Economic Forecasting
Exercise Sheet 4 Solutions

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1. (a) Figure 1 graphs the Canadian employment series. Note that there is no obvious trend in the series but the mean is clearly non-zero.

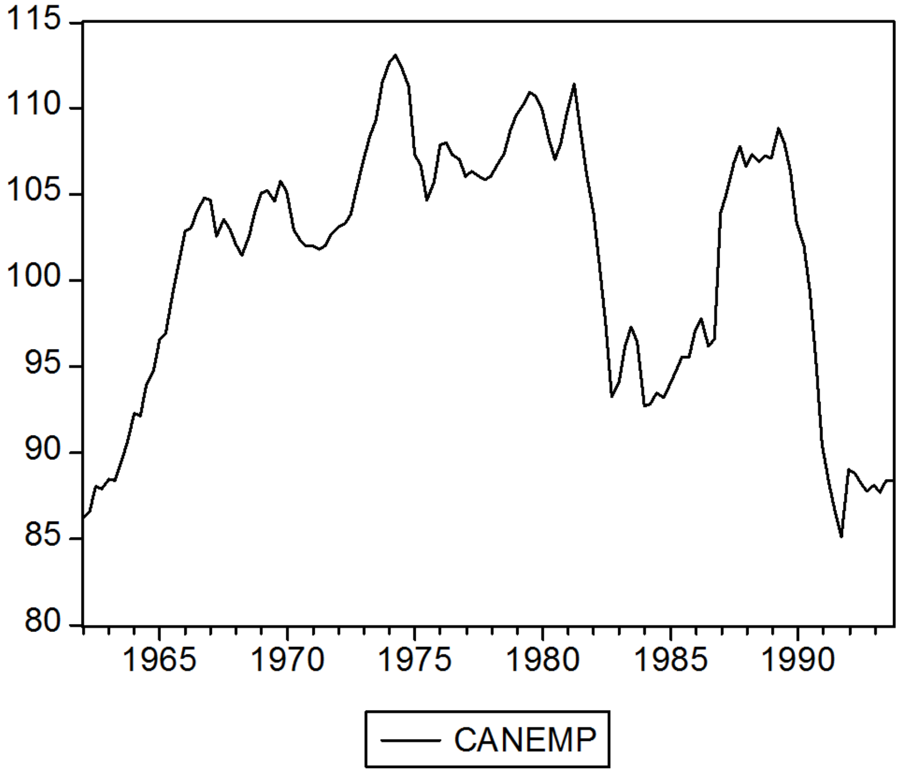

Figure 1: Canadian Employment Data: 1961-1994

(b) Results of the *ADF* test are reported below. An intercept was included because the series has non-zero mean but not a deterministic trend since there is no obvious trend in the series. The test fails to reject the null hypothesis of a unit root since the test statistic of -2.205591 is less negative than the one-sided critical values at the 1%, 5% or 10% levels. Despite this result, which suggest that the series needs to be differenced to achieve stationary, following Diebold we will proceed to search for an $ARMA$ model of the undifferenced series.

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Null Hypothesis: CANEMP has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=12)
================================================================
                                          t-Statistic   Prob.*
================================================================
Augmented Dickey-Fuller test statistic     -2.205591    0.2054
Test critical values:          1% level    -3.482035
                               5% level    -2.884109
                              10% level    -2.578884
================================================================
*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(CANEMP)
Method: Least Squares
Date: 03/10/13   Time: 16:19
Sample: 1962Q1 1993Q4
Included observations: 128
================================================================
Variable         Coefficient    Std. Error   t-Statistic  Prob.
================================================================
CANEMP(-1)        -0.037641      0.017066    -2.205591   0.0292
D(CANEMP(-1))      0.476451      0.077902     6.116042   0.0000
C                  3.810783      1.728134     2.205143   0.0293
================================================================
R-squared          0.244585   Mean dependent var       0.023315
Adjusted R-squared 0.232499   S.D. dependent var       1.651307
S.E. of regression 1.446663   Akaike info criterion    3.599554
Sum squared resid  261.6041   Schwarz criterion        3.666399
Log likelihood    -227.3715   Hannan-Quinn criter.     3.626713
F-statistic        20.23603   Durbin-Watson stat       2.067024
Prob(F-statistic)  0.000000
================================================================
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(c) The correlogram for CANEMP is reported below. The simple autocorrelations appear to die away gradually while the partal correlations appear to cut off abruptly at lag 3. This pattern suggests that the series might be a pure autoregressive process of order 2.

