0.07) |
| $SSR$ | 0.0586 | 0.00115 | 0.00368 | 0.00405 | 0.00419 |
| $SDR$ | 0.0100 | 0.0071 | 0.0126 | 0.0088 | 0.0086 |
| $R^2$ | 0.78 | 0.73 | 0.83 | 0.77 | 0.77 |
| $DW$ | 1.92 | 2.45 | 2.00 | 2.11 | 2.06 |
| $F_{p,q}$ | 1.50 | — | — | 0.89 | 0.65 |
| $p, q$ | 15, 43 | — | — | 6, 46 | 11, 46 |
| $\eta_1(4)$ | 5.69 | 2.77 | 4.03 | 2.96 | 3.48 |
| $\eta_2(4,k)$ | 0.98 | 0.54 | 0.54 | 0.53 | 0.64 |
| $k$ | 58 | 23 | 23 | 52 | 57 |
| $\eta_3(1)$ | 0.15 | 8.06 | 0.10 | 0.12 | 0.01 |
| $\eta_4(8)$ | 5.63 | 28.78 | 4.81 | 11.43 | 10.67 |
| $\eta_5(4)$ | 2.48 | 18.98 | 1.07 | 5.38 | 3.71 |
| $\eta_6(4,j)$ | 0.50 | 5.63 | 0.15 | 1.00 | 0.74 |
| $j$ | 54 | 19 | 19 | 48 | 53 |
| $\eta_7(1)$ | 0.007 | 3.292 | 0.593 | 0.907 | 0.245 |

*Notes*

[1] The numbers in parentheses are standard errors (corrected for degrees of freedom)

[2] Joint $F$-test of linear restrictions against the following alternatives: OLS: unrestricted initial equation. WLS: no common coefficient across the two subperiods (conditional on a variance ratio equal to 2.8).

(iii) The coefficient of $LR5$, which determines the direction of the long-run impact of interest rate on the velocity of circulation of money, has the "right" sign and is remarkably constant across the change of regimes;

(iv) None of the differences between the other coefficients appear to be statistically significant suggesting that we can impose common coefficient restrictions across the two regimes, gaining thereby precision on the point estimates.

*Step 3.* Equation (2.1) is finally reestimated by WLS over the entire sample period with multiplicative dummies accounting for the major coefficient changes. The results are reported in columns 3 and 4 of Table I. In both columns common coefficients for the variables $DLMP1$, $DLP$, $DDLP2$, $LMPY5$, $D4LY$ and $LR5$ have been imposed while in column 4 we have also imposed common seasonal coefficients and have deleted $DDLR3$ in the second subperiod.

The specification in Column 4 is the one that will be used for the exogeneity analysis. It contains 13 unrestricted coefficients (namely the 9 reported in Table I Column 4, togther with constant term $C = -0.15(0.05)$ and seasonals $C1 = -0.03(0.004)$, $C2 = m - 0.001(0.004)$ and $C3 = -0.01(0.003)$) leaving 57 degrees of freedom. Our analysis does not seem to provide significant statistical evidence against this equation. Despite data, period and adjustment differences, the actual values for M3 are in accordance with those in Hendry and Mizon (1979), except for the tiny ECM coefficient and for the long term unit elasticity of income (Hendry and Mizon (1979) found 1.6 while in the course of our specification search we have set it equal to 1). A discussion of some of the intriguing features of our equation is postponed until Section 5 since we first have to investigate whether our results suffer from simultaneous biases.

# 3 Bivariate Instrumental Variables Analysis and Exogeneity

## 3.1 Introduction

Let us first indicate how the framework developed in Engle et al. (1983) applies within the present context. For the sake of simplicity, we can restrict our attention to a stylised version[4] of the bivariate model we shall construct below for the variables $\dot{m}_t = \Delta(M/P)_t$ and $\dot{r}_t = \Delta \ln(1 + R_t)$. It consists of the money demand equation

$$\dot{m}_t = \beta \dot{r}_t + u_t \tag{3.1}$$

paired with the interest rate reaction function

$$\dot{r}_t = \rho z_t + v_t \tag{3.2}$$

where $z_t$ is an exogenous (instrumental) variable. It is assumed further that $(u_t, v_t)$ are jointly identically and independently normally distributed with mean zero and covariance matrix

$$\begin{pmatrix} u_t \\ v_t \end{pmatrix} \sim \mathrm{IN}(\mathbf{0}, \boldsymbol{\Sigma}) \quad \text{with} \quad \boldsymbol{\Sigma} = \begin{pmatrix} \sigma_u^2 & \sigma_{uw} \\ \sigma_{vu} & \sigma_v^2 \end{pmatrix}. \tag{3.3}$$

Let $\widehat{\beta}$ denote the OLS estimator of $\beta$ in (3.1).

The joint distribution of $(\dot{m}_t, \dot{r}_t | z_t)$ factorises[5] into the product of the marginal distribution of $(\dot{r}_t | z_t)$, as characterised by (3.1), and the conditional distribution of $(\dot{m}_t | \dot{r}_t, z_t)$, which is normal with conditional expectation

$$\mathrm{E}(\dot{m}_t | \dot{r}_t, z_t) = (\beta + \mu)\dot{r}_t - \mu \rho z_t \tag{3.4}$$

with $\mu = \sigma_{uv}\, \sigma_v^2$ and conditional variance $\tau^2 = \sigma_u^2 - \mu^2 \sigma_v^2$. Therefore, if $\sigma_{uv} = 0$ and if, furthermore, $\beta$ and $(\rho, \sigma_v^2)$ are not subject to cross-restrictions, then the conditional distribution of $(\dot{m}_t | \dot{r}_t, z_t)$ is fully characterised by the structural equation (3.1) on its own and the OLS estimator $\widehat{\beta}$ is BLUE. Equally, importantly, as long as $\beta$ is invariant with respect to intervention affecting $(\rho, \sigma_v^2)$ or, more generally, the distribution of $z_t$, the OLS estimator $\widehat{\beta}$ is not affected by these interventions.

The situation changes dramatically if $\sigma_{uv}$ differs from zero since the sampling distribution of $\widehat{\beta}$ depends heavily on the distribution of $\dot{r}_t$ and $z_t$. To take the simplest case, let us assume that $z_t$ is identically and independently normally distributed with zero mean and variance $\sigma_z^2$. In such a case, the distribution of $\dot{m}_t | \dot{r}_t$ (which is marginalised with respect to $z_t$ since we are discussing the properties $\widehat{\beta}$, the OLS estimator of $\dot{m}_t$ on $\dot{r}_t$ only) is normal with conditional expectation

$$\mathrm{E}(\dot{m}_t | \dot{r}_t) = (\beta + \mu_*)\dot{r}_t \tag{3.5}$$

with $\mu_* = \sigma_{uv}(\sigma_v^2 + \rho^2 \sigma_z^2)^{-1}$ and conditional variance $\tau_*^2 = \sigma_u^2 - \mu_*^2(\sigma_v^2 + \rho^2 \sigma_z^2)$. In such a case, interventions affecting the "nuisance" parameters $(\rho, \sigma_v^2, \sigma_z^2)$ will induce changes in the sampling properties of $\widehat{\beta}$, even when the underlying "structural" coefficient is invariant with respect to these interventions.

In our application we are confronted with the empirical finding that $\widehat{\beta}$ has changed with the introduction of CCC in 1971. Our analysis suggests immediately three possiible explanations for the lack of invariance:

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[4] All the regressors that are inessential to the argument are deleted for notational convenience. Therefore we are left with the simple model described by equations (3.1) and (3.2), which is, however, meant to be interpreted as a stylised version of a *short run dynamic model* (not to be confused with the static long run solution of the model we shall discuss in Section 5 below). Our discussion of weak exogeneity specifically refers to a property of (short term) dynamic model.

[5] Our more general model being dynamic it is essential to view this factorisation as a sequential one ($t : 1 \to T$) in the sense that, at time $t$, it is conditional on the past of all the variables in the model. Therefore, the concept under consideration here is that of weak exogeneity. Strong exogeneity requires in addition that $\dot{m}_t$ does not Granger-cause $\dot{r}_t$ (e.g. through $z_t$).

(i) $\beta$ itself has changed, a possibility that cannot be ruled out since a declared objective of the policy change was precisely to modify the market structure and since, more generally, the Lucas (1976) critique obviously requires our attention in the present context;

(ii) $\sigma_{uv} \neq 0$ and the interest rate setting process has changed;

(iii) the interest rate setting process has not changed but $\sigma_{uv}$ has (the institutional background suggests, in particular, that $\sigma_{uv}$ might be zero before the introduction of CCC and non-zero after that).

Obviously, these three possibilities are not mutually exclusive.

The analysis of our empirical findings relies heavily upon a correct interpretation of the "behavioural" content of the weak exogeneity assumption $\sigma_{uv} = 0$. Let, therefore, equation (3.1) be reformulated in terms of expectations, as in Florens et al. (1974, 1979):

$$\mathrm{E}(\dot{m}_t|z_t) = \beta\,\mathrm{E}(\dot{r}_t|z_t). \tag{3.6}$$

It appears that the condition $\sigma_{uv} = 0$ is necessary and sufficient for the equivalence of equation (3.4) as

$$\mathrm{E}(\dot{m}_t|\dot{r}_t, z_t) = \beta\rho z_t + (\beta + \mu)(\dot{r}_t - \rho z_t) \tag{3.7}$$

where $\rho z_t$ and $\dot{r} - \rho z_t$ are the "anticipated" and "unanticipated" components of $\dot{r}_t$. This indicates that when $\sigma_{uv} = 0$ ($\mu = 0$) economic agents treat in exactly the same way the anticipated and unanticipated components of $\dot{r}_t$. We shall describe such a situation as one of "effective" control to be contrasted with situations where economic agents might find ways of countering the (restrictive) measures that are enforced upon them.

Having set the basic framework, let us now outline the algebra of the exogeneity analysis for the simple model (3.1)–(3.3). Doing so enables us to motivate the introduction of the auxiliary parameters on which our analysis focuses and leaves the reader with the possibility of skipping the more technical details in the sections that follow. Unless we restrict our attention to deriving Lagrange Multiplier (LM) test-statistics for weak exogeneity, e.g. as in Engle (1982b), we need an operational factorisation of the llikelihood function that works even when $\sigma_{uv} \neq 0$ and that, as much as possible, enables us to deal analytically with the nuisance parameters $(\rho, \sigma_v^2)$ in equation (3.2) and to draw inference on $\sigma_{uv}$ or on appropriate functions thereof.

