Economic Forecasting
Exercise Sheet 5 Solutions

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1. (a) *Extend data set*

   (b) *Forecast AR(2) model*

   (c) Figure 1 graphs the forecasts from the *AR(2)* and *MA(8)* models. After 8 periods, the *MA(8)* forecast reverts to the unconditional mean. The *AR(2)* forecast has more persistence though it too will revert asymptotically to the unconditional mean.

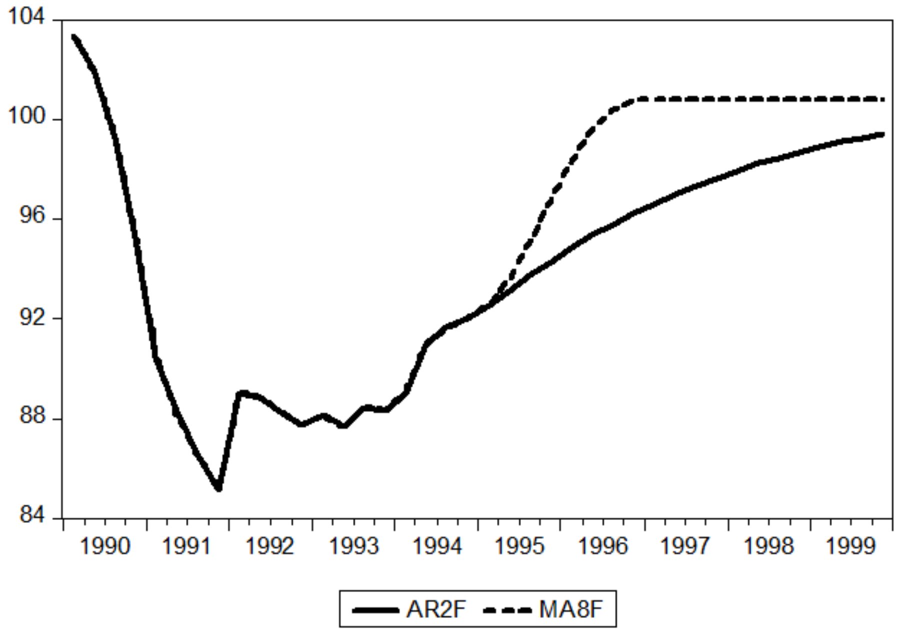

Figure 1: *AR(2)* and *MA(8)* forecasts for CANEMP: 1995q1-1999q4

(d) Figure 2 graphs the forecasts from the *ARMA(1,1)* models. Comparing to Figure 1, it is clear that this forecast is closer to the *AR(2)* forecast than the *MA(8)* forecast, only reverting asymptotically to the unconditional mean.

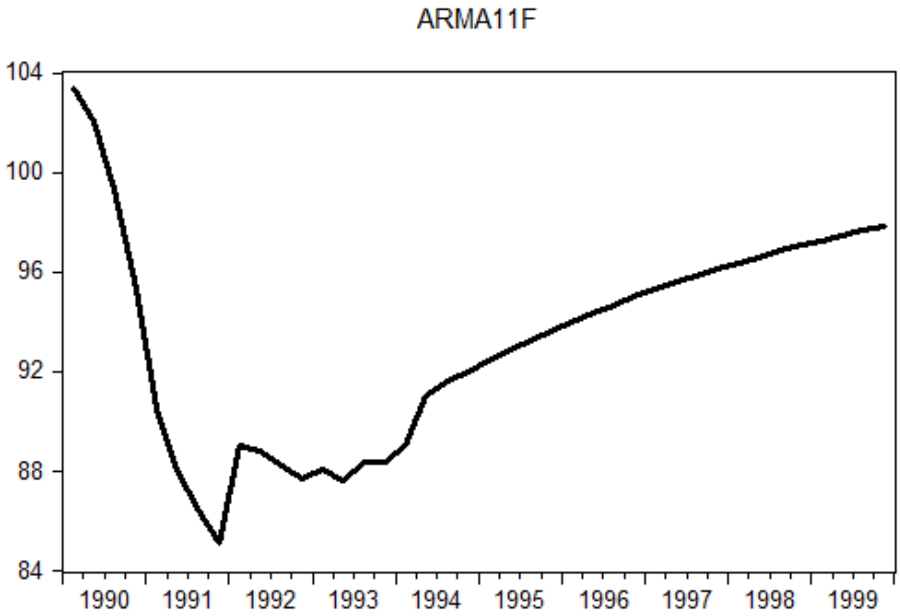

Figure 2: *ARMA(1,1)* forecast for CANEMP: 1995q1-1999q4

(e) (i) For the *MA(q)* model, the formula for $h < q$ is

$$\mathrm{E}(y_{t+h}|\boldsymbol{\Omega}_t) = c + \theta_h \varepsilon_t + \cdots + \theta_q \varepsilon_{t+h-q}$$

