Economic Forecasting
Exercise Sheet 3 Solutions

Richard G. Pierse

1. (a) Figure 1 graphs the pseudo-random variable U. The series has mean -0.073, standard deviation 0.259 (should be $\sqrt{0.1} = 0.316$), skewness 0.299 (should be 0) and kurtosis 3.4 (should be 3).

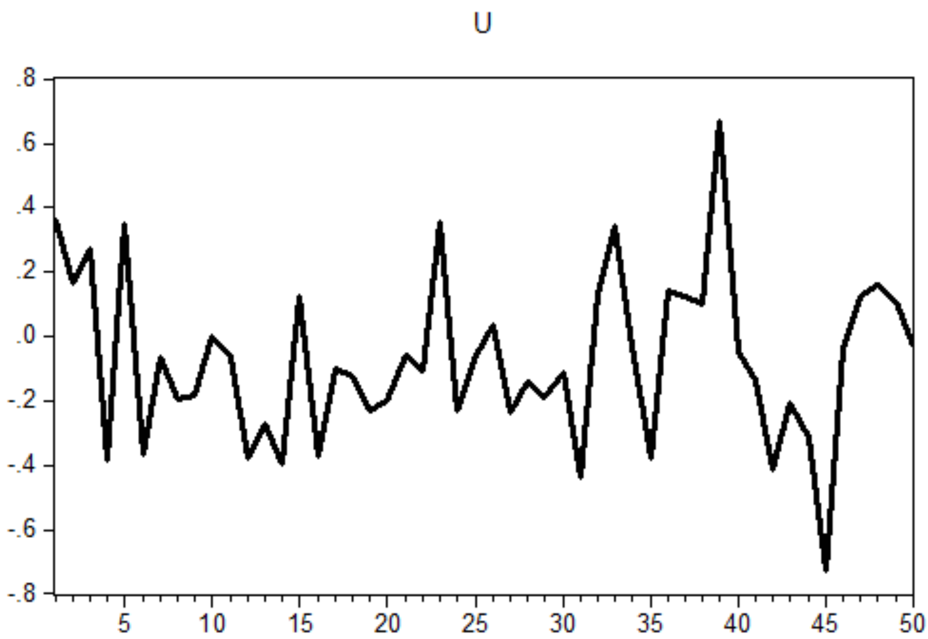

Figure 1: Pseudo-random variable U with variance 0.1

(b) *Create DC.*

(c) *Create DAR2.*

(d) Figure 2 graphs the deterministic cosine wave and also the deterministic *AR(2)* process with period 4 and damping factor r=0.9. Note that the

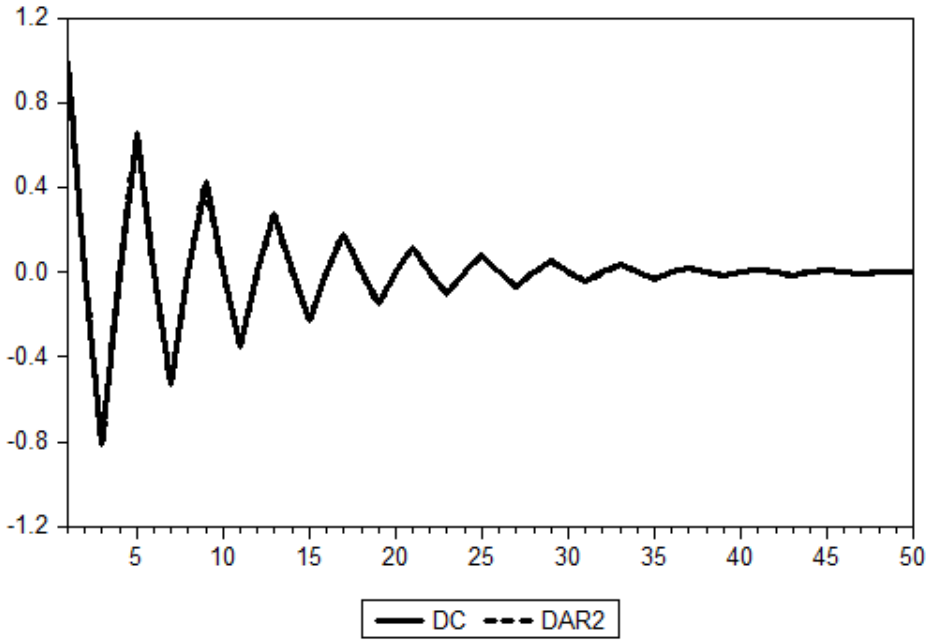

Figure 2: Deterministic cosine or *AR(2)* process with period 4 and r=0.9

AR(2) process

$$y_t = -0.81 y_{t-2}$$

can be written as

$$(1 + 0.81L^2)y_t = (1 - 0.9iL)(1 + 0.9iL)y_t$$

which has two purely imaginary roots of $\pm\frac{1}{0.9}i$ which lie outside the unit circle.

