Choice Between Disaggregate and Aggregate Specifications Estimated by Instrumental Variables Methods*

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Abstract

A choice criterion is proposed for discriminating between disaggregate and aggregate models estimated by the instrumental variables method. The criterion, based on prediction errors, represents a generalisation of criteria developed in the context of classical regression models. The article also derives general tests for aggregation bias in the instrumental variables context. The criterion and the tests are applied in an analysis of UK employment demand. It is shown that a model disaggregated by 40 industries predicts aggregate employment better than an aggregate model and that significant biases exist in estimates of the long-run wage and output elasticities obtained from the aggregate model.

Key Words: Aggregation; Instrumental variables; Labour demand; Model selection.
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The problem of aggregation over micro units has a long tradition in the econometrics literature stretching back to Theil (1954). Two issues in particular have attracted attention. The first concerns the prediction problem of choice between alternative disaggregate and aggregate specifications to predict aggregate variables. This issue was raised in the literature by Grunfeld and Griliches (1960) and reconsidered in a more general context by Pesaran et al. (1989) (henceforth PPK). The second issue concerns the problem of aggregation bias defined by the deviation of the macro parameters from the average of the corresponding micro parameters. This was first discussed by Theil (1954) and an indirect test proposed by Zellner (1962). Early empirical studies are reported by Boot and de Wit (1960), Orcutt et al. (1968), and Gupta (1971), for example, whereas more recent work includes that by Heckman and Sedlacek (1988), Keane et al. (1988), and contributions in Barker and Pesaran (1990). In the work of Lee et al. (1990a,b) some general direct tests were derived for the case where the subset of parameters of interest may be a (possibly nonlinear) function of the full vector of parameters.

This article reconsiders both of these issues in the context of models in which the assumption that model regressors and disturbances are uncorrelated cannot be maintained and, to obtain consistent parameter estimates, instrumental variables (IV) methods are used. This situation arises frequently in applied work either due to simultaneity or because expectations are replaced by their realisations under the rational expectations hypothesis in econometric equations. It also arises in models in which nonlinear relations (such as Euler equations) are derived as first-order conditions to optimisation problems at a microlevel (see for example Hansen (1982) and Hansen and Singleton (1982)). When regressors and disturbances are correlated, the usual criterion for choosing between models, the sum of squared residuals,

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is no longer an appropriate statistic in the sense that its use does not guarentee that the 'true' model will be chosen, even asymptotically. Since the criteria proposed by Grunfeld and Griliches (1960) and by PPK for choosing between alternative disaggregated and aggregated models are based on the sum of squared residuals, these criteria are also inappropriate in these circumstances. Statistics based on the prediction errors of alternative models that provide a valid model-selection criterion can be derived, however. Moreover, since the equation residuals coincide with the prediction errors in the least squares case, the criterion based on the prediction errors advanced in this article represents a generalisation of the criteria that have been considered previously in the literature.

In Section 1 of this article, new choice criteria are proposed for discriminating between disaggregate and aggregate specifications estimated by IV methods, and their validity in this context is established. In Section 2, the issue of aggregation bias is considered in the IV context. Here tests are derived that allow a statistical comparison to be made between different parameters of interest based on aggregate and disaggregate models in which the models are estimated using IV methods. Finally, in Section 3 the statistical tools that have been developed are applied in an analysis of employment demand for the UK economy. Labour-demand equations for 40 industrial sectors are estimated using the IV method and compared with their aggregate counterpart. It is established that the disaggregate model outperforms the aggregate model in terms of its ability to predict aggregate employment demand. Furthermore, key long-run elasticities of labour demand estimated by the aggregate and disaggregate models are shown to be significantly different, with elasticities based on the aggregate model overstating the extent of the responsiveness of labour demand to changes in wages and output when compared to estimated elasticities based on the disaggregate model.

1 A Choice Criterion under IV Estimation

Suppose we have a disaggregated multisectoral model, denoted H_d, consisting of m sectoral equations, where the dependent variable in the ith equation is $\mathbf{y}_i$, an $n \times 1$ vector of observations for the ith unit ($i = 1, \cdots, m$). We also have an aggregate model, denoted H_a, given by a single equation, the dependent variable of which is $\mathbf{y}_a = \sum_i \mathbf{y}_i$. Clearly, a disaggregate model can be used to address many questions that the aggregate model cannot. In this section, however, we assume that the primary focus of the analysis is the prediction of the aggregate variable $\mathbf{y}_a$ and consider the derivation of an appropriate selection criterion for choosing between the two models on this basis. This question was first addressed in the literature by Grunfeld and Griliches (1960), and a more general treatment was given by PPK. These works proposed selection criteria for choosing between disaggregate and aggregate models based on sums of squared residuals from the two models. The use of these selection criteria is justified on the grounds that, on average, their use would lead to the choice of the disaggregated model under the assumption that the micro equations are correctly specified. The use of the prediction criteria in the context of choice between models also has implications for model misspecification. When the disaggregate model fits worse than the aggregate model, this would indicate that the disaggregated model is misspecified. This suggests using a Durbin-Hausman type of misspecification test of the disaggregate model, and such a test is developed in the least squares context by Lee et al. (1990b)). A misspecification test of this type, however, serves a quite separate function to that served by the choice criterion. The way to think of the choice criterion is in situations in which an investigator is faced with two models, an aggregate and a disaggregate one, and *must* choose one of them for use in predicting the aggregate variable. The issues of model misspecification and aggregation errors were addressed in more detail by PPK, section 6.

The criteria proposed by Grunfeld and Griliches and by PPK are derived for models in which it could be assumed that regressors and disturbances are uncorrelated. In many instances, however, it is not reasonable to make this assumption, so ordinary least squares (OLS) estimation is no longer appropriate, and the IV estimation method is required to obtain consistent estimates. In these circumstances, the residual vectors obtained from the estimated model depend on the sign and magnitudes of the correlations between the dependent variable and the variables that are determined jointly with it. As a consequence, measures of goodness of fit that are based on the IV residuals cannot be guaranteed to choose a correct model even asymptotically, and the sum of squares of residuals is no longer an appropriate basis for developing model-selection criteria. (See Pesaran and Smith (1994) for further discussion of selection criteria appropriate for choice between models estimated by the IV method.)

In this section we consider alternative statistics, s_d^2 and s_a^2, relating to the disaggregate and aggregate models estimated by the IV method. These statistics are based on prediction errors, which are the appropriate measures for model comparison, and are not subject to the difficulties described previously.

Specifically, these statistics are shown to have the property that

$$\operatorname*{plim}_{n\to\infty}(s_d^2|H_d) \leq \operatorname*{plim}_{n\to\infty}(s_a^2|H_d),$$

where probability limits are taken under the hypothesis of the disaggregated model H_d, so that they are valid statistics for use in a choice rule. To this end, consider the general disaggregate model defined by

$$\begin{aligned} H_d: \quad \mathbf{y}_i &= \mathbf{X}_i\boldsymbol{\beta}_i + \mathbf{u}_i \quad i = 1,\ldots,m \\ \mathbf{X}_i &= \mathbf{Z}_i\boldsymbol{\Pi}_i + \mathbf{V}_i \end{aligned} \tag{1.1}$$

where $\mathbf{y}_i$ is the $n\times 1$ vector of observations on the dependent variable for the ith sector, $\mathbf{X}_i$ is the $n\times k_i$ matrix of observations on the regressors in (1.1) for the ith sector, assumed to have a full column rank, $\boldsymbol{\beta}_i$ is the $k_i\times 1$ vector of the coefficients associated with columns of $\mathbf{X}_i$, and $\mathbf{u}_i$ is the $n\times 1$ vector of disturbances for the ith sector. $\mathbf{Z}_i$ is a $n\times r_i$ matrix of IV's (where $r_i \geq k_i$), $\boldsymbol{\Pi}_i$ is an $r_i\times k_i$ matrix of parameters, and $\mathbf{V}_i$ is an $n\times k_i$ matrix of disturbances. The disturbances $\mathbf{u}_i$ and $\mathbf{V}_i$ are assumed to be serially uncorrelated within each sector but are contemporaneously correlated across sectors. Formally, the following standard assumptions are made:

Assumption A1: For all $i,j = 1,2,...,m$, the probability limits of $\mathbf{u}_i'\mathbf{u}_j/n$, $\mathbf{V}_i'\mathbf{V}_j/n$, and $\mathbf{V}_i'\mathbf{u}_j/n$ exist and are given by σ_{ij}, $\boldsymbol{\Sigma}_{ij}$, and $\boldsymbol{\delta}_{ij}$, respectively.

Assumption A2: For all $i,j = 1,2,...,m$, the instruments, $\mathbf{Z}_i$, are of full-column rank, and are asymptotically uncorrelated with the disturbances $\mathbf{u}_j$ and $\mathbf{V}_j$.

Assumption A3: For all $i,j = 1,2,...,m$, the matrices $\mathbf{Z}_i'\mathbf{X}_i/n$ and $\mathbf{Z}_i'\mathbf{Z}_j/n$ have finite probability limits, and the probability limits of $\mathbf{X}_i'\mathbf{X}_i/n$ and $\mathbf{Z}_i'\mathbf{Z}_i/n$ exist and are non-singular.