As is often the case with likelihood functions, it proves convenient to set the factorisation in terms of the distribution of the unobservable disturbance terms $(u_t, v_t)$. This can be done by using either of the following two auxiliary regression functions:

(i) the regression of $v_t$ on $u_t$:

$$v_t = \lambda u_t + \epsilon_{1t}, \quad \epsilon_{1t} \sim \mathrm{IN}(0, \omega^2) \tag{3.8}$$

with $\lambda = \sigma_{uv}\,\sigma_u^{-2}$ and $\omega^2 = \sigma_v^2 - \lambda^2\sigma_u^2$; or

(ii) the regression of $u_t$ on $v_t$:

$$u_t = \mu v_t + \epsilon_{2t}, \quad \epsilon_{2t} \sim \mathrm{IN}(0, \tau^2) \tag{3.9}$$

where $\mu$ and $\tau^2$ are defined as in (3.4).

The correspondence between $\boldsymbol{\Sigma}$ and the two sets of parameters $(\sigma_u^2, \lambda, \omega^2)$ and $(\sigma_v^2, \mu, \tau^2)$ is one-to-one and is characterised by the identitites

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_u^2 & \lambda\sigma_u^2 \\ \lambda\sigma_u^2 & \omega^2 + \lambda^2\sigma_u^2 \end{pmatrix} = \begin{pmatrix} \tau + \mu^2\sigma_v^2 & \mu\sigma_v^2 \\ \mu\sigma_v^2 & \sigma_v^2 \end{pmatrix}. \tag{3.10}$$

Starting with Wu (1973) most exogeneity tests are based on the auxiliary regression (3.9), whether implicitly or explicitly when, in a Lagrange Multiplier (LM) framework as described e.g. in Engle (1982b), they amount to including an estimated residual $\widehat{v}_t$ as an additional regessor in (3.1) and testing for its significance. In fact, as indicated by (3.4), the associated factorisation coincides with the sequential factorisation of the joint distribution of $(\dot{m}_t, \dot{r}_t|z_t)$ into the conditional distribution of $(\dot{m}_t|\dot{r}_t, z_t)$ and the marginal distribution of $(\dot{r}_t|z_t)$. However, when $\sigma_{uv} \neq 0$ this factorisation does not meet our requirements since, in particular, the nuisance parameters $(\rho, \sigma_u^2)$ appear on both sides. This is not the case in the factorisation associated with the auxiliary regression (3.8). This explains why our subsequent analysis is based on the following factorisation of the likelihood function

$$L(\mathbf{Y}; \boldsymbol{\theta}) = L_1(\mathbf{Y}; \boldsymbol{\theta}_1) \cdot L_2(\mathbf{Y}; \boldsymbol{\theta}_2), \tag{3.11}$$

with

$$L_1(\mathbf{Y};\boldsymbol{\theta}_1) = \prod_{t=1}^{T} f_N^1(u_t|0,\sigma^2) \tag{3.12}$$

$$L_2(\mathbf{Y};\boldsymbol{\theta}_2) = \prod_{t=1}^{T} f_N^1(v_t|\lambda u_t,\omega^2) \tag{3.13}$$

where $\mathbf{Y}$ denotes the $T \times 2$ matrix of observations on $(\dot{m}_t, \dot{r}_t)$, $u_t$ and $v_t$ are given in (3.1) and (3.2) respectively, $\boldsymbol{\theta}_1{}' = (\beta, \sigma^2)$ and $\boldsymbol{\theta}_2{}' = (\beta, \rho, \lambda, \omega^2)$. Also $f_N^1(x|\mu, \nu^2)$ denotes a univariate normal density function with mean $\mu$ and variance $\nu^2$, Its expression is given in Appendix B.

Conditionally on $\beta$, the submodel (3.13) takes the form of a standard regression model to which we can apply the usual classical and Bayesian techniques in order to derive *analytical* expressions for the conditional point estimates and posterior distributions of $(\rho, \lambda, \omega^2)$. They are to be marginalised with respect to $\beta$ at the final stage of the analysis. Technical details are provided in Sections 4.2 and 4.3. Note that $\lambda = 0$ if and only if $\sigma_{uv} = 0$ so that inference on the exogeneity of $r_t$ is a direct byproduct of our analysis.

The covariance matrix of $(\dot{m}_t, \dot{r}_t|z_t)$, say $\mathbf{V}$, is related to $\beta$ and $\boldsymbol{\Sigma}$ through the following identity:

$$\mathbf{V} = \mathbf{Q}'\boldsymbol{\Sigma}\mathbf{Q} \quad \text{with } \mathbf{Q} = \begin{pmatrix} 1 & 0 \\ \beta & 1 \end{pmatrix} = (\mathbf{b} : \mathbf{s}) \text{ say} \tag{3.14}$$

so that $\lambda$, as defined via (3.8), may be written as

$$\lambda = \mathbf{b}'\mathbf{V}\mathbf{s}(\mathbf{b}'\mathbf{V}\mathbf{b})^{-1}. \tag{3.15}$$

The question then arises of deciding whether prior information on $\lambda$ should be thought in terms of $\boldsymbol{\Sigma}$ or in terms of $\beta$ and $\mathbf{V}$. Technically, it makes little difference, in the present case at least, since, conditionally on $\beta$, the correspondence between $\Sigma$ and $\mathbf{V}$ is one-to-one and bilinear and since such prior densities as the inverted-Wishart are functionally invariant with respect to such transformations, Simply $\boldsymbol{\Sigma}$ and $\mathbf{V}$ cannot be both *a priori* independent of $\beta$. We have a definite preference for reasoning in terms of $\mathbf{V}$ since, from a statistical point of view, disturbances are merely "derived" unobservable quantities which *de facto* regroup all factors that have been omitted from the equations under consideration.[6] It seems, therefore, difficult to assume for example, prior independence between $\beta$ and $\boldsymbol{\Sigma}$, while we have no conceptual problems in doing so between $\beta$ and $\mathbf{V}$.

Three important issues remain to be clarified before we can concentrate on the more technical issues.

1. The interest rate equation (3.2), as well as the more general equation we shall introduce below, takes the form of an Instrumental Variables (IV) equation whereby the current value of money $\dot{m}_t$ is excluded from the list of regressors. The point is that we lack economic theories to support a complete specification search towards a genuine "structural" equation for the interest rate and that we are faced with limited sample sizes (35 observations in each regime). Considerations of robustness against the specification of the interest rate equation are, therefore, critical since the latter is only instrumental in the construction of the exogeneity tests.

2. Conventional Limited Information Maximum Likelihood (LIML) procedures, or approximations thereof such as Two-Stage Least Squares (2SLS), require that *all* the predetermined variables in the money demand equation should be included in $z_t$, viewing thereby equation (3.2) as an "unrestricted reduced form" equation. This requirement will not be imposed here since it renders a parsimonious selection of instruments impossible taking into account the facts that $z_t$ already includes 12 variables and that sample size is limited. Also hypotheses of interest such as the non-causality of money on interest rate cannot be dealt within an LI framework since, as we have seen, the money demand equation includes lagged values of money. In the present application instruments will, therefore, be selected on their own merits.

3. As discussed e.g. in Leamer (1978) there are a number of ways in which a Bayesian can approach the problem of "testing" a (point) hypothesis. A central issue is that of whether or not he should

[6] Furthermore, as discussed e.g. in Florens et al. (1979) or Richard (1984), the distribution of the disturbances no longer uniquely characterises the distribution of the observables as soon as the number of relationships under consideration is strictly less than the number of endogenous variables as is naturally the case with general linear models such as errors-in-variables models or so-called "incomplete" simultaneous equations models.

use continuous density functions whereby zero prior and posterior probabilities are attached to zero measure subsets of the parameter space. One route consists of attaching non-zero prior (discrete) probabilities to the hypothesis of interest and in analysing how the corresponding "prior odds" are revised into "posterior odds" in the light of sample evidence. In the specific context of exogeneity tests, this route has been adopted e.g. by Reynolds (1982) within an LI framework. It is our view that this approach can occasionally lead to questionable empirical results for example when it produces posterior odds which are much more extreme than one should be willing to accept on the basis of limited sample evidence. In fact, as argued e.g. by Kiefer and Richard (1979), it can easily lead to paradoxes when the (informative) prior odds are paired with prior densities which are otherwise "non-informative" within each hypothesis. We shall adopt here a "smoother" procedure whereby we rely upon continuous prior densities. Prior beliefs that a variable might be exogenous are then expressed in the form of an informative prior density for $\lambda$ which is centred around zero. Sample evidence will then either tighten the corresponding posterior density around zero (confirmation) or shift it away from zero (refutation). The complete posterior distribution of $\lambda$ is obviously far more informative than the scalar posterior odds and, for example, one can always examine whether or not an appropriate 95% posterior probability interval for $\lambda$ contains the origin.

Evidently conducting inference about a quantity such as $\lambda$ requires the choice of a metric whereby one can attach meaning to a non-zero value of $\lambda$. We see two ways of approaching this problem within the present context. Note first that, following its definition in (3.8), $\lambda$ is subject to the inequality constraint $|\lambda\,(\sigma_u/\sigma_v)| < 1$. Though $\lambda$ and $\sigma_u/\sigma_v$ are not independent this inequality can serve to have a rough appreciation of how far $\lambda$ is from zero. (The ratio of the OLS point estimates of $\sigma_u$ and $\sigma_v$ is of the order of 1.1 to 1.2 in both regimes.) We shall follow an alternative route, which seems more relevant to the object of our paper, whereby we shall compute the prior and posterior correlations bewteen $\beta$ and $\lambda$ since these enable us to translate approximately shifts in $\lambda$ into shifts in $\beta$ within a metric of standard deviations.