(e) *Create SC and SAR2.*

(f) Figure 3 graphs the stochastic cosine wave and *AR(2)* processes together.

Note that although the stochastic shock U is the same in both cases, it operates differently in the two processes. This is because each period's shock has *persistence* in the stochastic *AR(2)* case but not in the cosine case. The shocks are quite large compared with the scale of the underlying wave so the regular underlying pattern is obscured in both cases.

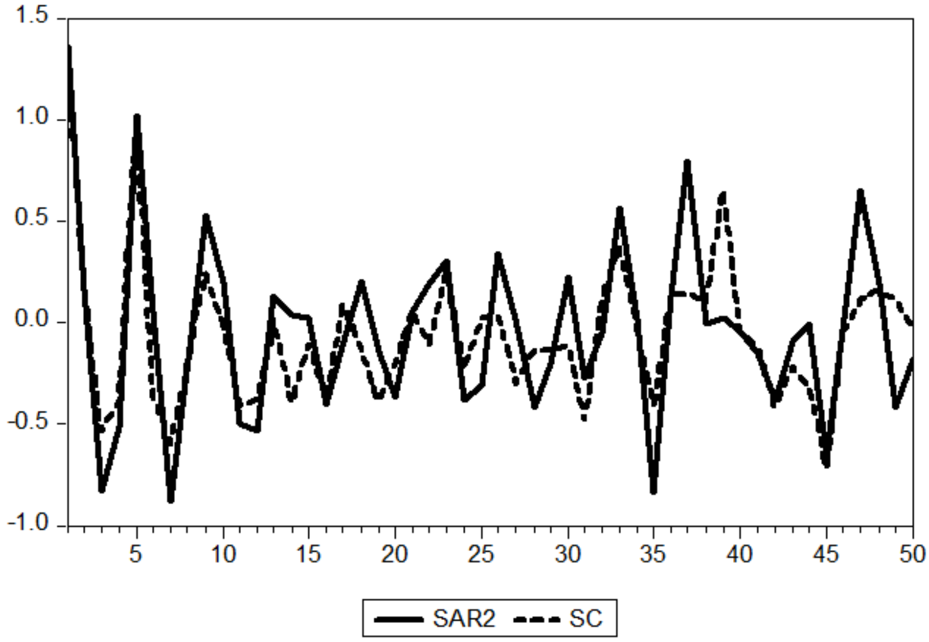

Figure 3: Stochastic cosine and *AR(2)* processes

(g) The following table is the correlogram for the deterministic *AR(2)* process DAR2 (or, equivalently, for the deterministic cosine wave DC).

```
Date: 03/06/13   Time: 14:26
Sample: 1 50
Included observations: 50
===============================================================
Autocorrelation Partial Correlation AC     PAC     Q-Stat  Prob
===============================================================
      . | .     |        . | .    |  1 0.002   0.002  0.0001  0.990
  ******| .     |    ******| .    | 2 -0.810 -0.810  35.545  0.000
      . | .     |         .| .    | 3 -0.002   0.007  35.545  0.000
      . |*****  |        . | .    | 4  0.656 -0.001  59.861  0.000
      . | .     |        . | .    | 5  0.001 -0.001  59.861  0.000
    ****| .     |        . | .    | 6 -0.532 -0.002  76.563  0.000
      . | .     |        . | .    | 7 -0.001 -0.002  76.563  0.000
      . |***    |        . | .    | 8  0.430 -0.003  88.012  0.000
      . | .     |        . | .    | 9  0.000 -0.003  88.012  0.000
```

```
     ***| .    |       . | .    |10 -0.349 -0.003  95.929  0.000
       . | .    |       . | .    |11 -0.001 -0.004  95.930  0.000
       . |**    |       . | .    |12  0.282 -0.004  101.36  0.000
       . | .    |       . | .    |13 -0.000 -0.005  101.36  0.000
      **| .    |       . | .    |14 -0.229 -0.005  105.16  0.000
       . | .    |       . | .    |15 -0.001 -0.005  105.16  0.000
       . |*.    |       . | .    |16  0.184 -0.006  107.76  0.000
       . | .    |       . | .    |17 -0.000 -0.006  107.76  0.000
      .*| .    |       . | .    |18 -0.151 -0.006  109.60  0.000
       . | .    |       . | .    |19 -0.001 -0.007  109.60  0.000
       . |*.    |       . | .    |20  0.120 -0.008  110.86  0.000
       . | .    |       . | .    |21 -0.001 -0.008  110.86  0.000
      .*| .    |       . | .    |22 -0.099 -0.008  111.77  0.000
       . | .    |       . | .    |23 -0.001 -0.009  111.77  0.000
       . |*.    |       . | .    |24  0.078 -0.009  112.38  0.000
===============================================================
```

Note that the simple autocorrelations die away gradually showing a clear damped cyclical pattern while the partial correlations cut off abruptly after the second order. The *Q-Stat* test for significant autocorrelation strongly rejects the null hypothesis of no autocorrelation.