In general, the matrix $\mathbf{X}_i$ is correlated with $\mathbf{u}_i$ and may include lagged values of the dependent variable, $\mathbf{y}_i$, as well as current and lagged values of other endogenous variables. It is possible, however, that $\mathbf{X}_i$ includes some exogenous variables, in which case we assume that these variables also appear in $\mathbf{Z}_i$, so $\mathbf{V}_i$ and consequently $\boldsymbol{\Sigma}_{ii}$ will not be of full rank.

The aggregate model is given by

$$H_a: \qquad \mathbf{y}_a = \mathbf{X}_*\boldsymbol{\beta}_* + \mathbf{u}_* \tag{1.2}$$

where $\mathbf{y}_a = \sum_{i=1}^m \mathbf{y}_i$ and $\mathbf{X}_*$ is a $n\times k_*$ matrix of regressors, $\boldsymbol{\beta}_*$ is a $k_*\times 1$ vector of the coefficients associated with the columns of $\mathbf{X}_*$ and $\mathbf{u}_*$ is an $n\times 1$ vector of disturbances. It will also be assumed that:

Assumption A4: There exists a set of "aggregate" instruments, $\mathbf{Z}_*$, of full-column rank that are asymptotically uncorrelated with the disturbances $\mathbf{u}_i$ and $\mathbf{V}_i$, and for which the matrices $\mathbf{Z}_*'\mathbf{X}_i/n$ and $\mathbf{Z}_*'\mathbf{Z}_*/n$ have finite probability limits for $i = 1,2,...,m$.

No assumption is made in (1.2) about the relationship between $\mathbf{X}_*$ and the $\mathbf{X}_i$'s. Model (1.2) is to be viewed here as a rival model to (1.1) for the purpose of predicting $\mathbf{y}_a$ and has not *necessarily* been derived from (1.1) through any formal aggregation procedure. (See Section 2, however, on testing for aggregation bias.) Similarly, the instruments of the aggregate model, $\mathbf{Z}_*$, are not *necessarily* related to the disaggregated instrument sets, $\mathbf{Z}_i$, except insofar as by Assumption A4 they would also be valid instruments under H_d. This condition would be satisfied, for example, when the $\mathbf{Z}_*$'s are restricted to include lagged variables only.

Now consider the statistics for the aggregate and disaggregate models, based on the prediction errors, given by

$$s_a^2 = \widehat{\mathbf{e}}_a'\widehat{\mathbf{e}}_a/n \tag{1.3}$$

and

$$s_d^2 = \widehat{\mathbf{e}}_d'\widehat{\mathbf{e}}_d/n, \tag{1.4}$$

respectively, where

$$\widehat{\mathbf{e}}_d = \sum_{i=1}^m \widehat{\mathbf{e}}_i \tag{1.5}$$

and where $\widehat{\mathbf{e}}_a = \mathbf{y}_a - \widehat{\mathbf{X}}_*\widetilde{\boldsymbol{\beta}}_*$, $\widetilde{\boldsymbol{\beta}}_* = (\mathbf{X}_*'\mathbf{P}_*\mathbf{X}_*)^{-1}\mathbf{X}_*'\mathbf{P}_*\mathbf{y}_a$, $\widehat{\mathbf{X}}_* = \mathbf{P}_*\mathbf{X}_*$, $\mathbf{P}_* = \mathbf{Z}_*(\mathbf{Z}_*'\mathbf{Z}_*)^{-1}\mathbf{Z}_*'$, and $\widehat{\mathbf{e}}_i = \mathbf{y}_i - \widehat{\mathbf{X}}_i\widehat{\mathbf{X}}e\boldsymbol{\beta}_i$, $\widetilde{\boldsymbol{\beta}}_i = (\mathbf{X}_i'\mathbf{P}_i\mathbf{X}_i)^{-1}\mathbf{X}_i'\mathbf{P}_i\mathbf{y}_i$, $\widehat{\mathbf{X}}_i = \mathbf{P}_i\mathbf{X}_i$, $\mathbf{P}_i = \mathbf{Z}_i(\mathbf{Z}_i'\mathbf{Z}_i)^{-1}\mathbf{Z}_i'$. The estimators $\widetilde{\boldsymbol{\beta}}_*$ and $\widetilde{\boldsymbol{\beta}}_i$ are the generalised IV estimators of the parameters of the aggregate and disaggregate models, respectively. These are consistent IV estimators although for the disaggregate model they are not fully efficient since they do not take into account the contemporaneous covariances between sectors characterised by the nonzero off-diagonal elements in σ_{ij} in Assumption A1. Clearly, the prediction errors of the two models, $\widehat{\mathbf{e}}_a$ and $\widehat{\mathbf{e}}_d$, are different from the usual single-equation residuals, $\mathbf{e}_a = \mathbf{y}_a - \mathbf{X}_*\widetilde{\boldsymbol{\beta}}_*$ and $\mathbf{e}_i = \mathbf{y}_i - \mathbf{X}_i\widetilde{\boldsymbol{\beta}}_i$ because they account for the fact that the regressors $\mathbf{X}_*$ and $\mathbf{X}_i$ are stochastic variables, which, for prediction, must be replaced by their predicted values $\widehat{\mathbf{X}}_*$ and $\widehat{\mathbf{X}}_i$ respectively. The two coincide only in a fixed regressor framework, where $\mathbf{V}_i = 0$ and where OLS is an appropriate estimator. From this perspective, (1.3) and (1.4) can be viewed as an obvious generalisation of the sum of squared residuals criterion proposed for OLS models by Grunfeld and Griliches (1960) and by PPK.

We now show that the statistics (1.3) and 1.4) have the desirable property that

$$\operatorname*{plim}_{n\to\infty}(s_d^2|H_d) \le \operatorname*{plim}_{n\to\infty}(s_a^2|H_d).$$

First note that

$$\widetilde{\boldsymbol{\beta}}_i = (\widehat{\mathbf{X}}_i'\widehat{\mathbf{X}}_i)^{-1}\widehat{\mathbf{X}}_i'\mathbf{y}_i \tag{1.6}$$

and

$$\widetilde{\boldsymbol{\beta}}_* = (\widehat{\mathbf{X}}_*'\widehat{\mathbf{X}}_*)^{-1}\widehat{\mathbf{X}}_*'\mathbf{y}_a. \tag{1.7}$$

Then we can write $\widehat{\mathbf{e}}_i = (\mathbf{I} - \widehat{\mathbf{Q}}_i)\mathbf{y}_i$, where $\widehat{\mathbf{Q}}_i = \widehat{\mathbf{X}}_i'(\widehat{\mathbf{X}}_i'\widehat{\mathbf{X}}_i)^{-1}\widehat{\mathbf{X}}_i'$. Hence, substituting from (1.1), $\widehat{\mathbf{e}}_i = (\mathbf{I} - \widehat{\mathbf{Q}}_i)\mathbf{X}_i\boldsymbol{\beta}_i + (\mathbf{I} - \widehat{\mathbf{Q}}_i)\mathbf{u}_i$, and, since $\widehat{\mathbf{X}}_i'\mathbf{X}_i = \widehat{\mathbf{X}}_i'\widehat{\mathbf{X}}_i$ and $\widehat{\mathbf{Q}}_i\widehat{\mathbf{X}}_i = \widehat{\mathbf{X}}_i$, we have

$$\widehat{\mathbf{e}}_i = (\mathbf{X}_i - \widehat{\mathbf{X}}_i)\boldsymbol{\beta}_i + (\mathbf{I} - \mathbf{Q}_i)\mathbf{u}_i. \tag{1.8}$$

However,

$$\begin{aligned}\mathbf{X}_i - \widehat{\mathbf{X}}_i &= \mathbf{X}_i - \mathbf{Z}_i(\mathbf{Z}_i'\mathbf{Z}_i)^{-1}\mathbf{Z}_i'\mathbf{X}_i \\ &= (\mathbf{I} - \mathbf{P}_i)\mathbf{X}_i \\ &= (\mathbf{I} - \mathbf{P}_i)(\mathbf{Z}_i\boldsymbol{\Pi}_i + \mathbf{V}_i) \\ &= (\mathbf{I} - \mathbf{P}_i)\mathbf{V}_i.\end{aligned}$$