## 3.2 Sampling theory analysis

The money demand equation we have obtained in Section 2.4 is rewritten as

$$\mathbf{b}'\mathbf{y}_t + \mathbf{c}'\mathbf{x}_t + u_t \tag{3.16}$$

where $\mathbf{b}' = (1 : \beta)$, $\mathbf{y}_t{}' = (\dot{m}_t : \dot{r}_t)$ and $\mathbf{x}_t \in \mathbb{R}^m$ regroups all the other variables entering the equation including lagged $\mathbf{y}$'s. The interest rate IV equation is written as

$$\dot{r}_t + \mathbf{p}'\mathbf{z}_t + v_t \tag{3.17}$$

where $\mathbf{z}_t \in \mathbb{R}^k$ represents the set of instruments. In order to single out the variables that are common to $\mathbf{x}_t$ and $\mathbf{z}_t$ let the corresponding data matrices be partitioned as

$$\mathbf{X} = (\mathbf{X}_1 : \mathbf{X}_2), \quad \mathbf{Z} = (\mathbf{X}_2 : \mathbf{X}_3) \tag{3.18}$$

where $\mathbf{X}_1$ is $T \times m_1$ with $m_1 + m_2 = m$ and $m_2 + m_3 = k$. Under a bivariate normality assumption the model consisting of equations (3.16) and (3.17) is rewritten as

$$\mathbf{y}_t|\mathbf{x}_t, \mathbf{z}_t \sim \mathrm{N}(\boldsymbol{\xi}_t, \mathbf{V}) \tag{3.19}$$

$$\mathbf{b}'\boldsymbol{\xi}_t + \mathbf{c}'\mathbf{x}_t = 0 \tag{3.20}$$

$$\mathbf{s}'\boldsymbol{\xi}_t + \mathbf{p}'\mathbf{z}_t = 0, \quad t = 1 \rightarrow T \tag{3.21}$$

where $\mathbf{s}' = (0 : 1)$ has been introduced for notational convenience. The model (3.19)–(3.21) belongs to a class of linear models discussed in Florens et al. (1974, 1979), Lubrano and Richard (1981) and Richard (1984) whose derivations serve as the basis of our analysis. The likelihood function associated with the model (3.19)–(3.21) factorises as in (3.11)–(3.13) except that $u_t$ and $v_t$ are now given by (3.16) and (3.17) respectively. Also $\theta_1$ and $\theta_2$ now include the additional parameter vector $\mathbf{c}$. Let $\mathbf{c}'$ be partitioned into $(\mathbf{c}_1{}' : \mathbf{c}_2{}')$ conformably with $X$ in (3.18). Additional notation is:

$$\mathbf{M}_2 = \mathbf{I}_T - \mathbf{X}_2(\mathbf{X}_2{}'\mathbf{X}_2)^{-1}\mathbf{X}_2{}', \quad \mathbf{M}_Z = \mathbf{I}_T - \mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}' \tag{3.22}$$

$$\mathbf{u}_1 = \mathbf{Y}\mathbf{b} + \mathbf{X}_1\mathbf{c}_1. \tag{3.23}$$

The superscripts $\tilde{}$ and $\hat{}$ denote IV and OLS estimators respectively.

The concentrated log-likelihood function of $(\beta, \mathbf{c}_1)$ and the corresponding stepwise IVML estimator of $\lambda$ are derived in Appendix C:

$$L^*_{IV}(\mathbf{Y}; \beta, \mathbf{c}_1) = -\frac{T}{2}\log\left[\frac{\mathbf{u}_1{}'\mathbf{M}_2\mathbf{u}_1}{\mathbf{u}_1{}'\mathbf{M}_z\mathbf{u}_1}\left|(\mathbf{u}_1 : \mathbf{Y}\mathbf{s})'\mathbf{M}_z(\mathbf{u}_1 : \mathbf{Y}\mathbf{s})\right|\right] \tag{3.24}$$

$$\widetilde{\lambda}(\beta, \mathbf{c}_1) = (\mathbf{u}_1{}'\mathbf{M}_z\mathbf{u}_1)^{-1}\mathbf{u}_1{}'\mathbf{M}_z\mathbf{Y}\mathbf{s} \tag{3.25}$$

where use has been made of the following identities:

$$\mathbf{M}_z\mathbf{u} = \mathbf{M}_z\mathbf{u}_1 \quad \text{and} \quad \min_{\epsilon_2}\mathbf{u}'\mathbf{u} = \mathbf{u}_1{}'\mathbf{M}_2\mathbf{u}_1. \tag{3.26}$$

Numerical optimisation of (3.24) yields the IVML estimators of $(\beta, \mathbf{c}_1)$ and, by subsitution in (3.25), that of $\lambda$. Under the null hypothesis of $\lambda = 0$, the equations (3.16) and (3.17) are estimated by OLS independently of each other and the corresponding log-likelihood function is given by

$$L^*_{OLS} = -\frac{T}{2}\log\left(\widehat{\mathbf{b}}'\mathbf{Y}'\mathbf{M}_x\mathbf{Y}\widehat{\mathbf{b}} \cdot \mathbf{s}'\mathbf{Y}'\mathbf{M}_z\mathbf{Y}\mathbf{s}\right) \tag{3.27}$$

with $\widehat{\mathbf{b}}' = (1 : \widehat{\beta})$. The log-likelihood ratio (LR) test statistic for the null hypothesis $\lambda = 0$ is

$$\eta_8(1) = 2\left[L^*_{IV}(\mathbf{Y}; \widetilde{\beta}, \widetilde{\mathbf{c}}_1) - L^*_{OLS}\right] \xrightarrow[\lambda=0]{\mathscr{L}} \chi^2_1. \tag{3.28}$$

### 3.3 Bayesian analysis

As usual we have to find a compromise between flexibility and tractability in the choice of a prior density. We wish to specify a prior density which is information on $(\mathbf{V}, \beta)$ and, thereby on $\lambda$. We might also think of useful prior information as regards $\mathbf{c}$ although, as discussed below, taking it into account would substantially increase the computational burden. It is anyway convenient to think of $\mathbf{V}$ and $(\beta, \mathbf{c})$ as being a priori independent.[7] We have little grounds for assessing an informative prior density in the form

$$D(\beta, \mathbf{c}, p, \mathbf{V}) = D(\beta, \mathbf{c}) \cdot D(p|\mathbf{V}) \cdot D(\mathbf{V}) \tag{3.29}$$

where $D(\beta, \mathbf{c})$ is left unspecified at the moment, $D(\mathbf{V})$ is an inverted-Wishart density

$$D(\mathbf{V}) = f^2_{iw}(\mathbf{V}|\mathbf{V}_0, \nu_0) \tag{3.30}$$

whose functional expression is given in Appendix B and $D(p|\mathbf{V})$ is a limiting non-informative natural conjugate prior density

$$D(p|\mathbf{V}) \propto \omega^{-k} \tag{3.31}$$

with $\omega^2$ being the variance associated with the partial likelihood function (3.13). A number of alternative forms of the prior densities (3.30) and (3.31) are discussed in Lubrano and Richard (1981).

*Conditionally on* $\beta$, the prior density of $\mathbf{\Sigma}$ is also inverted-Wishart

$$D(\mathbf{\Sigma}|\beta) = f^2_{iw}(\mathbf{\Sigma}|\mathbf{Q}'\mathbf{V}_0\mathbf{Q}, \nu_0) \tag{3.32}$$

where $\mathbf{Q}$ is defined in (3.14). The conditional distribution of $\lambda$ given $\beta$ is, therefore, a univariate $t$-density

$$D(\lambda|\beta) = f^1_t\left(\lambda|\lambda_0, \frac{h_0}{\omega_0^2}, \nu_0\right) \tag{3.33}$$

whose functional expression in given in Appendix B. The hyperparameters $\lambda_0$, $h_0$ and $\omega_0^2$ are functions of $\beta$ and are defined by the identity

$$\mathbf{Q}'\mathbf{V}_0\mathbf{Q} = \begin{pmatrix} h_0 & h_0\lambda_0 \\ h_0\lambda_0 & \omega_0^2 + h_0\lambda_0\lambda_0^2 \end{pmatrix}. \tag{3.34}$$

[7] We could accomodate prior dependence between $\mathbf{V}$ and $(\beta, \mathbf{c})$ by letting $\mathbf{V}_0$ in (3.30) be a function of $\beta$ and $\mathbf{c}$ since the analytical derivations in our analysis are mostly conditional on them. In particular, an independent prior density on $\mathbf{\Sigma}$ leads to replacing $\mathbf{Q}'\mathbf{V}_0\mathbf{Q}$ in (3.32) by, say, $\mathbf{\Sigma}_0$. The final numerical analysis of the posterior density of $(\beta, \mathbf{c}, \lambda)$ as well as the elicitation procedure described in Section 3.4 would have to be adapted in consequence.

The posterior densities of $(\beta, \mathbf{c})$ and $(\lambda|\beta, \mathbf{c}_1)$ are derived in Appendix D

$$D(\beta, \mathbf{c}|\mathbf{Y}) \propto \left(\frac{h_*}{\omega_*^2}\right)^{(1/2)(\nu-1)} |\mathbf{\Omega}|^{-(1/2)\nu_*} \cdot D(\beta, \mathbf{c}) \tag{3.35}$$

$$D(\lambda|\beta, \mathbf{c}_1, \mathbf{Y}) = f_t^1\left(\lambda \,\middle|\, \lambda_*, \frac{h_*}{\omega_*^2}, \nu_*\right) \tag{3.36}$$

where $\nu_* = \nu_0 + T$ and

$$\sigma_*^2 = \mathbf{b}'\mathbf{V}_0\mathbf{b} + \mathbf{u}'\mathbf{u} \tag{3.37}$$

$$\mathbf{\Omega}_* = \begin{pmatrix} h_* & h_*\lambda_* \\ h_*\lambda_* & \omega_*^2 + h_*\lambda_*^2 \end{pmatrix} = \mathbf{Q}'\mathbf{V}_0\mathbf{Q} + (\mathbf{u}_1 : \mathbf{Y}_s)'\mathbf{M}_Z(\mathbf{u}_1 : \mathbf{Y}_s). \tag{3.38}$$

If $D(\beta, \mathbf{c})$ is a mutivariate Student, then the posterior density (3.35) belongs to a class of so-called 3-1 poly-$t$ densities for which, as discussed in Richard and Tompa (1980), there exist efficient numerical methods of analysis. The evaluation of the *marginal posterior* denity (3.36) *jointly* with respect to $\beta$ and $\mathbf{c}_1$ proves tedious to implement. An operational alternative consists first in multiplying together the posterior densities (3.35) and (3.36), obtaining thereby the joint posterior density of $\beta$, $\mathbf{c}$ and $\lambda$ and taking advantage of a number of cancellations in the product. The evaluation of the posterior density of $(\beta, \lambda)$ at any given point then requires numerical integration with respect to $\mathbf{c}$ but, dince $D(\mathbf{c}|\beta, \lambda)$ is also poly-$t$, can be organised in such a way that the cost of computation does not critically depend on the dimension of $\mathbf{c}$. Finally, the marginal posterior densities of $\beta$ and $\lambda$ are obtained by means of coneventional bivariate numerical integration procedures paying attention to the fact that these densities can be extremely skewed. The details of this implementation are given in Appendix E where it is also shown that the use of a non-informative prior density on $\mathbf{c}$

$$D(\mathbf{c}|\beta) \propto 1 \tag{3.39}$$

results in a major reduction of the cost of computation. In contrast we can be fully flexible in the choice of $D(\beta)$.