The second table is the correlogram for the stochastic cosine process SC. The presence of the stochastic term U obscures the damped cyclical pattern we expect to see in the simple autocorrelations and the cutting off after lag 2 we expect to see in the partial correlations.

```
Date: 03/06/13   Time: 16:07
Sample: 1 50
Included observations: 50
===============================================================
Autocorrelation Partial Correlation AC    PAC     Q-Stat  Prob
===============================================================
       . | .    |       . | .    | 1  0.061  0.061  0.1993  0.655
      **| .    |      **| .    | 2 -0.284 -0.289  4.5785  0.101
       . | .    |       . | .    | 3 -0.025  0.016  4.6142  0.202
       . |**    |       . |**    | 4  0.310  0.250  10.040  0.040
      .*| .    |      .*| .    | 5 -0.110 -0.183  10.734  0.057
      **| .    |      .*| .    | 6 -0.274 -0.130  15.177  0.019
       . | .    |      .*| .    | 7 -0.047 -0.070  15.312  0.032
       . |*.    |       . | .    | 8  0.130 -0.038  16.350  0.038
       . | .    |       . | .    | 9 -0.049 -0.023  16.500  0.057
       . | .    |       . | .    |10 -0.052  0.064  16.679  0.082
```

```
       .*| .    |       .*| .   |11 -0.067  0.134  16.980  0.108
       . | .    |       . | .   |12  0.024 -0.044  17.018  0.149
       . | .    |       . | .   |13 -0.018 -0.047  17.039  0.198
       . | .    |       .*| .   |14 -0.042 -0.075  17.166  0.247
       . | .    |       . | .   |15 -0.013  0.025  17.178  0.308
       . |*.    |       . |*.   |16  0.149  0.144  18.879  0.275
       . | .    |       .*| .   |17 -0.030 -0.111  18.948  0.332
       . | .    |       . | .   |18 -0.053  0.000  19.176  0.381
       .*| .    |       .*| .   |19 -0.108 -0.166  20.162  0.385
       . | .    |       .*| .   |20 -0.005 -0.132  20.164  0.448
       . | .    |       . | .   |21  0.001  0.043  20.164  0.511
       . | .    |       . | .   |22 -0.049 -0.047  20.385  0.559
       . | .    |       . | .   |23 -0.018  0.018  20.416  0.617
       . |*.    |       . | .   |24  0.086  0.054  21.163  0.629
===============================================================
```

The third table is the correlogram for the stochastic *AR(2)* process SAR2. In this case, the damped cyclical pattern we expect to see in the simple autocorrelations is still quite visible as is the cutting off after lag 2 in the partial correlations so that the effect of the stochastic term U is less severe.

```
Date: 03/06/13   Time: 16:11
Sample: 1 50
Included observations: 50
===============================================================
Autocorrelation Partial Correlation AC    PAC     Q-Stat  Prob
===============================================================
       . | .    |       . | .   | 1  0.035  0.035  0.0640  0.800
   *****| .    |    *****| .   | 2 -0.694 -0.696  26.152  0.000
       . | .    |       . | .   | 3 -0.040  0.049  26.240  0.000
       . |****  |       . | .   | 4  0.514  0.060  41.160  0.000
       . | .    |       .*| .   | 5 -0.027 -0.141  41.203  0.000
      **| .    |       . |*.   | 6 -0.339  0.103  47.978  0.000
       . | .    |       .*| .   | 7 -0.007 -0.134  47.981  0.000
       . |*.    |       .*| .   | 8  0.150 -0.147  49.368  0.000
       . | .    |       . |*.   | 9  0.027  0.085  49.414  0.000
       . | .    |       . | .   |10  0.008  0.058  49.417  0.000
       .*| .    |       .*| .   |11 -0.083 -0.130  49.874  0.000
       .*| .    |       . | .   |12 -0.094  0.019  50.481  0.000
       . | .    |       .*| .   |13  0.067 -0.091  50.799  0.000
       . |*.    |       .*| .   |14  0.079 -0.134  51.249  0.000
```

```
      .*| .    |       . | .   |15 -0.113 -0.037  52.193  0.000
      . | .    |       . |*.   |16 -0.016  0.082  52.213  0.000
      . |*.    |       . | .   |17  0.140 -0.009  53.767  0.000
      . | .    |       . | .   |18  0.008  0.044  53.772  0.000
      **| .    |       .*| .   |19 -0.206 -0.181  57.327  0.000
      . | .    |       .*| .   |20 -0.010 -0.083  57.336  0.000
      . |*.    |       . | .   |21  0.173 -0.019  60.029  0.000
      . | .    |       .*| .   |22 -0.014 -0.076  60.047  0.000
      .*| .    |       . | .   |23 -0.149  0.010  62.187  0.000
      . |*.    |       . |*.   |24  0.078  0.104  62.797  0.000
================================================================
```