Hence, (1.8) can be rewritten as

$$\widehat{\mathbf{e}}_i = (\mathbf{I} - \mathbf{P}_i)\mathbf{V}_i\boldsymbol{\beta}_i + (\mathbf{I} - \widehat{\mathbf{Q}}_i)\mathbf{u}_i \tag{1.9}$$

so that

$$\widehat{\mathbf{e}}_d = \sum_i [(\mathbf{I} - \mathbf{P}_i)V_i\boldsymbol{\beta}_i + (\mathbf{I} - \widehat{\mathbf{Q}}_i)\mathbf{u}_i]$$

and

$$\begin{aligned}\widehat{\mathbf{e}}_d'\widehat{\mathbf{e}}_d = &\sum_{i,j} \boldsymbol{\beta}_i'\mathbf{V}_i'(\mathbf{I} - \mathbf{P}_i)(\mathbf{I} - \mathbf{P}_j)\mathbf{V}_j\boldsymbol{\beta}_j \\ &+ \sum_{i,j} \mathbf{u}_i'(\mathbf{I} - \widehat{\mathbf{Q}}_i)(\mathbf{I} - \widehat{\mathbf{Q}}_j)\mathbf{u}_j \\ &+ \sum_{i,j} \boldsymbol{\beta}_i'\mathbf{V}_i'(\mathbf{I} - \mathbf{P}_i)(\mathbf{I} - \widehat{\mathbf{Q}}_j)\mathbf{u}_j \\ &+ \sum_{i,j} \mathbf{u}_i'(\mathbf{I} - \widehat{\mathbf{Q}}_i)(\mathbf{I} - \mathbf{P}_j)\mathbf{V}_j\boldsymbol{\beta}_j.\end{aligned} \tag{1.10}$$

But, under Assumptions A1–A3,

$$\begin{aligned}\operatorname*{plim}_{n\to\infty}\left(\frac{\widehat{\mathbf{e}}_d'\widehat{\mathbf{e}}_d}{n}|H_d\right) &= \sum_{i,j}\boldsymbol{\beta}_i'\boldsymbol{\Sigma}_{ij}\boldsymbol{\beta}_j + \sum_{i,j}\sigma_{ij} + \sum_{i,j}\boldsymbol{\beta}_i'\boldsymbol{\delta}_{ij} + \sum_{i,j}\boldsymbol{\delta}_{ji}'\boldsymbol{\beta}_j \\ &= \mathrm{E}\left\{\left[\sum_i(\mathbf{u}_i + \mathbf{V}_i\boldsymbol{\beta}_i)\right]'\left[\sum_i \mathbf{u}_i + \mathbf{V}_i\boldsymbol{\beta}_i\right]\right\} \\ &= \mathrm{E}(\boldsymbol{\xi}_a'\boldsymbol{\xi}_a) > 0,\end{aligned} \tag{1.11}$$

where $\boldsymbol{\xi}_a = \sum_i \boldsymbol{\xi}_i$ is the vector of aggregate errors of the reduced form equations

$$\begin{aligned} \mathbf{y}_i &= \mathbf{Z}_i \boldsymbol{\Pi}_i \boldsymbol{\beta}_i + \mathbf{V}_i \boldsymbol{\beta}_i + \mathbf{u}_i \\ &= \mathbf{Z}_i \boldsymbol{\Pi}_i \boldsymbol{\beta}_i + \boldsymbol{\xi}_i \end{aligned} \tag{1.12}$$

and $\boldsymbol{\xi}_i = \mathbf{V}_i \boldsymbol{\beta}_i + \mathbf{u}_i$.

Consider now the aggregate prediction criterion. We have $\widehat{\mathbf{e}}_a = \mathbf{y}_a - \widehat{\mathbf{X}}_* \widetilde{\boldsymbol{\beta}}_*$, and, under H_d,

$$\widehat{\mathbf{e}}_a = \left(\mathbf{I} - \widehat{\mathbf{Q}}_*\right) \left(\sum_i \mathbf{X}_i \boldsymbol{\beta}_i + \mathbf{u}_i\right), \tag{1.13}$$

where

$$\begin{aligned} \widehat{\mathbf{Q}}_* &= \widehat{\mathbf{X}}_* (\widehat{\mathbf{X}}_*' \widehat{\mathbf{X}}_*)^{-1} \widehat{\mathbf{X}}_*' \\ &= \mathbf{P}_* \mathbf{X}_* (\mathbf{X}_*' \mathbf{P}_* \mathbf{X}_*)^{-1} \mathbf{X}_*' \mathbf{P}_*. \end{aligned}$$

Substituting from (1.12),

$$\begin{aligned} \widehat{\mathbf{e}}_a &= \left(\mathbf{I} - \widehat{\mathbf{Q}}_*\right) \left[\sum_i \mathbf{Z}_i \boldsymbol{\Pi}_i \boldsymbol{\beta}_i + \sum_i (\mathbf{u}_i + \mathbf{V}_i \boldsymbol{\beta}_i)\right] \\ &= \left(\mathbf{I} - \widehat{\mathbf{Q}}_*\right) (\mathbf{f}_a + \boldsymbol{\xi}_a) \end{aligned}$$

so that, taking probability limits under H_d,

$$\begin{aligned} \operatorname*{plim}_{n\to\infty} \left(\frac{\widehat{\mathbf{e}}_a' \widehat{\mathbf{e}}_a}{n} | H_d\right) &= \operatorname*{plim}_{n\to\infty} \left(\frac{\mathbf{f}_a' [\mathbf{I} - \widehat{\mathbf{Q}}_*] \mathbf{f}_a}{n}\right) + \operatorname*{plim}_{n\to\infty} \left(\frac{\boldsymbol{\xi}_a' [\mathbf{I} - \widehat{\mathbf{Q}}_*] \boldsymbol{\xi}_a}{n}\right) \\ &\quad + 2 \operatorname*{plim}_{n\to\infty} \left(\frac{\mathbf{f}_a' [\mathbf{I} - \widehat{\mathbf{Q}}_*] \boldsymbol{\xi}_a}{n}\right). \end{aligned} \tag{1.14}$$

But, since $\mathbf{u}_i$ and $\mathbf{V}_i$ are asymptotically distributed independently of $\mathbf{Z}_*$, by Assumption A4, it follows that

$$\begin{aligned} \operatorname*{plim}_{n\to\infty} \left(\frac{\boldsymbol{\xi}_a' [\mathbf{I} - \widehat{bfQ}_*] \boldsymbol{\xi}_a}{n}\right) &= \operatorname*{plim}_{n\to\infty} \left(\frac{\boldsymbol{\xi}_a' \boldsymbol{\xi}_a}{n}\right) \\ &= E(\boldsymbol{\xi}_a' \boldsymbol{\xi}_a) \end{aligned}$$

and

$$\operatorname*{plim}_{n\to\infty} \left(\frac{\mathbf{f}_a' [\mathbf{I} - \widehat{\mathbf{Q}}_*] \boldsymbol{\xi}_a}{n}\right) = 0.$$

Hence,

$$\begin{aligned} \operatorname*{plim}_{n\to\infty} \left(\frac{\widehat{\mathbf{e}}_a' \widehat{\mathbf{e}}_a}{n} | H_d\right) &= E(\boldsymbol{\xi}_a' \boldsymbol{\xi}_a) + \operatorname*{plim}_{n\to\infty} \left(\frac{\mathbf{f}_a' [\mathbf{I} - \widehat{\mathbf{Q}}_*] \mathbf{f}_a}{n}\right) \\ &\geq E(\boldsymbol{\xi}_a' \boldsymbol{\xi}_a), \end{aligned} \tag{1.15}$$

where the inequality follows because the second term in (1.15), namely

$$\operatorname*{plim}_{n\to\infty} \left(\frac{\mathbf{f}_a' [\mathbf{I} - \widehat{\mathbf{Q}}_*] \mathbf{f}_a}{n}\right),$$

is a positive semidefinite quadratic form. Comparing (1.15) with (1.11) establishes the result that

$$\operatorname*{plim}_{n\to\infty} \left(\frac{\widehat{\mathbf{e}}_d' \widehat{\mathbf{e}}_d}{n} | H_d\right) \leq \operatorname*{plim}_{n\to\infty} \left(\frac{\widehat{\mathbf{e}}_a' \widehat{\mathbf{e}}_a}{n} | H_d\right). \tag{1.16}$$

In general we have not made any assumptions about the relationship between the disaggregate model H_d and the aggregate model H_a. It is interesting, however, to look at the special case where the aggregate model has been derived from a formal aggregation of the disaggregate model so that $\mathbf{X}_* = \mathbf{X}_a = \sum_i \mathbf{X}_i$ and $\mathbf{Z}_* = \mathbf{Z}_a = \sum_i \mathbf{Z}_i$. In this case the best that the aggregate model can do is to predict as well as the disaggregate model so that the two criteria coincide, and the conditions under which this will occur are the conditions for perfect aggregation, discussed for the least squares case by PPK. In the fixed regressor context of PPK, it is well known that sufficient conditions are when either $\boldsymbol{\beta}_i = \boldsymbol{\beta}$, for all i, $i = 1, ..., m$ (the *microhomogeneity* hypothesis) or when $\mathbf{X}_i = \mathbf{X}_a \mathbf{\Lambda}_i$ for all i (the *compositional-stability* hypothesis) where $\mathbf{\Lambda}_i$ are square full-rank matrices satisfying $\sum_i \mathbf{\Lambda}_i = \mathbf{I}$. (See also Lewbel (1992) for the application of a stochastic version of the compositional-stability hypothesis in the context of aggregating log-linear microequations.) In the present IV framework, however, where there is more than one variable determined simultaneously, these two conditions are no longer sufficient to achieve perfect aggregation, and an additional condition on the $\mathbf{Z}_i$'s is also needed. One such condition is given by

$$\mathbf{Z}_i = \mathbf{Z}_a \mathbf{\Gamma}_i \tag{1.17}$$

for all i, $i = 1, ..., m$ where $\mathbf{\Gamma}_i$ are square full-rank matrices of fixed coefficients. This condition ensures that $\widehat{\mathbf{X}}_i = \mathbf{P}_a \mathbf{X}_i$, where $\mathbf{P}_a = \mathbf{Z}_a(\mathbf{Z}_a'\mathbf{Z}_a)^{-1}\mathbf{Z}_a'$, and, together with either the microhomogeneity hypothesis or the compositional-stability hypothesis, it is sufficient to ensure that $\widehat{\mathbf{e}}_d = \widehat{\mathbf{e}}_a$ so that disaggregate and aggregate criteria coincide. Condition (1.17) is a compositional stability hypothesis for the IV's of the disaggregate model. Clearly a special case is where $\mathbf{\Gamma}_i = \mathbf{I}$ for all i which is where a common set of instruments is used across all sectors.