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Date: 03/10/13   Time: 15:59
Sample: 1962Q1 1993Q4
Included observations: 128
===============================================================
Autocorrelation Partial Correlation AC    PAC     Q-Stat  Prob
===============================================================
       .|*******        .|******* 1 0.949  0.949  118.05  0.000
       .|******|      **|.      | 2 0.877 -0.244  219.60  0.000
       .|******|       *|.      | 3 0.795 -0.100  303.65  0.000
       .|*****  |       *|.      | 4 0.707 -0.070  370.75  0.000
       .|****   |       *|.      | 5 0.617 -0.066  422.20  0.000
       .|****   |       .|.      | 6 0.526 -0.047  459.91  0.000
       .|***    |       .|.      | 7 0.437 -0.031  486.23  0.000
       .|**     |       .|.      | 8 0.351 -0.049  503.32  0.000
       .|**     |       *|.      | 9 0.258 -0.151  512.62  0.000
       .|*      |       *|.      |10 0.163 -0.071  516.36  0.000
       .|.      |       .|.      |11 0.073 -0.010  517.12  0.000
       .|.      |       .|.      |12-0.005  0.016  517.13  0.000
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(d) The table below reports the *AIC* and *SIC* criteria and the *p-value* on the highest order coefficient for all eight *AR* models. The highest order coefficient is not significant for the models *AR(8)* to *AR(3)*, suggesting that the order can be reduced. However, it becomes very significant for the *AR(2)* model suggesting it cannot be reduced further. This result is reinforced by the information criteria which both select *AR(2)* as the preferred model.

	AR(p) models		
Order	*AIC*	*SIC*	*p-value*
8	3.70	3.91	0.599
7	3.69	3.87	0.557
6	3.67	3.83	0.961
5	3.65	3.78	0.491
4	3.63	3.74	0.803
3	3.61	3.70	0.424
2	3.60	3.67	0.000
1	3.85	3.89	0.000

(e) The table below reports the *AIC* and *SIC* criteria and the *p-value* on the highest order coefficient for all ten *MA* models. While the highest order coefficient is not significant for the *MA(10)* and *MA(9)* models, it becomes significant for the *MA(8)* model, suggesting that we need 8 *MA* terms to model the series. This result is reinforced by the information criteria which both select *MA(8)* as the preferred model.

	MA(q) models		
Order	*AIC*	*SIC*	*p-value*
10	3.80	4.05	0.1983
9	3.79	4.01	0.3531
8	3.78	3.98	0.0487
7	3.81	3.98	0.0003
6	3.87	4.03	0.0000
5	4.04	4.17	0.0000
4	4.15	4.26	0.0000
3	4.46	4.55	0.0000
2	4.90	4.97	0.0000
1	5.70	5.74	0.0000

Comparing the preferred *MA(8)* model with the preferred *AR(2)* model, the information criteria both strongly prefer the autoregressive model which is not surprising as it has only three parameters compared with 9 for the moving average model.

(f) The table below reports the *AIC* and *SIC* criteria, the *p-value* on the moving average coefficient and the (inverse) *MA* and *AR* roots from estimation of the *ARMA(1,1)*, *ARMA(2,1)* and *ARMA(3,1)* models. For the *ARMA(1,1)* model, the moving average coefficient is significant but the information criteria for this model are both higher than for the *AR(2)* model, suggesting that replacing the *AR(2)* term by an *MA(1)* term reduces the fit of the model. For the *ARMA(2,1)* model, the moving average coefficient is insignificant. Since this model is an *AR(2)* model with an extra *MA* coefficient, this suggests that the *MA* term is not needed and this result is borne out by the information criteria for this model which are both higher than for the *AR(2)* model.

	ARMA(p,q) models						
Order	*AIC*	*SIC*	*p-value*	*MA inv root*	*AR inverse roots*		
$1,1$	3.67	3.73	0.0000	-0.39	0.96		
$2,1$	3.61	3.70	0.3347	0.17	0.90	0.68	
$3,1$	3.60	3.71	0.0000	-0.97	-0.94	0.93	0.51

The previous results show that the *AR(2)* model is a strong model.

This suggests caution in interpreting the results for the *ARMA(3,1)* model since this is an *AR(2)* model with an extra *AR* term and an extra *MA* term. We have already seen that the extra *AR* term is not significant in the *AR(3)* model and the extra *MA* term is not significant in the *ARMA(2,1)* model. However, including both extra terms together introduces the possibility of a common factor cancelling out the effect of the extra terms but causing estimation problems since there are an infinite number of equivalent values for this common factor giving rise to a problem of *identification*. This means that the estimation results may be misleading as indeed is the case. Both the additional *AR* and *MA* terms seem to be extremely significant and the *AIC* information criterion rates this model as equal to the *AR(2)* model (though the *SIC* criterion still prefers the *AR(2)* model). A clue to what is really going on can be seen from the reported inverse roots of the process. The *MA* root of -0.97 is very close to one of the *AR* roots which is -0.94. This suggests a common root in which case the model reduces to the *AR(2)* model (which has roots 0.92 and 0.52).

Common roots can be a serious problem with over-specified mixed *ARMA* models and may not be easy to spot. However, by first searching for the best pure *AR* and pure *MA* model, an upper bound can be set for the maximum length of *ARMA* model to consider to avoid common factors. In this case since the *AR(2)* or *ARMA(2,0)* model seems sufficient, we should only consider the *ARMA(1,1)*, *ARMA(2,1)* and *ARMA(1,2)* mixed models since these can't nest our preferred pure *AR(2)* model.