## 3.4 Elicitation of the prior density

The prior density of $\mathbf{V}$ and $\beta$ has to be assessed in such a way that it reflects ones prior beliefs on the exogeneity of $\dot{r}_t$. the first and second order moments of $(\lambda|\beta)$ are central to this discussion. Following (3.33) they can be written as

$$\mathrm{E}(\lambda|\beta) = \lambda_0 = \phi_0 f_1(\beta\phi_0, \rho_0), \quad \nu_0 > 1 \tag{3.40}$$

$$\mathrm{V}(\lambda|\beta) = \frac{1}{\nu_0 - 2}\frac{\omega_0^2}{h_0} = \frac{1}{\nu_0 - 2}[\phi_0 \cdot f_2(\beta\phi_0, \rho_0)]^2, \quad \nu_0 > 2 \tag{3.41}$$

together with

$$f_1(x, \rho) = (\rho - x) \cdot (1 - 2\rho x + x^2)^{-1} \tag{3.42}$$

$$f_2(x, \rho) = (1 - \rho^2)^{1/2} \cdot (1 - 2\rho x + x^2)^{-1} \tag{3.43}$$

$$\phi_0 = (\nu_{22}^0/\nu_{11}^0)^{1/2}, \quad \rho_0 = \nu_{12}^0(\nu_{11}^0 \cdot \nu_{22}^0)^{-1/2}. \tag{3.44}$$

Figure 4 and 5 reproduce charts of the functions $f_1$ and $f_2$ for different values of $\rho$ and $x > 0$. Their values for $x < 0$ are obtained by symmetry since $f_1(-x, \rho) = -f_1(x, \rho)$ and $f_2(-x, \rho) = -f_2(x, \rho)$. We note that $f_1$ and $f_2$ are *bounded* functions of $x$ for any given $\rho$ such that $|\rho| < 1$.

$$|f_1(x, \rho)| < \frac{1}{2}(1 - \rho^2)^{-1/2} \quad \text{and} \quad 0 < f_2(x, \rho) < (1 - \rho^2)^{-1/2}. \tag{3.45}$$

It follows that the marginal prior and posterior moments of $\lambda$ are *finite* (up to the order $\nu_0$ and $\nu_*$ respectively) on the sole condition that the prior distribution of $\beta$ is integrable even if the prior and posterior moments of $\beta$ themselves do not exist, as with the Cauchy prior used below. In contrast the

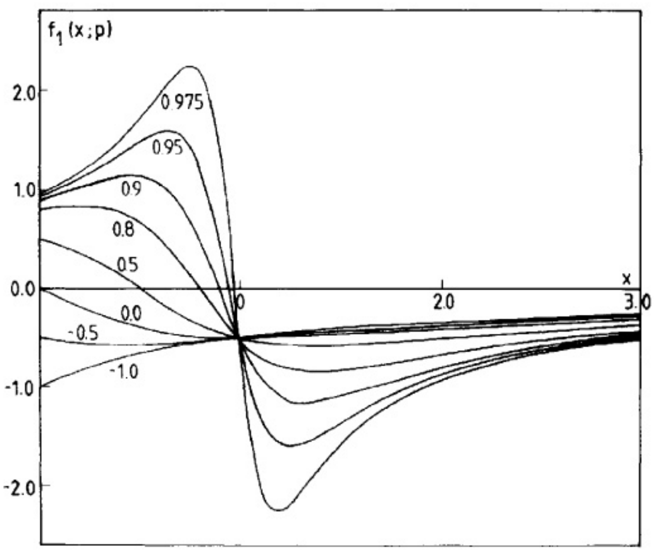


Figure 4: Function $f_1(x, \rho)$ in (3.42) for different $\rho$.

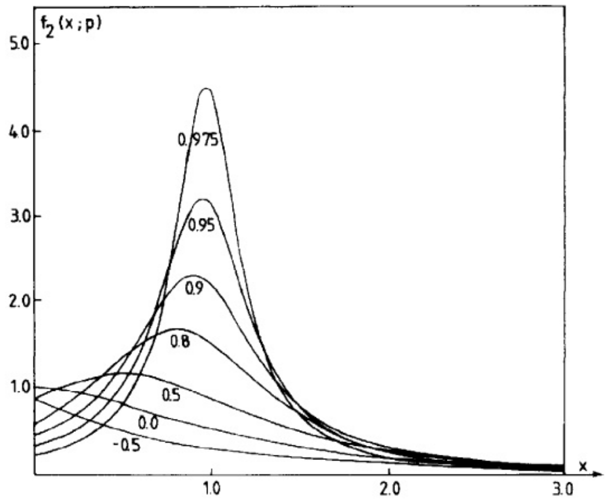


Figure 5: Function $f_2(x, \rho)$ in (3.43) for different $\rho$.

existence of prior and posterior moments of $\mu$, as defined in (3.9), require sharper *prior* information on $\beta$ (as discussed e.g. in Dréze and Richard (1983), the sample information itself typically does not contribute to the existence of moments for $\beta$). This is, in our view, a major argument for conducting inference on the exogeneity of $\dot{r}_t$ in terms of $\lambda$ instead of $\mu$.

The above discussion suggests the following procedure for specifying a prior density on $\mathbf{V}$ and $\beta$ which approximately reflects our prior beliefs on the exogeneity of $\dot{r}_t$:

1. We first specify a *proper* prior density $D(\beta)$, e.g. in the form of a Cauchy density or of a more "informative" $t$-density;

2. The prior expectations of $v_{11}$ and $v_{22}$, the diagonal elements of $\mathbf{V}$, are then elicitated on such heuristic considerations as the expected "goodness of fit" of our model. The choice of $\nu_0$ determines the prior squared variation coefficient $\mathrm{E}^2(v_{ii})/\operatorname{Var}(\sigma_{ii})$ for $i = 1, 2$;

3. $\rho_0$ is then selected in such a way that $\mathrm{E}(\lambda)$ takes the desired value, possibly at the cost of trying different values and computing the corresponding $\mathrm{E}(\lambda)$. If, in particular, $\rho_0 = \phi_0 M(\beta)$, where $M(\beta)$

denotes the prior median of $\beta$, we expect $E(\lambda)$ to be near zero given the symmetry of $D(\beta)$ and the shape of $f_1$, as depicted in Figure 4.

### 3.5 Shifts of regime

The above analysis can be applied as such to the pre- and post-1971 periods separately. However, hypotheses about the constancy of the coefficients of the demand for money equation over the complete sample period are of major interest to us. Joint tests for the exogeneity of subsets of variables can be conducted within a sampling theory framework along the lines discussed in Richard (1980) by means of the computer program PERSEUS developed by Pierse (1982). The sampling theory results which are reported in Section 4 have been computed with PERSEUS.

However, PERSEUS has no Bayesian counterpart since it is obvious from the discussion in Section 3.3 that the analysis of posterior densities combining together sample information from the two subperiods would prove analytically tedious and numerically very costly. This explains why the Bayesian results which are reported in Section 4 have been computed for each period separately.

## 4 Bivariate Analysis of the Demand for Money

### 4.1 Specification of the reaction functions

We have already mentioned in Section 3.1 the difficulties we encountered in the specification of the interest rate reaction function. For each subperiod, the selection of instruments has been conducted by OLS estimation. The choice is restricted to lagged values of money ($LM$) and interest rate ($LR$) together with current and lagged values of prices ($LP$), reserves ($LB$) and unemployment ($LU$) since these are likely targets of monetary policies. The results which are reported in Table II are less than fully satisfactory though the signs are generally in accordance with common sense. Neither of the two equations has a constant growth long-run solution and, in line with our description of the institutional background, money does not enter significantly into the first period reaction function.

Table II: OLS estimators of the interest rate equations
Dependent variable $DLR$

| | | | |
|---|---|---|---|
| | Period A | | |
| $DLR2$ : | −0.28 (0.16) | $D2LU$ : | −0.013 (0.007) |
| $DLR4$ : | −0.21 (0.18) | $DDLU3$ : | −0.035 (0.013) |
| $DDLP2$ : | 0.27 (0.18) | $D4LB$ : | −0.017 (0.007) |
| $DLM2$ : | 0.10 (0.16) | $DDLB1$ : | −0.021 (0.011) |

$SSR = 0.00083$ $\quad SDR = 0.0060$ $\quad R^2 = 0.59$ $\quad DW = 1.67$
$\eta_1(4) = 32.13$ $\quad \eta_2(4,23) = 3.33$ $\quad \eta_3(1) = 1.65$ $\quad \eta_4(8) = 9.09$
$\eta_5(4) = 3.44$ $\quad \eta_6(4,19) = 0.52$ $\quad \eta_7(1) = 3.66$

| | | | |
|---|---|---|---|
| | Period B | | |
| $D4LR1$ : | −0.13 (0.10) | $DLPU$ : | 0.74 (0.26) |
| $LR2$ : | −0.19 (0.10) | $DLM1$ : | 0.60 (0.21) |
| $DLR3$ : | 0.28 (0.15) | $DLB$ : | −0.028 (0.008) |
| $DDLP$ : | −0.38 (0.15) | $DLB4$ : | −0.012 (0.007) |
| $DLP2$ : | −0.31 (0.21) | $D2LU1$ : | −0.051 (0.019) |

$SSR = 0.00257$ $\quad SDR = 0.111$ $\quad R^2 = 0.80$ $\quad DW = 2.22$
$\eta_1(4) = 16.60$ $\quad \eta_2(4,23) = 1.66$ $\quad \eta_3(1) = 2.24$ $\quad \eta_4(8) = 7.47$
$\eta_5(4) = 6.78$ $\quad \eta_6(4,19) = 1.02$ $\quad \eta_7(1) = 2.80$

## 4.2 IVML estimation and exogeneity tests

The IVML estimators of the coefficients of the demand for money equation have been obtained with PERSEUS. Some of our empirical findings in section 3 have been reexamined within this framework including evaluating the constancy of several coefficients, particularly those of $LMPY5$ and $LR5$. the main results are reported in Table III except for the seasonal coefficients and are numbered conformably with their OLS equivalents in Table I.