The prediction criteria (1.3) and 1.4) can be modified to incorporate degrees-of-freedom corrections. Clearly, such corrections will not affect the asymptotic properties of the statistics but we conjecture that they might improve their performance in finite samples. PPK derived corrections that ensured unbiasedness of the criteria in the least squares context. While no formal proof can be given in the present context, by analogy, we suggest using similar correction factors. This has the advantage of ensuring consistency with the criteria of PPK in the limiting case where $\mathbf{X}_i \subset \mathbf{Z}_i$ and $\mathbf{X}_* \subset \mathbf{Z}_*$ and the models (1.3) and (1.4) collapse to the fixed regressor models considered by PPK. Hence, the following modified criteria are suggested:

$$s_a^2 = \widehat{\mathbf{e}}_a'\widehat{\mathbf{e}}_a/(n - k_*) \tag{1.18}$$

and

$$s_d^2 = \sum_i \sum_j \widehat{\mathbf{e}}_i'\widehat{\mathbf{e}}_j/\{n - k_i - k_j + \mathrm{tr}(\widehat{\mathbf{Q}}_i\widehat{\mathbf{Q}}_j)\}, \tag{1.19}$$

where, as before, $\widehat{\mathbf{Q}}_i = \widehat{\mathbf{X}}_i(\widehat{\mathbf{X}}_i'\widehat{\mathbf{X}}_i)^{-1}\widehat{\mathbf{X}}_i'$.

2 Testing for Aggregation Bias under IV Estimation

Another important aspect in the comparison of aggregate and disaggregate models is the issue of *aggregation bias*. This concept was originally formalised by Theil (1954), who defined aggregation bias as the deviations of the parameters of a *macro* equation from the average of the corresponding parameters of the *micro* equations. Other definitions of aggregation bias are also used in the literature. For example, in his analysis of aggregating log-linear relations with fixed slope coefficients, Lewbel (1992) defined aggregation bias as the percentage difference between the common slope coefficient of the micro relations and the probability limit of the slope coefficient in the analogue aggregate equation and showed that this bias depends on the extent of the dependence between the regressors and the disturbances of the aggregate model. In the context of our application, where the micro equations are linear but have *different* slope coefficients, an adaptation of Lewbel's condition for no aggregation bias requires the micro coefficients, $\boldsymbol{\beta}_i$, to be distributed with a common mean, $\boldsymbol{\beta}_a$, such that $\boldsymbol{\beta}_i - \boldsymbol{\beta}_a$ are distributed independently of the regressors in all the micro equations. This condition yields the familiar random-coefficients model discussed by Zellner (1969). The condition that $\boldsymbol{\beta}_i - \boldsymbol{\beta}_a$ and the regressors of the micro equations are independently distributed is not, however, likely to be satisfied if the micro equations contain lagged dependent variables. (On this, see Pesaran and Smith (1992)). In general, however, where the slope coefficients differ across the micro equations, Theil's definition will still be appropriate for dynamic models, and will therefore be adopted in the rest of the article.

Here we generalise the tests for aggregation bias derived by Lee et al. (1990a,b) to the case where the macro and micro models are estimated by the IV method. In the application of the tests of aggregation bias, it is only meaningful to consider the case where the macro model is defined to be an analogue of the micro relations (1.1), given by

$$H_a: \qquad \mathbf{y}_a = \mathbf{X}_a\beta_a + \mathbf{u}, \tag{1.2´}$$

where $\mathbf{X}_a = \sum_{i=1}^m \mathbf{X}_i$. Here, the coefficients β_a can be interpreted as the 'average' counterparts of β_i. Such an interpretation of (1.2′) arises naturally in the case of random-coefficients models mentioned previously. The familiar method of testing for aggregation bias in the context of the micro relations (1.1) is to test directly the micro homogeneity hypothesis—namely, $H_\beta : \beta_1 = \beta_2 = \cdots = \beta_m$. An alternative approach, which is less restrictive, would be to test the equality of β_a from the macro equation with the average of the coefficients of the micro equations—namely,

$$H_0 : \eta_\beta = \beta_a - m^{-1} \sum_{i=1}^{m} \beta_i = 0. \tag{2.1}$$

Clearly H_0 implies H_β, but not *vice versa*. In what follows, we focus on tests of H_0 and its generalisation (which was discussed in detail by Lee et al. (1990b)), when the micro and macro equations are estimated by the IV method. The generalisation of H_0 covers situations in which the parameters of interest are (possibly nonlinear) functions of the micro parameters and their macro counterparts. In this general case, the hypothesis of no aggregation bias may be defined as

$$H_0 : \widehat{\mathbf{e}}ta_h = \mathbf{g}(\boldsymbol{\beta}_a) - \mathbf{h}(\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_m) = 0, \tag{2.2}$$

where $\mathbf{h}$ and $\mathbf{g}$ are assumed to be continuous and differentiable vector functions of dimension s, and where $\mathbf{g}(\boldsymbol{\beta}_a) = \mathbf{h}(\boldsymbol{\beta}_a, \ldots, \boldsymbol{\beta}_a)$. This formulation includes the hypothesis expressed at (2.1) as a special case and also allows the possibility of defining bias as the deviation of a function of the macro parameters from an average of the same function of the micro parameters or from a function of the average of the micro parameters or some other general form. In all cases, the null hypothesis that there is no aggregation bias would not be rejected under the micro homogeneity hypothesis H_β. On the other hand, it would be possible that no evidence of aggregation bias is found even when micro homogeneity does not hold, so that testing H_0 provides a less restrictive test for the presence of aggregation bias than the familiar test of the micro homogeneity hypothesis H_β. (Clearly this approach to testing for the presence of aggregation errors is distinct from that based on tests of mispecification in an aggregate model in which measures of distributional effects, calculated across the micro units, are employed (e.g., see Stoker (1986)).)

Two test statistics are derived corresponding to two different assumptions about the vector of *macro* parameters $\boldsymbol{\beta}_a$. First assume that $\boldsymbol{\beta}_a$ is a vector of known parameters given *a priori* from some 'consensus' view, for example. A test statistic can be constructed based on the vector

$$\widetilde{\boldsymbol{\eta}}_h = \mathbf{g}(\boldsymbol{\beta}_a) - \mathbf{h}(\widetilde{\boldsymbol{\beta}}_1, \ldots, \widetilde{\boldsymbol{\beta}}_m). \tag{2.3}$$

On the null hypothesis H_0:

$$\operatorname*{plim}_{n\to\infty} \widetilde{\boldsymbol{\eta}}_h = \boldsymbol{\eta}_h = 0, \tag{2.4}$$

and

$$\widehat{\text{Avar}}(\widetilde{\boldsymbol{\eta}}_h) = \sum_{i=1}^{m} \sum_{j=1}^{m} \widetilde{\mathbf{H}}_i \widehat{\text{Avar}}(\widetilde{\boldsymbol{\beta}}_i, \widetilde{\boldsymbol{\beta}}_j) \widetilde{\mathbf{H}}_j' = \widetilde{\boldsymbol{\Omega}}_n, \tag{2.5}$$

where $\widetilde{\mathbf{H}}_i = \partial \mathbf{h} / \partial \widetilde{\boldsymbol{\beta}}_i'$, and the variance-covariance matrix of $\boldsymbol{\beta}_i$ in model (1.1) is estimated consistently by $\widehat{\text{Avar}}(\widetilde{\boldsymbol{\beta}}_i, \widetilde{\boldsymbol{\beta}}_j) = \widetilde{\sigma}_{ij} (\widehat{\mathbf{X}}_i' \widehat{\mathbf{X}}_i)^{-1} \widehat{\mathbf{X}}_i' \widehat{\mathbf{X}}_j (\widehat{\mathbf{X}}_j' \widehat{\mathbf{X}}_j)^{-1}$, where $\widetilde{\sigma}_{ij}$ is any consistent estimator of σ_{ij}. Then the test statistic for the hypothesis (2.2) is given by

$$q_1^* = \widetilde{\boldsymbol{\eta}}_h' \widetilde{\boldsymbol{\Omega}}_n \widetilde{\boldsymbol{\eta}}_h, \tag{2.6}$$

and on the null hypothesis, H_0, $q_1^* \overset{a}{\sim} \chi_s^2$.