For the *MA(8)* case this gives a one-step ahead forecast

$$\mathrm{E}(y_{t+1}|\boldsymbol{\Omega}_t) = c + \theta_1 \varepsilon_t + \cdots + \theta_8 \varepsilon_{t-7}$$

and a two-step ahead forecast

$$\mathrm{E}(y_{t+2}|\boldsymbol{\Omega}_t) = c + \theta_2 \varepsilon_t + \cdots + \theta_8 \varepsilon_{t-6}$$

The estimated parameter values and residuals are given in the following table:

parameter	value	residual	value
$\widehat{\theta}_1$	1.514681	$\widehat{\varepsilon}_t$	-0.156477
$\widehat{\theta}_2$	1.825121	$\widehat{\varepsilon}_{t-1}$	-1.690104
$\widehat{\theta}_3$	1.906466	$\widehat{\varepsilon}_{t-2}$	-0.144025
$\widehat{\theta}_4$	1.831035	$\widehat{\varepsilon}_{t-3}$	-1.298424
$\widehat{\theta}_5$	1.533896	$\widehat{\varepsilon}_{t-4}$	-1.155967
$\widehat{\theta}_6$	1.119613	$\widehat{\varepsilon}_{t-5}$	0.160877
$\widehat{\theta}_7$	0.571330	$\widehat{\varepsilon}_{t-6}$	-0.715611
$\widehat{\theta}_8$	0.176659	$\widehat{\varepsilon}_{t-7}$	-0.076160
$\widehat{c}$	100.4795		

Plugging these values into the expressions, we derive the forecasts:

$$\widehat{y}_{t+1} = \widehat{c} + \sum_{i=1}^{8} \widehat{\theta}_i \widehat{\varepsilon}_{t-i+1} = 92.49049$$

$$\widehat{y}_{t+2} = \widehat{c} + \sum_{i=2}^{8} \widehat{\theta}_i \widehat{\varepsilon}_{t-i+2} = 93.38768$$

which are identical with the forecast values for periods 1995q1 and 1995q2 generated by *EViews* in variable **MA8F**.

(ii) For the *AR(2)* model, the recursive formulae are

$$\mathrm{E}(y_{t+1}|\mathbf{\Omega}_t) = c + \phi_1 y_t + \phi_2 y_{t-1}$$

and

$$\mathrm{E}(y_{t+2}|\mathbf{\Omega}_t) = c + \phi_1 \,\mathrm{E}(y_{t+1}|\mathbf{\Omega}_t) + \phi_2 y_t.$$

The estimated parameters are given in the following table:

parameter	value
y_t	92.01492
y_{t-1}	91.67328
$\widehat{\phi}_1$	1.429843
$\widehat{\phi}_2$	-0.469551
$\widehat{c}$	4.025728

Plugging these values into the expressions, we derive the forecasts:

$$\begin{aligned}\widehat{y}_{t+1} &= 4.025728 + 1.429843 * 92.01492 - 0.469551 * 91.67328 \\ &= 92.54734\end{aligned}$$

and

$$\begin{aligned}\widehat{y}_{t+2} &= 4.025728 + 1.429843 * 92.54734 - 0.469551 * 92.01492 \\ &= 93.14821.\end{aligned}$$

These values correspond exactly to the *EViews* forecast values for periods 1995q1 and 1995q2 in variable **AR2F**. However, for reasons I don't understand, if the equation is estimated using **AR(1)** and **AR(2)** as regressors, the resulting forecasts are slightly different. I suspect that this is due to a bug in *EViews*.

(iii) For the *ARMA(1,1)* model, the formulae are

$$\mathrm{E}(y_{t+1}|\mathbf{\Omega}_t) = c + \phi_1 y_t + \theta_1 \varepsilon_t$$

and

$$\mathrm{E}(y_{t+2}|\mathbf{\Omega}_t) = c + \phi_1 \,\mathrm{E}(y_{t+1}|\mathbf{\Omega}_t).$$

If the model is estimated in the form

$$y_t = c + \phi_1 y_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1}$$

including a lagged dependent variable and MA(1) term, then the estimated parameters and residual are given in the following table:

parameter	value
y_t	92.01492
$\widehat{\varepsilon}_t$	0.011693
$\widehat{\phi}_1$	0.954873
$\widehat{\theta}_1$	0.386176
$\widehat{c}$	4.589509