Table III: FIML (PERSEUS) estimation

| Case | $2_A$ | $2_B$ | 3 | | 4 | |
|---|---|---|---|---|---|---|
| Period | A | B | A | B | A | B |
| | | | Demand for money equation | | | |
| $DLMP1$ | 0.04 (0.088) | 0.29 (0.085) | 0.19 (0.061) | | 0.17 (0.061) | |
| $DLP$ | −0.49 (0.131) | −0.41 (0.124) | −0.44 (0.093) | | −0.39 (0.082) | |
| $DDLP2$ | −0.29 (0.131) | −0.11 (0.105) | −0.15 (0.084) | | −0.18 (0.076) | |
| $LMPY5$ | −0.06 (0.034) | −0.04 (0.010) | −0.04 (0.010) | | −0.04 (0.008) | |
| $D4LY$ | 0.17 (0.036) | 0.01 (0.048) | 0.11 (0.029) | | 0.11 (0.028) | |
| $DLR$ | −0.51 (0.125) | 0.30 (0.099) | −0.55 (0.125) | 0.24 (0.092) | −0.41 (0.120) | 0.23 (0.088) |
| $DDLR3$ | −0.25 (0.088) | −0.25 (0.076) | −0.28 (0.089) | −0.03 (0.058) | −0.32 (0.086) | — |
| $LR5$ | −0.13 (0.062) | −0.12 (0.044) | −0.15 (0.045) | | −0.17 (0.044) | |
| | | | Reaction functions | | | |
| DLR2 | −0.25 (0.081) | | −0.23 (0.082) | | −0.25 (0.085) | |
| DLR4 | −0.18 (0.094) | | −0.18 (0.094) | | −0.18 (0.097) | |
| DDLP2 | 0.26 (0.101) | | 0.31 (0.093) | | 0.32 (0.095) | |
| D2LU | −0.02 (0.004) | | −0.02 (0.004) | | −0.02 (0.004) | |
| DDLU3 | −0.03 (0.007) | | −0.03 (0.007) | | −0.04 (0.007) | |
| D4LB | −0.01 (0.004) | | −0.02 (0.004) | | −0.02 (0.004) | |
| DDLB1 | −0.01 (0.006) | | −0.01 (0.006) | | −0.01 (0.006) | |
| DLM2 | 0.03 (0.075) | | 0.05 (0.077) | | 0.06 (0.081) | |
| D4LR1 | | −0.14 (0.055) | | −0.13 (0.054) | | −0.13 (0.054) |
| LR2 | | −0.19 (0.055) | | −0.19 (0.054) | | −0.20 (0.054) |
| DLR3 | | 0.28 (0.083) | | 0.29 (0.083) | | 0.29 (0.080) |
| DDLP | | −0.43 (0.078) | | −0.42 (0.077) | | −0.42 (0.077) |
| DLP2 | | −0.26 (0.112) | | −0.24 (0.110) | | −0.23 (0.110) |
| DLP4 | | 0.71 (0.138) | | 0.70 (0.137) | | 0.70 (0.136) |
| DLM1 | | 0.62 (0.111) | | 0.62 (0.109) | | 0.62 (0.109) |
| DLB | | −0.03 (0.004) | | −0.03 (0.004) | | −0.03 (0.004) |
| DLB4 | | −0.01 (0.004) | | −0.01 (0.004) | | −0.01 (0.004) |
| D2LU1 | | −0.06 (0.010) | | −0.06 (0.010) | | −0.06 (0.010) |
| Log-likelihood | −48.29 | | −51.78 | | −53.39 | |
| Joint ex. tests | 5.3 | | 6.0 | | 5.1 | |
| Indiv. ex. tests | 3.5 | 1.8 | 5.0 | 1.0 | 3.9 | 1.2 |
| $\lvert\boldsymbol{\Sigma}\rvert$ | $7.1E-10$ | $7.3E-9$ | $8.0E-10$ | $8.1E-9$ | $8.3E-10$ | $8.8E-9$ |
| $\sigma_u$ | 0.0066 | 0.0105 | 0.0067 | 0.0110 | 0.0064 | 0.0114 |
| $\sigma_v$ | 0.0050 | 0.0087 | 0.0050 | 0.0087 | 0.0050 | 0.0087 |
| $\sigma_{uv}$ | $1.9E-5$ | $-3.2E-5$ | $1.8E-5$ | $-3.4E-5$ | $1.4E-5$ | $-3.5E-5$ |

*Note*: The numbers in parentheses are asymptotic standard errors (*uncorrected for degrees of freedom*)

These results clearly indicate that the shift in the OLS estimate of $\beta$ is not caused by simultaneity biases since the IVML estimate of $\beta$ exhibits an even larger shift with the introduction of CCC! Also the weak exogenity of the interest rate suffers a borderline rejection at the 5% level in the first period while it is accepted in the second period. We shall elaborate upon these results in our conclusions. In the meantime we should take due account of the fact that the small sample properties of the LR test statistic (3.26) for weak exogeneity are largely unknown. We might of course use degree of freedom adjustments as in Kiviet (1985) but the application of the Bayesian procedures we have developed in Section 3.3 and 3.4 should provide us with more useful information as regards the exact (finite sample) information content of our data set.

## 4.3 Elicitation of the prior densities

The elicitation procedure described in Section 3.4 is now applied separately to the pre- and post-1971 period. In both cases, several specifications, including "non-informative" ones, are considered in order to conduct a sensitivity analysis. All the informative prior densities are constructed in such a way that $\mathrm{E}(\lambda) = 0$.

### 4.3.1 The period 1963(i)–1971(iii)

Our prior beliefs are that $\beta$, the short-term elasticity of $\dot{m}_t$ with respect to $\dot{r}_t$, probably lies between $-1.0$ and 0. Since inferences on $\lambda$ are likely to be sensitive to the choice of $D(\beta)$—see Figures 4 and 5—two different specifications are considered:

(i) The Cauchy density:

$$D(\beta) \propto [1.0 + 4.0(\beta + 0.5)^2]^{-1} \tag{4.1}$$

is invariant with respect to the normalisation of the money demand equation and is relatively "non-informative" with $\Pr(-1 \le \beta \le 0) = 0.5$;

(ii) The Student density:

$$D(\beta) \propto [1.0 + 0.75(\beta + 0.5)^2]^{-1/2} \tag{4.2}$$

is more informative with standard deviation $\sigma_\beta \simeq 0.41$ and $\Pr(-1 \le \beta \le 0) = 0.8$. The prior means of $v_{11}$ and $v_{22}$, the conditional variances of $(\dot{m}_t, \dot{r}_t | x_t, z_t)$, can usefully be thought of as fractions of the corresponding unconditional variances. For the first period, $\mathrm{E}(v_{11})$ is set equal at 20% of the sampling variance of $\dot{m}_t$ and $\mathrm{E}(v_{22})$ at 40% of the sampling variance of $\dot{r}_t$. The corresponding numerical values are:

$$\mathrm{E}(v_{11}) = 0.243 \times 10^{-4}, \quad \mathrm{E}(v_{22}) = 0.233 \times 10^{-4}. \tag{4.3}$$

We have little grounds on which to select $\nu_0$, which can be interpreted as the size of the "hypothetical sample" on which prior beliefs are based. Three different values will be considered: $\nu_0 = 0$ (non-informative on $\mathbf{V}$), $\nu_0 = 15$ and $\nu_0 = 30$. Note that $\phi_0$ and $\rho_0$, as defined in (3.45) are invariant with respect to the choice of $\nu_0 > 0$ and so is $\mathrm{E}(\lambda|\beta)$ in (3.40).

The discussion in Section 3.4 suggests taking $\rho_0 = -0.5\,\phi_0$ so that, following (3.42), $v_{12}^0 = -0.5\,v_{22}^0$. This completes the first period elicitation of $\mathbf{V}_0$ which is set at zero if $\nu_0 = 0$ and is otherwise given by

$$\mathbf{V}_0 = (\nu_0 - 2)\begin{pmatrix} 0.243 & -0.117 \\ -0.117 & 0.233 \end{pmatrix} \times 10^{-4}, \quad \nu_0 = 15, 30 \tag{4.4}$$

Numerical integration of the bivariate prior density $D(\lambda, \beta)$—with $\nu_0 > 0$—reveals that in all cases $|\,\mathrm{E}(\lambda)| < 0.01\sigma(\lambda)$ as intended (see Table IV).

### 4.3.2 The period 1971(iv)–1980(ii)

The fact that we already know that $\beta$ has changed sign after the introduction of CCC creates an obvious problem in our assessment of the "prior" density of $\beta$. In order to cope with this problem two different sets of prior densities are introduced.

(i) It is unlikely that in 1971 many economists would have predicted the change in the sign of $\beta$. The prior densities (4.1) and (4.2) are taken as representative of such "pre-1971" prior beliefs:

(ii) As an alternative we can put ourselves in the in the position of an economist who would have correctly inferred the positive sign of $\beta$ after 1971, e.g. on the grounds that the initial impact of an *unexpected* rise in interest rates will be to increase money holdings if money is a buffer financial asset and if agents take time to adjust towards long-run equilibrium. Changing the sign of the median of $\beta$ in the prior densities (4.1) and (4.2) yields densities which are representative of such "post-1971" prior beliefs.

For the rest the elicitation procedure is conducted as in Section 4.3.1, except that $\mathrm{E}(v_{11})$ is now set equal at 10% of the sampling variance of $\dot{m}_t$. $\mathbf{V}_0$b is then given by

$$\mathbf{V}_0 = (\nu_0 - 2)\begin{pmatrix} 0.647 & 0.716\mathscr{S} \\ 0.716\mathscr{S} & 1.432 \end{pmatrix} 10^{-4}, \quad \nu_0 = 15, 30 \tag{4.5}$$

Table IV: First period prior and posterior moments of $\beta$ and $\lambda$

| $D(\beta)$ | $\nu_0$ | $\beta$ | $\lambda$ | $\rho_{\beta_\lambda}$ |
|---|---|---|---|---|
| | | Prior | | |
| $S$ | 15 | −0.50 | 0.00 | −0.74 |
| | | (0.41) | (0.44) | |
| $S$ | 30 | −0.50 | 0.00 | −0.84 |
| | | (0.41) | (0.39) | |
| | | Posterior ($DLM2$ included) | | |
| $C$ | 0 | — | 0.37 | — |
| | | | (0.11) | |
| $S$ | 0 | −0.80 | 0.41 | −0.25 |
| | | (0.35) | (0.11) | |
| $S$ | 15 | −0.67 | 0.35 | −0.60 |
| | | (0.33) | (0.15) | |
| $S$ | 30 | −0.56 | 0.27 | −0.77 |
| | | (0.31) | (0.19) | |
| | | Posterior ($DLM2$ excluded) | | |
| $S$ | 0 | −0.67 | 0.41 | −0.31 |
| | | (0.25) | (0.11) | |
| $S$ | 15 | −0.61 | 0.36 | −0.60 |
| | | (0.24) | (0.14) | |
| $S$ | 30 | −0.54 | 0.28 | −0.75 |
| | | (0.23) | (0.17) | |

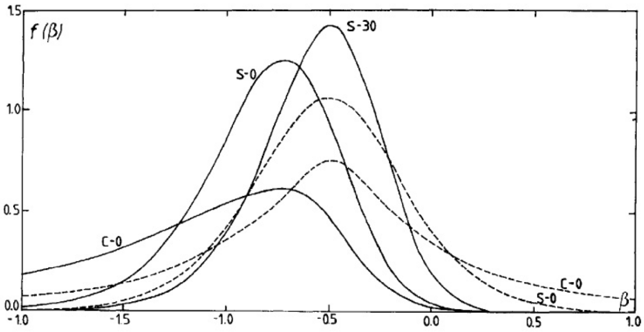


Figure 6: First period prior and posterior densities of $\beta$

with $\mathscr{S} = \text{sign}(M(\beta))$. In the rest of the paper we use a two-character notation to identify the prior sign on $\beta$ and $\mathbf{V}$: the first character refers to the prior density on $\beta$ ($C$ for a Cauchy-density and $S$ for a $t$-density) and the second one indicates the value of $\nu_0$.