Second, consider the case in which there is no consensus view on $\boldsymbol{\beta}_a$, so that, instead of being given *a priori*, the parameter vector $\boldsymbol{\beta}_a$ is estimated from the aggregate model (1.2′). From Assumption A4,

$$\operatorname*{plim}_{n\to\infty} (\widetilde{\boldsymbol{\beta}}_a | H_d) = \sum_{i=1}^{m} \mathbf{C}_i \boldsymbol{\beta}_i, \tag{2.7}$$

where

$$\mathbf{C}_i = \operatorname*{plim}_{n\to\infty} \{ (\widehat{\mathbf{X}}_a' \widehat{\mathbf{X}}_a / n)^{-1} (\widehat{\mathbf{X}}_a' \mathbf{X}_i / n) \}.$$

In this case, a test of (2.2) can be based on the vector

$$\widetilde{\boldsymbol{\eta}}_h = \mathbf{g}(\widetilde{\boldsymbol{\beta}}_a) - \mathbf{h}(\widetilde{\boldsymbol{\beta}}_1, \ldots, \widetilde{\boldsymbol{\beta}}_m). \tag{2.8}$$

On the null hypothesis of no aggregation bias, H_0,

$$\operatorname*{plim}_{n\to\infty}(\widetilde{\boldsymbol{\eta}}_h|H_d) = \mathbf{g}\left(\sum_{i=1}^{m}\mathbf{C}_i\boldsymbol{\beta}_i\right) - \mathbf{h}(\boldsymbol{\beta}_1,\ldots,\boldsymbol{\beta}_m) = 0. \tag{2.9}$$

The test statistic for this case is given by

$$q_2^* = (\mathbf{g}(\widetilde{\boldsymbol{\beta}}_a) - \mathbf{h}(\widetilde{\boldsymbol{\beta}}_1,\ldots,\widetilde{\boldsymbol{\beta}}_m))'\widetilde{\boldsymbol{\Phi}}_n^{-1}(\mathbf{g}(\widetilde{\boldsymbol{\beta}}_a) - \mathbf{h}(\widetilde{\boldsymbol{\beta}}_1,\ldots,\widetilde{\boldsymbol{\beta}}_m)), \tag{2.10}$$

where

$$\widetilde{\boldsymbol{\Phi}}_n = \sum_{i=1}^{m}\sum_{j=1}^{m}\hat{\sigma}_{ij}\widetilde{\boldsymbol{\Psi}}_i\widetilde{\boldsymbol{\Psi}}_j', \tag{2.11}$$

and the matrix $\widetilde{\boldsymbol{\Psi}}_i$ (which corresponds to equation (21) of Lee et al. (1990b)) is defined by

$$\widetilde{\boldsymbol{\Psi}}_i = \widetilde{\mathbf{G}}_a(\widehat{\mathbf{X}}_a'\widehat{\mathbf{X}}_a)^{-1}\widehat{\mathbf{X}}_a' - \widetilde{\mathbf{H}}_i(\widehat{\mathbf{X}}_i'\widehat{\mathbf{X}}_i)^{-1}\widehat{\mathbf{X}}_i', \tag{2.12}$$

where $\widetilde{\mathbf{G}}_a = \partial\mathbf{g}/\partial\widetilde{\boldsymbol{\beta}}_a'$. On the null hypothesis, H_0, $q_2^* \overset{a}{\sim} \chi_s^2$.

3 An Application to Sectoral Labour-Demand Determination

In this section, the statistics that have been developed are applied to aggregate and sectorally disaggregated labour demand functions for the UK economy. This is an area of research that has received considerable attention recently as economists have attempted to understand and explain the causes of the historically high unemployment levels experienced recently in the United Kindom and elsewhere (e.g., see Layard et al. (1991), and the references therein). In particular, much applied research has focused on the responsiveness of labour demand to changes in real wages and in output levels in an effort to evaluate the efficacy of different policies designed to reduce unemployment. Much of this analysis, however, has been carried out using aggregate data, and it is of some interest to consider whether conclusions drawn on the basis of this work are affected by the choice of the level of aggregation used in the analysis.

PPK and Lee et al. (1990b) investigated this question empirically, using annual data for 40 industrial sectors over the period 1956-1984. The data cover the whole of the private sector, excluding the mineral oil and natural gas sector (sector 4) for which data are available only since 1971 when North Sea oil production started to come on line. (Full details of sources and definitions can be found in the data appendix to PPK.) In these works, the following general log-linear dynamic specifications for the sectoral labour demand equations were adopted:

$$\begin{aligned} LE_{it} &= \beta_{i1}/m + \beta_{i2}(T_t/m) + \beta_{i3}LE_{i,t-1} + \beta_{i4}LE_{i,t-2} + \beta_{i5}LY_{it} + \beta_{i6}LY_{i,t-1} + \beta_{i7}LW_{it} \\ &\quad + \beta_{i8}LW_{i,t-1} + \beta_{i9}\overline{LY}_{at} + \beta_{i10}\overline{LY}_{a,t-1} + u_{it}, \\ &\qquad i = 1,2,3,5,6,\ldots,41, \quad t = 1956,\ldots,1984, \end{aligned} \tag{3.1}$$

where LE_{it} = log of man-hours employed in sector i at time t, T_t = time trend ($T_{1980} = 0$), LY_{it} = log of sector i output at time t, LW_{it} = log of average product real wage rate per man-hours employed in sector i at time t, $\overline{LY}_{at}$ = average of LY_{it} over the 40 sectors and m = number of sectors, ($m = 40$). This specification can be justified theoretically when employment decisions are made within an industry by cost minimising firms with identical production functions and the same given demand and factor price expectations. The inclusion of lagged employment variables can be justified on the grounds of inertia in revision of expectations, adjustment costs involved in hiring and firing of workers, or aggregation over different labour types (see, for example, Nickell (1984) and Pesaran (1991)). The variable $\overline{LY}_{at}$, which measures the level of aggregate output (in logs), is a proxy measure intended to capture changes in demand expectations arising from the perceived interdependence of demand in the economy by the firms in the industry. The time trend is included in the specification in order to allow for the effect of neutral technical progress on labour productivity.

OLS estimates of the disaggregated model in (3.1), and a restricted version of the model (in which linear parameters restrictions are imposed as a means of avoiding over-parameterisation), were presented in tables I and II of PPK. Using these, evidence is found to suggest that a disaggregate model is superior to its aggregate counterpart in terms of its ability to predict fluctuations in aggregate labour demand and that statistically significant differences exist between estimates of labour-demand elasticities obtained from the estimated aggregate and disaggregated models. In many models of supply-side behaviour, however, it is acknowledged that employment, wage, price, and output levels are determined simultaneously. Furthermore, in these circumstances, it is not clear how aggregate output levels, themselves an aggregation of the outcomes of sectoral output decisions, could be known with certainty prior to the time when sectoral employment decisions are made. Consequently, it might be argued that all of the current-dated explanatory variables in (3.1) are potentially correlated with the u_{it}, and that instruments for these variables are required. It is important, therefore, that we establish whether the previous findings are robust to the choice of estimation method, and to this end we have reestimated both the aggregate and disaggregate models using the IV method, and employed the techniques developed in the preceding sections to evaluate their relative performance.

As a first step in the empirical work of this paper, we estimated model (3.1) by the generalised IV method using the instruments $Z_{it} = \{1,\, T_t,\, LE_{i,t-1},\, LE_{i,t-2},\, LW_{i,t-1},\, LW_{i,t-2},\, LY_{i,t-1},\, LY_{i,t-2},\, \overline{LE}_{a,t-1},\, \overline{LE}_{a,t-2},\, \overline{LW}_{a,t-1},\, \overline{LW}_{a,t-2},\, \overline{LY}_{a,t-1},\, \overline{LY}_{a,t-2}\}$. This choice of instruments is a natural one given the preceding discussion; the simultaneity of the employment-, price-, output-, and wage-setting decisions in each sector, and the possibility of intersectoral interdependencies, exerted directly through product or labour-market competition, or indirectly through the expectations-formation process, means that lagged sectoral and aggregate variables are likely to provide valid instruments for the current-valued explanatory variables in (3.1). Moreover, it is important to be as comprehensive as possible in the choice of instruments for the sectoral regressions; if for any sector i, the included instruments are only a subset of those variables that determine $\mathbf{X}_i$ in model (1.1), then the assumed independence of the $\mathbf{Z}_i$ and the $\mathbf{V}_j$ is likely to be violated for $i \neq j$. Similar comments are likely to be true for the assumed independence of the $\mathbf{Z}_*$ and the $\mathbf{V}_i$ if the $\mathbf{Z}_*$ include aggregated values of the $\mathbf{Z}_i$. The second step in the empirical study was to calculate the Wu (1973) T_2 statistic, also known as the Wu-Hausman statistic, for each of the sectoral equations to test for the exogeneity of the current-dated explanatory variables in (3.1), and to investigate the relevence of the IV estimation method in this context. In those sectors in which the null of exogeneity was not rejected, we reestimated the labour-demand equations by the OLS method. Finally, for each sector, we undertook a specification search in which variables with t ratios that were less than unity (in absolute terms) were dropped from the list of explanatory variables to obtain a more parsimonious set of employment equations. At each stage of the specification search, a joint test of the parameter restrictions and a test of the exogeneity of the regressors were also carried out. In the case of industries where the exogeneity hypothesis was not rejected, the employment equations were estimated by OLS.