Plugging these values into the expressions, we derive the forecasts:

$$\begin{aligned}\widehat{y}_{t+1} &= 4.589509 + 0.954873 * 92.01492 + 0.386176 * 0.011693 \\ &= 92.45654\end{aligned}$$

and

$$\begin{aligned}\widehat{y}_{t+2} &= 4.589509 + 0.954873 * 92.45654 \\ &= 92.87372.\end{aligned}$$

These values correspond exactly to the *EViews* forecast values for periods 1995q1 and 1995q2 in variable **ARMA11F**. Note that if the model is estimated using **AR(1)** and **MA(1)** as regressors, the resulting estimators and forecasts are slightly different. In this case the reported intercept is the unrestricted mean $\widehat{\mu}$ and the estimator $\widehat{c}$ needs to be calculated by hand using the formula

$$\widehat{c} = (1 - \widehat{\phi}_1)\widehat{\mu}.$$

(f) (i) For the *MA(q)* model, the formula for the forecast error variance for $h < q$ is

$$\mathrm{var}(e_{t+h,t}) = \sigma_h^2 = (1 + \cdots + \theta_{h-1}^2)\sigma^2.$$

For the *MA(8)* case this gives a one-step ahead forecast error variance of

$$\mathrm{var}(e_{t+1,t}) = \sigma_1^2 = \sigma^2$$

and a two-step ahead forecast error variance of

$$\mathrm{var}(e_{t+2,t}) = \sigma_2^2 = (1 + \theta_1^2)\sigma^2.$$

The estimated value of σ, the reported standard error of the *MA(8)* regression, is

$$\widehat{\sigma} = 1.530204$$

so we have

$$\widehat{\sigma}_1 = \widehat{\sigma} = 1.530204$$

and

$$\widehat{\sigma}_2 = \widehat{\sigma}\sqrt{1 + \widehat{\theta}_1^2} = 2.777333.$$

These are the same as the forecast standard errors saved by *EViews* in variable **MA8SE** *provided* that these are generated with the option to include coefficient uncertainty in the standard error calculation *unchecked* in the forecast menu. However, if this option is left checked, the standard errors calculated by *EViews* are considerably larger with $\widehat{\sigma}_1 = 2.319150$ and $\widehat{\sigma}_2 = 3.287321$. This reflects the additional uncertainty caused by the fact that the estimated *MA* parameters are themselves uncertain.

(ii) For the *AR(2)* model, the one-step and two-step forecast errors are

$$e_{t+1,t} = \epsilon_{t+1}$$

and

$$e_{t+2,t} = \epsilon_{t+2} + \phi_1\epsilon_{t+1}$$

so

$$\mathrm{var}(e_{t+1,t}) = \sigma_1^2 = \sigma^2$$

and

$$\mathrm{var}(e_{t+2,t}) = \sigma_2^2 = (1 + \phi_1^2)\sigma^2.$$

The estimated value of σ, the reported standard error of the *AR(2)* regression, is

$$\widehat{\sigma} = 1.424510$$

so we have

$$\widehat{\sigma}_1 = \widehat{\sigma} = 1.424510$$

and

$$\widehat{\sigma}_2 = \widehat{\sigma}\sqrt{1 + \widehat{\phi}_1^2} = 2.485534.$$

These are the same as the forecast standard errors generated by *EViews* with coefficient uncertainty excluded. However, when coefficient uncertainty is included, the standard errors are slightly larger with $\widehat{\sigma}_1 = 1.436086$ and $\widehat{\sigma}_2 = 2.505496$.

(iii) For the *ARMA(1,1)* model, the formulae for the one-step ahead and two-steps ahead forecast error variances are given by

$$\text{var}(e_{t+1,t}) = \sigma_1^2 = \sigma^2$$

and

$$\text{var}(e_{t+2,t}) = \sigma_2^2 = (1 + (\phi_1 + \theta_1)^2)\sigma^2$$

respectively. The estimated value of σ, the reported standard error of the *ARMA(1,1)* regression, is

$$\widehat{\sigma} = 1.468548$$

so we have

$$\widehat{\sigma}_1 = \widehat{\sigma} = 1.468548$$

and

$$\widehat{\sigma}_2 = \widehat{\sigma}\sqrt{1 + (\widehat{\phi}_1 + \widehat{\theta}_1)^2} = 2.456655.$$

These are identical to the forecast standard errors generated by *EViews* with coefficient uncertainty excluded. However, when coefficient uncertainty is included, the standard errors are slightly larger with $\widehat{\sigma}_1 = 1.490156$ and $\widehat{\sigma}_2 = 2.480693$.