## 4.4 Posterior densities

### 4.4.1 The period 1963(i)-1971(iii)

Two sets of posterior densities have been computed. In the first set, $DLM2$ is included in the reaction function and it is, therefore the weak exogeneity of $\dot{r}_t$ which is under investigation. In the second set $DLM2$ is excluded on the basis of the results given by the OLS specification search, in which case the weak exogeneity of $\dot{r}_t$ implies its strong exogeneity. The posterior means of $\beta$ and $\lambda$ are reported in Table IV together with prior moments. Graphs of the prior and posterior densities of $\beta$ and $\lambda$ are reproduced in Figures 6 and 7. The results obtained under a non-informative prior density for $\mathbf{V}$ ($\nu_0 = 0$) confirm the rejection of the exogeneity of $\dot{r}_t$. The C-O graph of the posterior densities of $\beta$ reveals that the

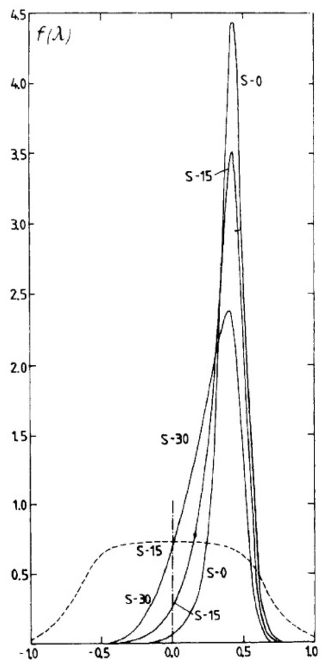


Figure 7: First period prior and posterior densities of $\lambda$

(marginal) likelihood function is highly skewed towards large negative values of $\beta$. Note, furthermore that $\beta$ and $\lambda$ are negatively correlated. the introduction of prior information on $\beta$ in the form of the $t$-density (4.2) reduces the skewness and its impact increases with $\nu_0$.

### 4.4.2 The period 1971(iv)-1980(ii)

Table V: Second period prior and posterior moments of $\beta$ and $\lambda$

| (a): $\beta_0 = 0.5$ | | | | | (b): $\beta_0 = -0.5$ | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $D(\beta)$ | $\nu_0$ | $\beta$ | $\lambda$ | $\rho_{\beta_\lambda}$ | $D(\beta)$ | $\nu_0$ | $\beta$ | $\lambda$ | $\rho_{\beta_\lambda}$ |
| | | Prior | | | | | Prior | | |
| $S$ | 15 | 0.50 | 0.00 | −0.76 | $S$ | 15 | −0.50 | 0.00 | −0.76 |
| | | (0.41) | (0.97) | | | | (0.41) | (0.97) | |
| $S$ | 30 | 0.50 | 0.00 | −0.81 | $S$ | 30 | −0.50 | 0.00 | −0.81 |
| | | (0.41) | (0.91) | | | | (0.41) | (0.91) | |
| | | Posterior | | | | | Posterior | | |
| $C$ | 0 | — | −0.29 | | | | | | |
| | | | (0.18) | | | | | | |
| $S$ | 0 | 0.45 | −0.30 | −0.54 | $S$ | 0 | 0.16 | −0.12 | −0.73 |
| | | (0.21) | (0.17) | | | | (0.26) | (0.26) | |
| $S$ | 15 | 0.34 | −0.12 | −0.78 | $S$ | 15 | 0.33 | −0.45 | −0.49 |
| | | (0.20) | (0.26) | | | | (0.20) | (0.14) | |
| $S$ | 30 | 0.25 | 0.13 | −0.84 | $S$ | 30 | 0.38 | −0.59 | −0.22 |
| | | (0.20) | (0.31) | | | | (0.19) | (0.10) | |

The posterior means and variances of $\beta$ and $\lambda$ under the two sets of prior densities we have introduced in Section 4.3.2 are reported in Tables V(a) and V(b) together with prior moments. Graphs of the posterior densities of $\beta$ and $\lambda$ are found in Figures 8 to 11.

The results in Table V(a) are essentially unambiguous and lead to the acceptance of the weak exogeneity of $\dot{r}_t$ under minimal prior information. We note simply that the second period sample is

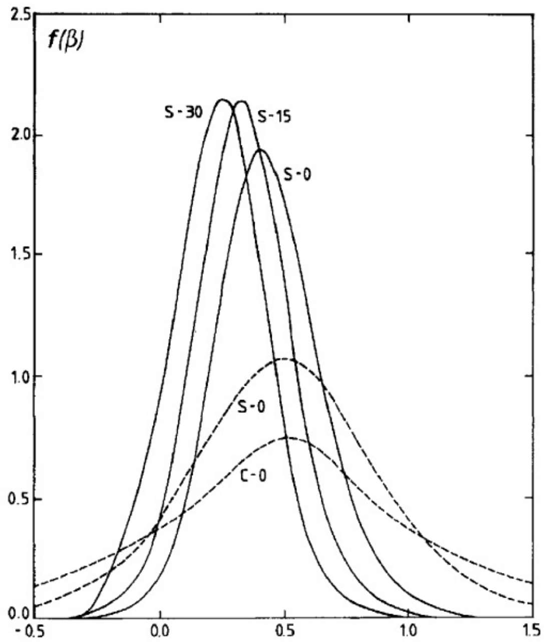


Figure 8: Second period prior and posterior densities of $\beta$ ($\beta_0 = 0.5$)

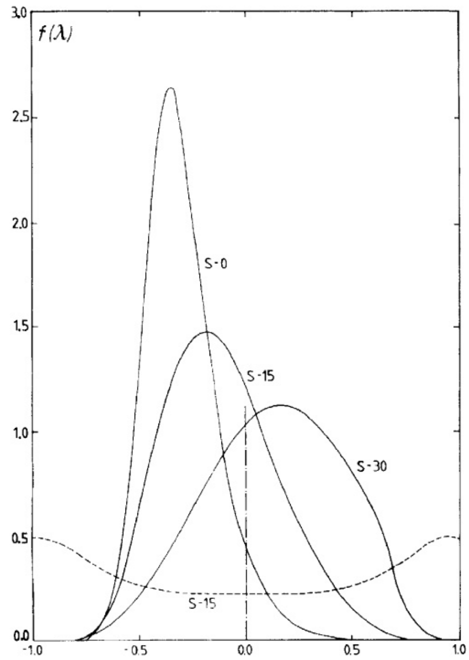


Figure 9: Second period prior and posterior densities of $\lambda$ ($\beta_0 = 0.5$)

comparatively more informative on $\beta$ and less informative on $\lambda$ than the first period sample. Also the negative correlation between $\beta$ and $\lambda$ is more pronounced.

The results in Table V(b) have been derived under a prior density which is in conflict with the sample evidence. The introduction of a prior density for $\beta$ centred around $-0.5$ shifts the posterior density of $\beta$ towards negative values and the posterior density of $\lambda$ towards positive values (in accordance with the negative correlation between $\beta$ and $\lambda$). It is, therefore, a mere coincidence that the posterior expectation of $\lambda$ is close to zero when $\nu_0 = 0$. The cases where $\nu_0 = 15$ or 30 clearly indicate that a conflict between the prior and sample information can totally distort the evidence relative to exogeneity.

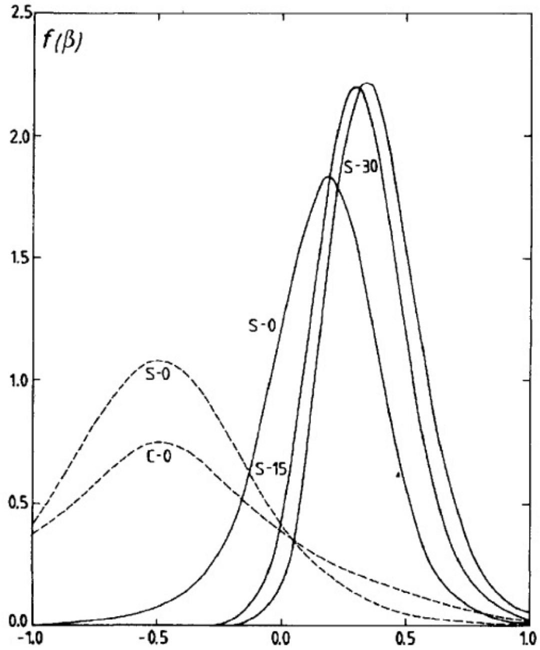


Figure 10: Second period prior and posterior densities of $\beta$ ($\beta_0 = -0.5$)

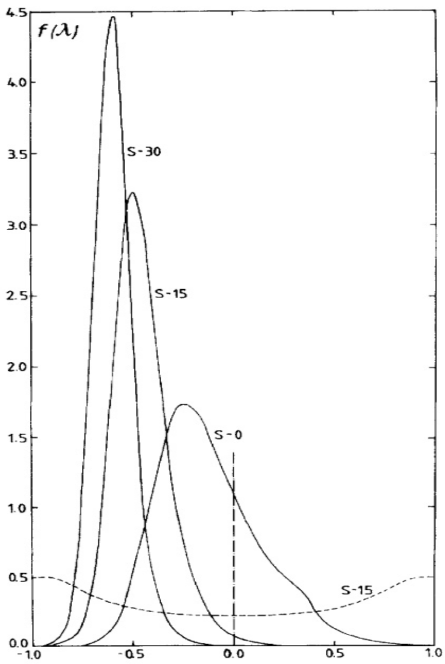


Figure 11: Second period prior and posterior densities of $\lambda$ ($\beta_0 = -0.5$)

## 5 Conclusions

A first set of conclusions concerns the applicability of the Bayesian methods to the class of problems we have discussed and, more specifically, to inference on exogeneity. The limited information maximum likelihood framework whereby all the explanatory variables in the structural equation of interest have to be included in the list of "instrumental variables" has proved inpractical for the sample sizes we were confronted with. This has led us to develop instead an "instrumental variables" approach whereby the instrumental variables are selected solely on their own contribution to the reaction function. We have demonstrated that this more general approach remains fully tractable. Prior information on the exogeneity of a variable is easily taken into account.

The second set of conclusions concerns our money demand equation itself.

1. The evidence on the weak exogeneity of the interest rate is, at first sight, counterintuitive since it leads to rejection in the first period and acceptance in the second period while we might have expected just the opposite in the light of our description of the institutional background. However, as discussed in Section 3.1, weak exogeneity measures the *effectiveness* of a control policy. Situations in which agents find ways of bypassing the restrictions which are enforced upon them naturally lead to a rejection of the weak exogeneity assumption either by linking together the coefficients in the relevant equations or by inducing a significant correlation between the corresponding disturbances. Such may well have been the case in the pre-CCC period where changes in the Bank Rate were rare and carried important signalling effects (hence the presence of a term such as $DDLR3$ in our money demand equation) and where banks could probably find ways of countering the restrictive "requests" they were confronted with. In contrast the more erratic behaviour of interest rates after the introduction of CCC might have made it more difficult for the economic agents to react differently to anticipated and non-anticipated variations in the interest rates. These are, however, mere conjectures that would have to be supported by a more detailed analysis of the economic background.