The estimates of the sectoral labour-demand equations obtained through this procedure are given in Table 1 and Table 2 provides some of the associated summary and diagnostic statistics. Included also in Table 2, in the columns headed $\chi^2_{MS}(4)$ and $F_{WH}(3,16)$, are the Sargan (1964) general misspecification test statistics and Wu-Hausman test statistics, respectively, carried out on the (unreported) unrestricted versions of the equations in (3.1). The Sargan test statistics serve as a general misspecification test of the joint validity of the model specification and the instruments, and are below their 95% critical values in all sectors. Turning to the Wu-Hausman test results, note that, conservatively working at the 10% level of significance, these statistics suggest the rejection of the exogeneity hypothesis in 6 of the 40 industries—namely, mechanical engineering (15), office goods (16), electrical engineering (17), rubber goods (31), hotels and catering (35), and communications (39). Furthermore, in the course of the specification search procedure, exogeneity of regressors in the restricted version of the labour-demand equation for the office goods sector could not be rejected either. Consequently, in all but five industries the parameter estimates reported in Table 1 are obtained by OLS and are equivalent to those in table II of PPK (in which OLS methods were employed throughout). In these five industries, however, exogeneity cannot be assumed to hold, and the IV estimation method has been employed; Wu-Hausman statistics for the test of the exogeneity of regressors in the restricted regressions reported in Table 1 for sectors 15, 17, 31, 35, and 39 were 3.14 (2,18), 11.87 (2,22), 3.39 (1,23), 8.83 (2,19), and 18.39 (2,21), respectively, where the relevent degrees of freedom of the F distribution are given in parentheses.

The parameter estimates presented in Table 1 are generally of the expected sign and, following the specification search, are generally well determined. In particular, it is worth noting that a second lagged dependent variable is included in 17 of the 40 industrial equations, and its coefficient takes a negative sign, as suggested by the theory, in all cases in which the coefficient is statistically different from 0 (see Pesaran (1991)). The need to include a variable to capture the effects of changes in demand expectations arising from interdependencies in the economy is confirmed by the presence of aggregate output terms in 19 of the 40 sectors. And the signs of the coefficients on the wage and output terms are generally as expected: the sum of the coefficients on current and lagged wage terms is negative in 31 of the sectors

Table 1: Disaggregate Labour-Demand Functions (restricted) 1956–1984

Industry	$c/40$	$T/40$	LY_{it}	$LY_{i,t-1}$	$LE_{i,t-1}$	$LE_{i,t-2}$	LW_{it}	$LW_{i,t-1}$	$\overline{LY}_{at}$	$\overline{LY}_{a,t-1}$
1. Agriculture	152.152	—	.2687	.1752	.5312	—	−.4312	—	−.2437	−.1729
	(64.95)		(.14)	(.11)	(.06)		(.08)		(.10)	(.11)
2. Mining	41.200	−.3502	.2734	4 − .4181	1.1604	−.2848	−.2018	—	—	—
	(14.34)	(.07)	(.04)	(.06)	(.09)	(.08)	(.03)			
3. Coke	−351.57	−1.3100	—	.6330	—	—	−.3005	—	1.0448	—
	(44.66)	(.18)		(.15)			(.04)		(16)	
4. Oil	—	—	—	—	—	—	—	—	—	—
5. Petroleum Products	−70.796	−.5087	.3640	—	.5185	—	−.3144	—	—	—
	(71.77)	(.13)	(.13)		(.13)		(.09)			
6. Electric	18.523	—	.1614	—	1.2739	−.5958	−.1732	—	—	—
	(14.70)		(.08)		(.17)	(.16)	(.07)			
7. Gas	−47.110	−.6014	—	.0611	.4191	—	−.1507	—	.5379	—
	(97.22)	(.20)		(.07)	(.15)		(.05)		(.18)	
8. Water	8.168	—	.6536	−.6536	.8112	—	−.4027	.4027	−.6415	.7906
	(18.92)		(.40)	(.40)	(.08)		(.11)	(.11)	(.31)	(.31)
9. Minerals	172.916	—	.2655	—	.6931	—	−.1494	—	−.5337	—
	(79.12)		(.13)		(.08)		(.06)		(.26)	
10. Iron	−349.96	−.9045	.1083	—	.4978	—	−.3873	—	1.1803	—
	(58.87)	(.27)	(.09)		(.08)		(.08)		(.29)	
11. Other metals	−84.826	−.5749	.1817	−.3091	1.2461	−.4796	−.0756	.0756	.5854	—
	(30.72)	(.15)	(.13)	(.13)	(.15)	(.12)	(.05)	(.05)	(.18)	
12. Mineral products	−280.57	−.3729	.3101	—	.6919	—	−.2356	−.2214	.5170	—
	(60.64)	(.21)	(.15)		(.09)		(.11)	(.10)	(.29)	
13. Chemicals	−125.06	—	—	—	.6205	—	−.2810	—	.6049	—
	(23.83)				(.07)		(.03)		(.08)	
14. Metal goods	−32.245	−1231	.4365		.5798	—	−.1671	—	—	—
	(25.53)	(.10)	(.04)		(.05)		(.08)			
15. Mechanical engineering†	−140.40	.2775	.5872	−.2910	.5309	−.1529	−.3090	−.3407	—	.3966
	(63.92)	(.15)	(.10)	(.11)	(.22)	(.16)	(.15)	(.13)		(.21)
16. Office goods	−3.4674	—	.1694	−.1694	1.2748	−.3244	−.3884	.3123	—	—
	(22.75)		(.09)	(.09)	(.20)	(.18)	(.14)	(.13)		
17. Electrical engineering†	−64.345	—	.4199	—	.5345	—	−.9220	.5053	—	—
	(29.12)		(.09)		(.09)		(.18)	(.17)		
18. Motor vehicles	−184.62	−.2365	.4908	−.3811	.9237	−.1783	—	−.1843	.5856	—
	(50.06)	(.11)	(.06)	(.11)	(.16)	(.09)		(.07)	(.18)	
19. Aerospace	200.392	−.6788	.0732	—	.7560	−.4659	—	−.1252	—	—
	(53.12)	(.16)	(.06)		(.17)	(.14)		(.07)		
20. Ships	−.7667	—	.4809	−.4809	1.4717	−.4717	—	—	.5103	−.5103
	(.31)		(.12)	(.12)	(.15)	(.15)			(.20)	(.20)
21. Other vehicles	−132.16	−.4754	.3130	—	.7270	—	−.1432	—	—	.2845
	(54.39)	(.17)	(.07)		(.09)		(.05)			(.11)
22. Instrument engineering	−11.357	−.3580	.3611	—	.5319	—	−.2624	—	—	—
	(47.49)	(.14)	(.10)		(.13)		(.11)			
23. Food	−172.16	−.4510	.6697	—	.3177	.2237	−.1962	—	—	.1157
	(76.05)	(.20)	(.17)		(.17)	(.16)	(.06)			(.12)
24. Drink	−15.180	−.4844	.2933	—	.7283	—	−.0945	.0591	—	—
	(73.49)	(.14)	(.12)		(.12)		(.09)	(.09)		
25. Tobacco	−213.37	−.3959	.7424	—	.7367	.2633	—	—	—	—
	(80.84)	(.12)	(.28)		(.22)	(.22)				
26. Textiles	−68.150	—	.5278	−.1236	.5880	—	−.3428	—	—	—
	(10.02)		(.05)	(.08)	(.06)		(.05)			
27. Clothing	−68.949	—	.4514	—	.5364	—	−.3756	—	—	—
	(11.96)		(.04)		(.04)		(.03)			
28. Timber	60.3106	−.3017	.3769	—	.4312	—	−.2460	.1493	—	—
	(20.95)	(.08)	(.04)		(.06)		(.07)	(.07)		
29. Paper	−44.740	−.3259	.4680	.1585	.3644	—	−.2503	—	—	—
	(13.29)	(.10)	(.07)	(.09)	(.08)	(.04)				
30. Books	58.9249	—	.2973	−.2575	1.4842	−.7029	−.0454	—	—	—
	(20.82)		(.06)	(.06)	(.17)	(.15)	(.05)			

(continued)

Table 1: (Continued)