2. In Section 3.1 we proposed three alternative explanations for the shift in the OLS estimate of $\beta$, the impact coefficient of the interest rate on money. Sample evidence unambiguously suggests that the introduction of CCC has jointly induced a large shift in the structural coefficient $\beta$, a shift from a significantly positive $\sigma_{uv}$ to a moderately negative one and a substantial change in the interest equation itself. The last two effects combine together in such a way that the OLS estimate underestimates the shift in $\beta$! It is, however, comforting to discover that simultaneity biases do not seem to have much effect on the other coefficients in the money demand equation, including those of $LMPY5$ and $LR5$ which determines the long-term impact of interest rate on the velocity of circulation of money. This empirical finding also seems to suggest that the preliminary specification search based on OLS estimation in Section 2 is unlikely to have severely biased our choice of a functional form for the money demand equation and, therefore, to have distorted the evidence on the exogeneity of the interest rate variable.

3. Our money demand equation presents a number of intriguing features which might deserve further investigation. Two which are specific to the second subperiod are the positive sign of the impact coefficient of interest rate, for which we have ventured a possible explanation in the course of Section 4.3.2, and the lack of significance of the own rate for which we have no explanation. A number of problems probably hinge around the existence of a long-term solution characterised by a constant velocity of circulation of money: the long lag associated with the disequilibrium feedback variable $LMPY$ (though the precise lag is essentially unidentified), the tiny ECM coefficient of $LMPY5$ itself and possibly also the unit income elasticity[8]

In fact we suspect that the two subperiods are probably more distinct than our analysis seems to suggest. The functional form we have selected originates from an overall specification search which may have been heavily influenced by the second subperiod, hence the overall positive sign of $\beta$. It did prove convenient for our purposes to have a common functional form across the two regimes for the ease of comparison and, more importantly, in order to gain degrees of freedom that were critically needed. With larger sample sizes we might have conducted independent specification searches over the two regimes but CCC has been abolished since 1981 and we might well be faced since with new coefficient changes (though the 1982 data and our reading of recent economic indicators seem to confirm the positive impact coefficient of interest rate on M3).

We would guess that the problem lies mostly with the CCC regime and that we need an ECM formulation which is coherent both in level, as the present one is, and in differences, the latter requirement

[8] We did compute $t$-test statistics for the addition of $LY5$ to the first equations in Table I. The results are

| Column | 1 | 2A | 2B | 3 | 4 |
|---|---|---|---|---|---|
| $t$-value: | 2.18 | 0.22 | 0.53 | 1.72 | 1.89 |

This variable $LY5$ is clearly not significant for runs on separate periods but the imposition of coefficient restrictions across the two periods increases its significance. This might contribute towards explaining the non-unit elasticity found by Hendry and Mizon (1979) and suggests (ex post) that we might usefully consider deleting the common coefficient restrictions for $D4LY$. Such a modificatoion would, however, not affect our findings as regards the exogeneity of interest rates (compare cases 2 and 4 in Table III).

being critical with such money aggregates as M3, a substantial part of which is now bearing interest.[9] Also the concept of cointegrability recently developed by Granger and Engle (1985) provides us with another route of investigation worth exploring for the second regime.[10]

The comforting message in our analysis is that such additional investigation can probably be conducted by means of OLS estimation if one's attention is restricted to the CCC regime as it should be at this level of investigation. Also the procedures we have developed in our paper are now fully operational and could easily be applied in these and other contexts.

# A The Data Sources

All data are quarterly and seasonally unadjusted. The following abbreviations are used:

| | |
|---|---|
| *ETAS* | Economic Trends Annual Supplement (1982 edition) |
| *BESA1* | Bank of England Statistical Abstract No. 1 (1970) |
| *BESA2* | Bank of England Statistical Abstract No. 2 (1975) |
| *FS* | Financial Statistics (various issues) |

| | |
|---|---|
| $M$ | Personal Sector M3. Cumulated from changes from the Flow of Funds accounts. Source: *BESA2* (1963–1973), *FS* (1974–1981) |
| $R$ | Local Authority 3 month deposit rate (last working day). Source: *BESA1* (1963–1973), *BESA2* (1970–1974), *FS* (1975–1981) |
| $Y$ | Real Personal Disposable Income (£million. 1975 prices. Source: *ETAS* |
| $P$ | Implied deflator for Personal Disposable Income. Souce: *ETAS* |
| $U$ | Unemployment rate (Total unemployed / Working population). Source: *ETAS* |
| $B$ | Real value of UK Official Reserves (£million. 1975 prices). Source: *ETAS* |

# B Notation for Density Functions

The properties of the distribution which are presented here are found e.g. in Zellner (1971) or in Dréze and Richard (1983).

1. Multivariate Normal Distribution

$$f_N^n(\mathbf{x}|\boldsymbol{\mu}, \boldsymbol{\Sigma}) = (2\pi)^{-(1/2)n}|\boldsymbol{\Sigma}|^{-1/2} \exp \frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu}).$$

2. Multivariate $t$-Distribution

$$f_t^n(\mathbf{x}|\boldsymbol{\mu}, \mathbf{H}, \nu) = \pi^{-(1/2)n}\Gamma\left(\frac{\nu+n}{2}\right)/\Gamma\left(\frac{\nu}{2}\right)|\mathbf{H}|^{1/2}[1+(\mathbf{x}-\boldsymbol{\mu})'\mathbf{H}(\mathbf{x}-\mu)]^{-\mathbf{(1/2)}(\nu+\mathbf{p})}.$$

3. Inverted Gamma Distribution

$$f_{i\gamma}(\sigma^2|s^2, \nu) = \left[\Gamma\left(\frac{\nu}{2}\right)\right]^{-1}\left(\frac{s^2}{2}\right)^{\nu/2}(\sigma^2)^{-(1/2)(\nu+2)} \exp -\frac{1}{2}\frac{s^2}{\sigma^2}.$$

4. Inverted Wishart Distribution

$$f_{iW}^n(\boldsymbol{\Sigma}|\mathbf{S}, \nu) = \left[2^{(1/2)\nu}\pi^{(1/4)n(n-1)}\prod_{i=1}^{n}\Gamma\left(\frac{\nu+1-i}{2}\right)\right]^{-1}$$
$$\cdot\,|\mathbf{S}|^{(1/2)\nu}|\boldsymbol{\Sigma}|^{-(1/2)(\nu+q+1)} \exp -\frac{1}{2}\operatorname{tr}\boldsymbol{\Sigma}^{-1}\mathbf{S}.$$

[9] We are grateful to D. F. Hendry for this suggestion.

[10] Though as illustrated in figure 3 a "long-run" OLS regression of $LMPY$ on $LR$ and a constant does not seem to support an hypothesis of cointegrability of $M$ and $R$.

# C Derivation of Formulae (3.24) and (3.25)

The likelihood fiunction of our model is given in (3.11)–(3.13) with respect to $(\mathbf{c}_2, \sigma^2)$ and $(\rho, \lambda, \omega^2)$ respectively yields the following expressions:

$$\widetilde{\mathbf{c}}_2\ (\beta, \mathbf{c}_1) = -\mathbf{X}_2{}'\mathbf{X}_2)^{-1}\mathbf{X}_2{}'\mathbf{u}_1 \tag{C.1}$$

$$\widetilde{\sigma}^2(\beta, \mathbf{c}_1) = \frac{1}{T}\mathbf{u}_1{}'\mathbf{M}_2\mathbf{u}_1 \tag{C.2}$$

$$\widetilde{\rho}\,(\beta, \mathbf{c}_1) = -(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'[\mathbf{Y}_s - \widetilde{\lambda}(\beta, \mathbf{c}_1)\cdot \mathbf{M}_2\mathbf{u}_1] \tag{C.3}$$

$$\widetilde{\lambda}\,(\beta, \mathbf{c}_1) = (\mathbf{u}_1{}'\mathbf{M}_Z\mathbf{u}_1)^{-1}\mathbf{u}_1{}'\mathbf{M}_Z\mathbf{Y}_s \tag{C.4}$$

$$\begin{aligned}\widetilde{\omega}^2\,(\beta, \mathbf{c}_1) &= \frac{1}{T}\left[\mathbf{s}'\mathbf{Y}'\mathbf{M}_Z\mathbf{Y}_s - \mathbf{u}_1{}'\mathbf{M}_Z\mathbf{u}_1 \cdot \widetilde{\lambda}^2(\beta, \mathbf{c}_1)\right] \\ &= \left[\widehat{\theta}\,(\beta, \mathbf{c}_1)\right]^{-1} \cdot \left|\widetilde{\mathbf{\Omega}}\,(\beta, \mathbf{c}_1)\right|\end{aligned} \tag{C.5}$$

together with

$$\widetilde{h}\,(\beta, \mathbf{c}_1) = \frac{1}{T}\mathbf{u}_1{}'\mathbf{M}_Z\mathbf{u}_1 \tag{C.6}$$

$$\widetilde{\Omega}\,(\beta, \mathbf{c}_1) = \frac{1}{T}(\mathbf{u}_1 : \mathbf{Y}_s)'\mathbf{M}_Z(\mathbf{u} : \mathbf{Y}_s). \tag{C.7}$$

The concentrated log-likelihood function is then given by

$$L^*_{IV}(\mathbf{Y}; \beta, \mathbf{c}_1) \propto -\frac{T}{2}\log\left[\widetilde{\sigma}^2(\beta, \mathbf{c}_1)\cdot \widetilde{\omega}^2(\beta, \mathbf{c}_1)\right] \tag{C.8}$$

$$\propto -\frac{T}{2}\log\left[\frac{\mathbf{u}_1{}'\mathbf{M}_2\mathbf{u}_1}{\mathbf{u}_1{}'\mathbf{M}_Z\mathbf{u}_1} - |(\mathbf{u}_1 : \mathbf{Y}_s)'\mathbf{M}_Z(\mathbf{u}_1 : \mathbf{Y}_s)|\right]. \tag{C.9}$$

More detail and generalisations to systems of equations are found in Richard (1984).

# D Derivation of Formulae (3.25) and (3.26)

By application of the properties of inverted-Wishart densities as described e.g. in (Dréze and Richard, 1983, Appendix) the prior densities of $(\sigma^2|\beta)$ and $(\lambda, \delta^2|\beta)$ as derived from (3.32) are given by

$$D(\sigma^2|\beta) = f_{i\gamma}(\sigma^2|\sigma_0^2, \nu_0 - 1) \tag{D.1}$$

$$D(\lambda, \omega^2|\beta) = f_N^1(\lambda|\lambda_0, \omega^2 h_0^{-1})\cdot f_{i\gamma}(\omega^2|\omega_0^2, \nu_0) \tag{D.2}$$

where $(h_0, \lambda_0, \omega_0^2)$ are defined in (3.34) and $\sigma_0^2 = h_0$ (a distinct notation is used for $\sigma_0^2$ and $h_0$ because their posterior counterparts $\sigma_*^2$ and $h_*$, as defined below, differ and because it proves notationally convenient to have common functional forms for the prior and posterior moments of $(\sigma^2, \lambda, \omega^2|\beta)$). Let $l_i(\mathbf{Y}; \beta, \mathbf{c})$ for $i = 1, 2$ denote the "marginalised" likelihood function as derived from $L_i(\mathbf{Y}; \boldsymbol{\theta}_i)$ under the relevant prior density.

Combining together the partial likelihood function (3.12) and the prior density (D.1) yields the following expressions

$$D(\sigma^2|\mathbf{Y}, \beta, \mathbf{c}) = f_{i\gamma}(\sigma^2|\sigma_*^2, \nu_* - 1) \tag{D.3}$$

$$l_1(\mathbf{Y}; \beta, \mathbf{c}) \propto (\sigma_*^2)^{-1/2(\nu_* - 1)}, \tag{D.4}$$

with

$$\sigma_*^2 = \sigma_0^2 + \mathbf{u}'\mathbf{u} + \mathbf{b}'\mathbf{V}_0\mathbf{b} + \mathbf{u}'\mathbf{u} \quad \text{and} \quad \nu_* = \nu_0 + T. \tag{D.5}$$

The product of the partial likelihood function (3.13) and the prior densities (3.21) and (D.2) is handled in a smilar way, except that it proves convenient to derive sequentially the posterior densities of $(p|\lambda, \omega^2, \cdot)$ and of $(\lambda, \omega^2|\cdot)$ which are respectively

$$D(p|\mathbf{Y}, \lambda, \omega^2, \beta, \mathbf{c}) = f_N^k(p|p_*, \omega^2(\mathbf{Z}'\mathbf{Z})^{-1}) \tag{D.6}$$

$$D(\lambda, \omega^2|\mathbf{Y}, \beta, \mathbf{c}) = f_N^1(\lambda|\lambda_*, \omega^2 h_*^{-1}) \cdot f_{i\gamma}(\omega^2|\omega_*^2, \nu_*) \tag{D.7}$$

where

$$p_* = -(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'(\mathbf{Y}s - \lambda\mathbf{u}) \tag{D.8}$$

and, in parallel with (3.34),

$$\mathbf{\Omega}_* = \begin{pmatrix} h_* & h_*\lambda_* \\ h_*\lambda_* & \omega_*^2 + h_*\lambda_*^2 \end{pmatrix} = \mathbf{\Omega}_0 + (\mathbf{u}_1 : \mathbf{Ys})'\mathbf{M}_Z(\mathbf{u}_1 : \mathbf{Ys}). \tag{D.9}$$

Also $l_2(\mathbf{Y}; \beta, \mathbf{c})$ is given by

$$l_2(\mathbf{Y}; \beta, \mathbf{c}) = l_2(\mathbf{Y}; \beta, \mathbf{c}_1) \propto (\omega_*^2)^{-(1/2)\nu_*} h_*^{-1/2}. \tag{D.10}$$

The posterior density of $(\beta, \mathbf{c})$ is given by the product of $D(\beta, \mathbf{c})$ and of the two marginalised likelihood functions (D.4) and (D.10) and may be rewritten as (3.35).

More detail and generalisations to systems of equations are found in Richard (1984).

# E Implementation of the Bayesian Analysis in Section 3.3

Combining together formulae (3.35), (3.36) and (D.2), and taking advantage of the identity $|\mathbf{\Omega}_*| = h_* \cdot \omega_*^2$ we can write the joint posterior density of $\beta$, $\mathbf{c}$ and $\lambda$ as

$$D(\beta, \mathbf{c}, \lambda|\mathbf{Y}) \propto D(\beta, \mathbf{c}) \cdot (\sigma_*^2)^{-(1/2)(\nu_*-1)} \left[\omega_*^2 + h_*(\lambda - \lambda_*)^2\right]^{-1/2(\nu_*+1)}. \tag{E.1}$$

Throughout the rest of the discussion it is assumed that $D(\beta, \mathbf{c})$ is a $t$-density. In all generality $D(\mathbf{c}\,|\,\beta, \lambda, \mathbf{Y})$ is then a product of three kernels of $t$-densities, i.e. a so-called 3-0 poly-$t$ density whose evaluation requires a bivariate numerical integration on an auxiliary random variable—see Richard and Tompa (1980) for details. All together the analysis of the posterior density (E.1) requires, therefore a four-dimensional numerical integration. For the integration of $\beta$ and $\lambda$ we use a bivariate iterative Simpson procedure, as described in Tompa (1973), which has proved far more reliable than the other methods we have tested (such as Gaussian rules) given that the posterior density of $\beta$ and $\lambda$ can be extremely skewed. It implies, however, that the algorithm has to be run twice to obtain the marginal densities of $\beta$ and $\lambda$ since the use of a bivariate iterative Simpson rule is essentially incompatible with a complete analysis of the marginal density associated with the inner integration loop. In compensation this repetition provides a very useful check of numerical accuracy since the integrating constants and the moments are evaluated twice on different grid points. For a relative precision of the order of 1% a complete run of computation may require up to 200 minutes of CPU time on a DGMV/8000 mini computer equipped with a floating point accelerator.

The cost of computation can be divided by a factor of 20 if we use the non-informative prior density (3.39) on $\mathbf{c}$. In such a case $\sigma_*^2$ is the sole factor in (E.1) which still depends on $\mathbf{c}_2$. It can be rewritten as

$$\sigma_*^2 = \mathbf{b}'\mathbf{V}_0\mathbf{b} + \mathbf{u}_1{}'\mathbf{M}_2\mathbf{u}_1 + (\mathbf{c}_2 - \mathbf{c}_2^*)'\mathbf{Z}_2{}'\mathbf{Z}_2(\mathbf{c}_2 - \mathbf{c}_2^*) \tag{E.2}$$

with

$$\mathbf{c}_2^* = (\mathbf{Z}_2{}'\mathbf{Z}_2)^{-1}\mathbf{Z}_2{}'\mathbf{u}_1. \tag{E.3}$$

Therefore, the conditional posterior density of $(\mathbf{c}_2|\beta, \mathbf{c}_1)$ is Student whence

$$D(\beta, \mathbf{c}_1, \lambda) \propto D(\beta)(\sigma_0^2 + \mathbf{u}_1{}'\mathbf{M}_2\mathbf{u}_1)^{-1/2(\nu_*-m_2-1}[\omega_*^2 + h_*(\lambda - \lambda_*)^2]^{-1/2(\nu_*+1)}. \tag{E.4}$$

The numerical analysis of $D(\lambda, \beta, \mathbf{c}_1|\mathbf{Y})$ is then based on the following identities

$$\begin{aligned}\sigma_0^2 + \mathbf{u}_1{}'\mathbf{M}_2\mathbf{u}_1 &= (\mathbf{c}_1 - \mathbf{c}_{1a}^*)'\mathbf{X}_1{}'\mathbf{M}_2\mathbf{X}_1(\mathbf{c}_1 - \mathbf{c}_{1a}^*) \\ &\quad + \mathbf{b}'[\mathbf{V}_0 + \mathbf{Y}'\mathbf{M}_2\mathbf{Y} - \mathbf{Y}'\mathbf{M}_2\mathbf{X}_1(\mathbf{X}_1{}'\mathbf{M}_2\mathbf{X}_1)^{-1}\mathbf{X}_1{}'\mathbf{M}_2\mathbf{Y}]\mathbf{b}\end{aligned} \tag{E.5}$$

$$\omega_*^2 + h_*(\lambda - \lambda_*)^2 = \lambda^2(\mathbf{c}_1 - \mathbf{c}_{1b}^*)'\mathbf{X}_1{}'\mathbf{M}_Z\mathbf{X}_1(\mathbf{c}_1 - \mathbf{c}_{1b}^*) + \boldsymbol{\phi}'[\mathbf{V}_0 + \mathbf{Y}'\mathbf{M}_Z\mathbf{Y} - \mathbf{Y}'\mathbf{M}_Z\mathbf{X}_1(\mathbf{X}_1{}'\mathbf{M}_Z\mathbf{X}_1)^{-1}\mathbf{X}_1{}'\mathbf{M}_Z\mathbf{Y}]\boldsymbol{\phi} \tag{E.6}$$

together with

$$\mathbf{c}_{1a}^* = -(\mathbf{X}_1{}'\mathbf{M}_2\mathbf{X}_1)^{-1}\mathbf{X}_1{}'\mathbf{M}_2\mathbf{Y}\mathbf{b} \tag{E.7}$$

$$\mathbf{c}_{1b}^* = -\frac{1}{\lambda}(\mathbf{X}_1{}'\mathbf{M}_Z\mathbf{X}_1)^{-1}\mathbf{X}_1{}'\mathbf{M}_Z\mathbf{Y}\boldsymbol{\phi} \tag{E.8}$$

$$\boldsymbol{\phi} = \lambda\mathbf{b} - \mathbf{s}. \tag{E.9}$$

It follows that the conditional posterior density of $(\mathbf{c}_1|\beta, \lambda)$ is a product of two kernels of Student densities, i.e. a so-called 2-0 poly-$t$ density whose evaluation requires only a one-dimensional numerical integration on an auxiliary random variable. The analysis of the posterior density (E.4) then require altogether a tridimensional numerical integration. The numerical procedure we have just described has proved reliable and numerically efficient. It is now part of a Bayesian Regression Computer Program (BRP) developed at CORE.

We mention finally that, as discussed in Lubrano and Richard (1981), equally efficient numerical procedures apply to the case where the independent prior density $D(\beta, \mathbf{c})$ in (3.29) is replaced by a conditional prior density $D(\beta, \mathbf{c}|\sigma^2)$ where $\sigma^2$ is the variance of $u_t$, in the form of a conventional natural conjugate prior density for the parameters of the sole equation (3.16). The posterior density of $(\beta, \mathbf{c}, \lambda)$, as given in (E.1), then takes a simpler expression in that $D(\beta, \mathbf{c})$ is incorporated within $\sigma_*^2$ in the form of an additional quadratic term in (3.37) and a non-informative prior density on $\mathbf{c}$ is no longer required to obtain an expression similar to (E.4). We decided, however, against using such a conditional prior density which suffers the major drawback of imposing a spurious dependence between $(\beta, \mathbf{c})$ and $\sigma^2$.

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