Industry	$c/40$	$T/40$	LY_{it}	$LY_{i,t-1}$	$LE_{i,t-1}$	$LE_{i,t-2}$	LW_{it}	$LW_{i,t-1}$	$\overline{LY}_{at}$	$\overline{LY}_{a,t-1}$
31. Rubber†	−54.009	−.6246	.6726	−.2123	.7116	—	—	—	—	—
	(13.15)	(.12)	(.09)	(.12)	(.09)					
32. Other manufacturing	60.3555	−.3233	.2345	—	.6028	—	—	—	.4274	−.4274
	(20.03)	(.07)	(.04)		(.09)				(.13)	(.13)
33. Construction	7.2409	—	.5490	−.4527	1.0813	−.2453	−.4434	.3376	—	—
	(20.56)		(.08)	(.09)	(.16)	(.11)	(.08)	(.11)		
34. Distribution	109.986	.3892	—	.5034	.5641	—	−.3036	—	—	−.5678
	(43.33)	(.21)		(.20)	(.09)		(.12)			(.17)
35. Hotels†	388.377	.4384	−.8857	1.6050	—	−.4753	−.8499	.3938	—	−.4509
	(205.9)	(.18)	(.52)	(.60)		(.40)	(.31)	(.24)		(.28)
36. Rail	−65.107	—	—	.4077	.8047	—	−.0729	—	—	—
	(26.28)			(.10)	(.05)		(.05)			
37. Land transportation	146.432	−.4542	—	.2451	.9023	−.4855	—	—	—	—
	(37.81)	(.10)		(.07)	(.19)	(.18)				
38. Sea transportation	48.5901	−.1921	.1924	—	1.1919	−.5542	−.0853	—	—	—
	(104.9)	(.11)	(.16)		(.17)	(.22)	(.07)			
39. Communications†	195.189	—	−.2816	—	.8450	−.3766	—	.2841	.5211	—
	(48.50)		(.10)		(.17)	(.17)		(.10)	(.13)	
40. Business	209.651	—	.3108	—	.6781	−.3104	—	—	—	−.1633
	(49.15)		(.08)		(.18)	(.17)				(.05)
41. Services	−39.904	—	.2123	—	.8264	—	−.1408	—	—	—
	(33.31)		(.08)		(.10)		(.07)			

Note: Equations are estimated using the OLS method, *except* in the case of industries denoted † (i.e. industries numbered 15, 17, 31, 35 and 39), in which the IV method was employed. For these five sector, the following variables were included in the instrument set for the ith industry: $c/40$, $T/40$, $LY_{i,t-1}$, $LY_{i,t-2}$, $LE_{i,t-1}$, $LE_{i,t-2}$, $LW_{i,t-1}$, $LW_{i,t-2}$, $\overline{LY}_{a,t-1}$, $\overline{LY}_{a,t-2}$, $\overline{LE}_{a,t-1}$, $\overline{LE}_{a,t-2}$, $\overline{LW}_{a,t-1}$, $\overline{LW}_{a,t-2}$. Variables definitions are provided in the text, and data sources are provided in PPK. Values in parentheses are standard errors.

(and is not significantly different from 0 in a further 8), but the sum of the coefficients on the sectoral output terms is positive in 33 sectors (and is not significantly different from 0 in a further 4).

Table 2 reports the generalised $\bar{R}^2$ as measures of the 'fit' of the IV regressions and also several diagnostic statistics, denoted $\chi^2_{SC}(1)$, $\chi^2_{FF}(1)$, $\chi^2_N(2)$, and $\chi^2_H(1)$, and distributed approximately as chi-squared variates (with degrees of freedom in parentheses), for tests of residual serial correlation, functional form misspecification, nonnormal errors, and heteroscedasticity, respectively. (For more details of the tests, see Pesaran and Pesaran (1991)). These statistics indicate that there is evidence of misspecification in only a few cases. For example, there is evidence of residual serial correlation only in the chemicals (13) and construction (33) industries, and this is weak in the former case. The $\chi^2_R(r)$ statistics reported in Table 2 for testing the restrictions imposed on the unrestricted labour-demand equations in (3.1) to obtain the specifications given in Table 1 are below their 95% critical values in all industries other than office goods (16), thus reaffirming the plausibility of our search procedure. In summary, the results of Tables 1 and 2 indicate that, although there may be room for improving the results—by including industry-specific variables, for example—the specifications considered here provide a reasonable model of labour-demand determination at the sectorally disaggregated level.

Consider now the aggregate employment equation obtained as an analogue of (3.1):

$$\begin{aligned} LE_{at} &= b_1 + b_2 T_t + b_3 LE_{a,t-1} + b_4 LE_{a,t-2} + b_5 LY_{at} \\ &\quad + b_6 LY_{a,t-1} + b_7 LW_{at} + b_8 LW_{a,t-1} + u_{at}, \end{aligned} \tag{3.2}$$

where

$$LE_{at} = \sum_{i=1,\ i\neq 4}^{41} LE_{it} \quad , \quad LY_{at} = \sum_{i=1,\ i\neq 4}^{41} LY_{it} \quad \text{and} \quad LW_{at} = \sum_{i=1,\ i\neq 4}^{41} LW_{it}.$$

Here the dependent variable of interest is assumed to be LE_{at}—that is, the sum of the logarithms of industry employment (in man-hours). Clearly, this is not the dependent variable usually considered in aggregate labour-demand equations (which tend to consider the logarithm of the sum of industry employment). The issue of consistent aggregation in the context of log-linear models has been discussed in the literature (e.g., Lovell (1973); van Daal and Merkies (1981)), and here we simply note that the

Table 2: Summary and Diagnostic Test Statistics for Restricted Labour-Demand Equations of Table 1

Industry	$\overline{GR}^2$	$\widehat{\sigma}$	$\chi^2_R(r)$	$\chi^2_{SC}(1)$	$\chi^2_{FF}(1)$	$\chi^2_N(2)$	$\chi^2_H(1)$	$F_{WH}(3,16)$	$\chi^2_{MS}(4)$
1. Agriculture	.9983	.0137	.04(3)	.00	7.40**	.39	2.55	1.03	5.96
2. Mining	.9986	.0158	3.72(3)	.83	.91	.32	.05	.82	5.96
3. Coke	.9771	.0449	5.20(5)	.24	.67	.27	1.87	.02	4.66
4. Oil	—	—	—	—	—	—	—	—	—
5. Petroleum Products	.9178	.0566	4.89(5)	.48	.01	1.83	.85	.78	6.68
6. Electric	.9876	.0190	2.19(5)	.17	1.26	.18	.12	.26	5.62
7. Gas	.9719	.0322	3.97(4)	1.29	.00	4.88*	1.42	1.09	5.91
8. Water	.9279	.0412	.73(4)	1.67	.00	.47	1.05	.36	4.94
9. Minerals	.9760	.0318	2.40(5)	1.36	.16	32.7**	.00	.39	4.21
10. Iron	.9933	.0265	2.49(4)	.08	.19	1.42	.43	1.07	6.88
11. Other metals	.9864	.0250	2.63(2)	.00	3.45*	.20	1.89	1.29	5.63
12. Mineral products	.9935	.0177	3.44(3)	1.11	.23	.76	3.15*	.34	6.38
13. Chemicals	.9795	.0156	6.27(6)	3.51*	1.49	.96	1.14	.44	8.96*
14. Metal goods	.9877	.0192	2.37(5)	.09	.27	.38	1.00	.36	9.02*
15. Mechanical engineering†	.9918	.0148	.49(1)	1.21	6.21*	.77	1.49	2.60*	5.85
16. Office goods	.8922	.0345	10.5(4)**	.05	2.49	7.24**	5.05**	3.00*	3.38
17. Electrical engineering†	.9683	.0224	2.95(5)	.00	7.90**	.69	3.48*	5.40**	1.90
18. Motor vehicles	.9874	.0186	1.29(2)	1.55	8.71**	3.89	.00	.92	5.92
19. Aerospace	.9878	.0268	2.21(4)	.90	.30	1.81	1.30	1.04	3.69
20. Ships	.9818	.0323	9.70(6)	.45	.43	.40	6.26**	.67	2.58
21. Other vehicles	.9973	.0241	1.69(4)	.00	.81	.17	.04	.91	1.99
22. Instrument engineering	.9250	.0257	7.92(5)	.47	3.05*	.00	.84	.29	6.88
23. Food	.9837	.0164	.85(3)	1.69	1.76	1.33	4.38**	.75	8.92
24. Drink	.9232	.0269	2.56(4)	1.32	.02	.94	2.06	.80	6.79
25. Tobacco	.8796	.0497	7.09(6)	.25	.70	.65	6.33**	1.66	4.36
26. Textiles	.9981	.0175	3.18(5)	.05	4.44**	.74	5.09**	.70	5.62
27. Clothing	.9984	.0110	3.76(6)	.36	1.91	.62	.03	.20	3.46
28. Timber	.9864	.0138	4.24(4)	.00	2.56	1.34	.30	.41	2.85
29. Paper	.9927	.0192	2.86(4)	1.09	1.32	1.74	4.41**	1.93	2.30
30. Books	.9306	.0123	4.69(4)	1.70	.00	.14	.44	1.01	2.38
31. Rubber†	.9570	.0193	.67(5)	.11	3.34*	.56	2.48	3.71**	3.43
32. Other manufacturing	.9570	.0137	7.18(5)	.37	.21	1.12	.00	.91	3.45
33. Construction	.9689	.0179	5.54(3)	5.00**	2.13	1.62	1.00	.43	3.83
34. Distribution	.9589	.0143	5.44(4)	.49	.00	.94	2.06	2.12	3.61
35. Hotels†	.9202	.0316	.38(2)	.03	6.48**	2.21	1.77	3.18*	1.01
36. Rail	.9960	.0230	2.36(6)	.28	.00	1.27	1.98	.12	7.10
37. Land transportation	.9747	.0163	4.04(5)	.02	1.75	.64	2.71*	.95	4.00
38. Sea transportation	.9155	.0229	7.87(4)*	.27	6.06**	.39	2.75*	.22	8.64
39. Communications†	.9232	.0203	.33(4)	1.87	1.48	1.01	2.75*	6.39**	1.17
40. Business	.9940	.0128	1.81(5)	.98	2.01	1.98	.17	.49	2.90
41. Services	.9512	.0222	3.21(6)	.06	.53	.39	1.91	.34	6.63

Note: Equations are estimated using the OLS method, *except* in the case of industries denoted † (i.e. industries numbered 15, 17, 31, 35 and 39), in which the IV method was employed. See footnotes to Table 1. $\overline{GR}^2$ refers to the generalised R^2 statistic (cf. Pesaran and Smith (1994)). $\widehat{\sigma}$ is the estimate of the of the equation's standard error. $\chi^2_R(r)$ is the chi-squared statistic for the Lagrange multiplier test for r linear restrictions imposed on the parameters of the unrestricted equation (where r is given in parentheses). $\chi^2_{SC}(1)$, $\chi^2_{FF}(1)$, $\chi^2_N(2)$ and $\chi^2_H(1)$ are diagnostic statistics, distributed approximately as chi-squared variates (with degrees of freedom in parentheses) for tests of residual serial correlation, functional form misspecification, nonnormal errors, and heteroscedasticity, respectively. (See Pesaran and Pesaran (1991)). $F_{WH}(3,16)$ is the Wu-Hausman test for the exogeneity of $LYit$, $LWit$ and $\overline{LY}_{at}$ carried out on the unrestricted version of the model (cf. $F(3,16)$). $\chi^2_{MS}(4)$ is Sargan's general misspecification test carried out on the unrestricted version of the model. This latter statistic is the same as the J statistic in the generalised method of moments proposed by Hansen (1982). ** denotes significance at the 5% level and * denotes significance at the 10% level.

aggregates employed in (3.2) may have some theoretical advantages over standard aggregate measures (i.e., the logarithm of the sum of sectoral employment, wages, or output) when the issue of interest is the analysis of sectoral employment growths. Of course, for our purposes, the specification (3.2) also has the advantage of fitting directly within the linear framework of the article.

A restricted version of (3.2) was estimated by the IV method using the instrument set $Z_{at} = \{1, LE_{a,t-1}, LE_{a,t-2}, LY_{a,t-1}, LY_{a,t-2}, LW_{a,t-1}, LW_{a,t-2}\}$, and the following results were obtained:

$$\begin{array}{lllll} LE_{at} = & -137.01 & +0.6840 LE_{a,t-1} & +0.4745 LY_{at} & -0.3830 LW_{at} + \hat{u}_{at} \\ & (20.70) & (0.0569) & (0.0708) & (0.0540) \end{array}$$

$$\hat{\sigma} = 0.3487, \quad s(LE_{at}) = 5.75, \quad \text{Sample} = 1956 - 1984 \ (n = 29)$$

$$\chi^2_{SC}(1) = 0.56, \quad \chi^2_{FF}(1) = 0.07, \quad \chi^2_{N}(2) = 4.15, \quad \chi^2_{H}(1) = 2.18, \quad \chi^2_{MS}(3) = 2.68. \tag{3.3}$$

Here, standard errors of the estimated parameters are given in brackets, $\hat{\sigma}$ is the estimate of the equation's standard error, $s(LE_{at})$ is the standard deviation of the dependent variable, and the remaining diagnostic statistics are as described in relation to Table 1. These IV estimates differ only marginally from those previously obtained using the OLS procedure and reported by PPK, and indeed the Wu-Hausman test fails to reject the exogeneity of the regressors LW_{at} and LY_{at} in this equation. While this finding might appear to suggest that the use of the OLS estimation method would be acceptable, it is in fact most important that the presence of simultaneity is taken into account here. If there is simultaneity in any of the sectoral equations, then it is clear that the aggregate model will be affected by such simultaneity so long as the matrix of regressors in the aggregate model includes aggregated values of the $\mathbf{X}_i$'s. The IV method will be the appropriate estimation procedure for the aggregate model in these circumstances, even if tests of exogeneity of regressors in the aggregate model fail to reflect this, possibly because of lack of power. Given the presence of simultaneity in the determination of employment, wages, and output in 5 of the 40 sectors, it is not appropriate to estimate the aggregate equation using OLS methods, and the IV results reported here are the relevent ones for use in comparison of the aggregate and disaggregate models.

For the two models (3.1) and (3.2), the statistics s_d^2 and s_a^2 of Section 1 were computed in both the uncorrected form and the modified form making an adjustment for degrees of freedom. (These calculations, as with all those described in this section, were carried out using the GAUSS programming language. Copies of the procedures as well as the data used for this analysis, are available from the authors on request.) For the disaggregated model, the uncorrected and modified values of s_d^2 were found to be 0.0742 and 0.1000, respectively, whereas for the aggregate model, the uncorrected and corrected values of s_a^2 were 0.3584 and 0.4158, respectively. It is clear that the criteria favour the disaggregate model, both in the uncorrected forms of (1.3) and (1.4) and in the corrected forms of (1.18) and (1.19), which include the degrees-of-freedom adjustments. These results are consistent with the findings of PPK based on the OLS estimates.

Models (3.1) and (3.2) were also used to test for aggregation bias in the estimates of the long-run elasticities of UK labour demand with respect to wages and output. (For this analysis sectors 20 and 25 had to be excluded because the restricted specifications estimated for these sectors do not seem to possess long-run solutions. The two sectors were consequently also removed from the definition of the aggregate variables entering equation (3.2).) For the ith sector, the long-run elasticities of interest are defined by

$$\epsilon_{iw} = \frac{\beta_{i7} + \beta_{i8}}{1 - \beta_{i3} - \beta_{i4}}, \quad \epsilon_{iy} = \frac{\beta_{i5} + \beta_{i6} + \beta_{i9} + \beta_{i10}}{1 - \beta_{i3} - \beta_{i4}},$$

and in considering aggregation bias, we aim to compare the average of each of these sectoral elasticities with the corresponding estimates based on the aggregate specification. As noted previously, these elasticities have been the subject of considerable interest because of their implications for macroeconomic policy. Various aggregate studies (many of which were reviewed by Treasury (1985)), have found a significant effect for real wages on employment demand, although the estimated size of the effect has varied considerably across studies, depending on the coverage of the data and on the specification of the employment equation that is considered. A consensus view has emerged on the basis of these aggregate studies, however, that the elasticity is close to -1, and hence this is the figure employed in the test of aggregation bias when comparison is made with an aggregate measure that is assumed known *a priori*. Similarly, a unit elasticity is used as the consensus figure for the output elasticity. Note that the wage and output elasticities obtained based on the aggregate model of (3.3) are -1.2792 (0.2121) and 1.5189

(0.2676), respectively. (Asymptotically valid standard errors are in parentheses.) These estimates are consistent with the hypothesis of wage and output elasticities of −1 and +1, respectively.

To examine whether these estimates for the aggregate wage and output elasticities are subject to aggregation bias, we use the restricted versions of IV models (3.1) and (3.2) and apply the tests defined by the statistics (2.6) and (2.10) of Section 2. A value of −0.5112 was obtained for the average of the sectoral wage elasticities estimated in Model (3.1), but the average of the sectoral output elasticities is found to be 0.9094. The q_1^* statistics of (2.6) that correspond to these figures, testing the null hypotheses that the wage and output elasticities are equal to their 'consensus' values of −1 and +1 respectively, are 29.25 and 0.48. Since both statistics are compared to the χ_1^2 distribution, these results provide strong evidence with which to reject the null hypothesis of no aggregation bias in the case of the wage elasticity but no evidence to reject the null in the case of the output elasticity. In contrast, when the q_2^* test statistics of (2.10) are calculated in which aggregation bias is defined with respect to the aggregate elasticities obtained from the estimated version of (3.2), the test statistic takes the value of 6.82 in the case of the wage elasticity, and 3.36 in the case of the output elasticity. Again each statistic is to be compared to the χ_1^2 distribution, so that there remains strong evidence with which to reject the null hypothesis of no aggregation bias in the case of the wage elasticity, and there is now some marginal evidence with which to reject this hypothesis for the output elasticity. These findings are also in line with those reported by Lee et al. (1990b), using the OLS method.

The results just described, obtained using the statistics appropriate for models estimated using the IV method derived in the previous sections of the article, confirm the findings of PPK and Lee et al. (1990b) that a disaggregate model of employment demand in the United Kingdom outperforms an aggregate model in terms of its predictive power and that there is significant aggregation bias in the estimation of the key wage elasticity of employment demand. The results substantiate the conclusions drawn previously by demonstrating that they cannot be attributed simply to some neglected simultaneity bias. The implications of those findings may be important for policy formulation, and certainly the results indicate that further work at the disaggregate level may be worthwhile.

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