

Economic Forecasting

Lecture 7: Macroeconomic Forecasting

Richard G. Pierse

1 Introduction

All the forecasting models that we have looked at so far in these lectures have been *univariate* and have been purely statistical in the sense that they are not based on any underlying economic theory. In this lecture we look at multivariate forecasting models based on economic theory that jointly model the inter-relationships between several variables. The main application of these models has been in the area of macroeconomic forecasting. The first empirical macroeconomic model is usually taken to be a model of the Dutch economy built by Jan Tinbergen in 1936. Laurence Klein was an important figure in the development of macroeconomic modelling after the Second World War and he built the first model of the US economy (the famous *Klein Model 1*, Klein (1950)) and helped to build (along with Sir James Ball and others) the first model of the UK economy in 1961 (Klein *et al.* 1962). Another important UK model was the *Cambridge Growth Project* model set up by Sir Richard Stone (Stone and Brown (1962)) which was concerned with forecasting long run trends in the UK economy and which ran until the mid 1980s.

Today macroeconomic models of the UK are maintained by the *National Institute of Economic and Social Research*, *Her Majesty's Treasury* and the *Bank of England*. The National Institute uses its model to provide quarterly forecasts of the UK economy, published in its journal, the *National Institute Economic Review* (*NIER*). Other commercial forecasts based on macro-models and sold to clients are produced by city firms such as *Phillips and Drew* and other groups such as *Oxford Economic Forecasting* and *Cambridge Econometrics*. In addition, multi-country macroeconomic models have been built at the National Institute (*NiGEM*), the *International Monetary Fund* (*MULTIMOD*), the *OECD* (*INTERLINK*) and the *European Central Bank* (*MCM*) among others.

2 A simple macroeconomic model

We will look at *Klein's Model 1*: the first macroeconomic forecasting model of the United States. While this model is now extremely out-of-date and the economics underlying it is old-fashioned, it does serve to illustrate some of the main principles that remain valid for any macroeconomic model. The model equations are:

$$\begin{aligned}C_t &= a_0 + a_1\Pi_t + a_2\Pi_{t-1} + a_3(W_t + S_t) + \varepsilon_{1t} \\I_t &= b_0 + b_1\Pi_t + b_2\Pi_{t-1} + b_3K_{t-1} + \varepsilon_{2t} \\W_t &= c_0 + c_1Y_t + c_2Y_{t-1} + c_3t + \varepsilon_{3t} \\Y_t &= C_t + I_t + G_t \\ \Pi_t &= Y_t - W_t - T_t \\K_t &= K_{t-1} + I_t\end{aligned}\tag{1}$$

where, for period t , C_t is consumption, Π_t is corporate profits, W_t is the private sector wage bill, S_t is the government wage bill, I_t is investment, K_t is the capital stock, Y_t is output (production), G_t is government expenditure, T_t is indirect taxes and t is a time trend. The terms ε_{1t} , ε_{2t} and ε_{3t} appearing in the first three equations are disturbances. The model has six equations and eleven variables. The six variables that are explained by model equations $\{C_t, I_t, W_t, Y_t, \Pi_t, K_t\}$ are called *endogenous* variables. The remaining four variables $\{G_t, T_t, S_t, t\}$ are known as *exogenous* variables. The exogenous variables are not explained within the model. When the model is used for forecasting it will generate forecasts for the endogenous variables but the exogenous variables need to be known or must be forecast outside the model.

The first three equations in (1) are *behavioural* equations. These equations represent the outcomes of decisions by economic agents: consumers, investors and wage-setting firms respectively, and they involve unknown coefficients a_i , b_i and c_i that need to be estimated. These equations have associated disturbances ε_{1t} , ε_{2t} and ε_{3t} . The last three equations in (1) are *identities*. Identities are definitions and accounting relationships and have no unknown parameters or disturbances. For example, the fourth equation is the accounting identity that output is the sum of consumption, investment and government expenditure (in this simple model there are no exports or imports). The sixth equation is the definition of the capital stock as the accumulation of past investment (notice that there is no capital depreciation). Identities are very important to any model and ensure that model forecasts will satisfy the accounting relationships and definitions that link the variables. If, for example, we left out the output identity from this model then there would be nothing to ensure that, when consumption and investment are forecast to increase, output must also be forecast to rise.

2.1 Simultaneity and Estimation

Klein's Model 1 is a system of *simultaneous* equations. In a simultaneous system, current dated *endogenous* variables appear as explanatory variables in other equations. For example, in (1), the current dated endogenous variable I_t appears on the right-hand side of the equations for Y_t and for K_t and the current dated variable W_t appears as an explanatory variable in the equations for C_t and Π_t . When estimating the parameters of the model with a data set of observations on the model variables, this simultaneity has an important consequence because it means that the current dated *endogenous* variables on the right-hand side of equations will be correlated with the disturbance in the equation which violates a basic assumption of the standard regression (*OLS*) model.

Consider the effect of a positive unit shock to the disturbance of the consumption equation, ε_{1t} . This increases C_t by one unit through the first equation. However, this increase in C_t also leads to an increase in Y_t through the fourth equation and this in turn increases W_t through the fifth equation. Thus the shock to ε_{1t} in the first equation affects W_t which is one of the explanatory variables in the same equation. This means that the explanatory variable W_t is correlated with the disturbance ε_{1t} which violates a basic assumption of the *OLS* model. In order to estimate the coefficients in a simultaneous system like (1), we cannot use ordinary least squares but need to use an appropriate estimator such as *Two Stage Least Squares (2SLS)*. Denoting the *2SLS* parameter estimates with hats, we can write the estimated model as

$$\begin{aligned}
 C_t &= \hat{a}_0 + \hat{a}_1\Pi_t + \hat{a}_2\Pi_{t-1} + \hat{a}_3(W_t + S_t) + e_{1t} \\
 I_t &= \hat{b}_0 + \hat{b}_1\Pi_t + \hat{b}_2\Pi_{t-1} + \hat{b}_3K_{t-1} + e_{2t} \\
 W_t &= \hat{c}_0 + \hat{c}_1Y_t + \hat{c}_2Y_{t-1} + \hat{c}_3t + e_{3t} \\
 Y_t &= C_t + I_t + G_t \\
 \Pi_t &= Y_t - W_t - T_t \\
 K_t &= K_{t-1} + I_t
 \end{aligned} \tag{2}$$

where e_{1t} , e_{2t} and e_{3t} are regression residuals that are known as *Single Equation Residuals* or *SERs*. The other three equations in which there are no estimated parameters should hold exactly in the data set and so have zero *SERs*.

In order to forecast one step ahead with the model in this *structural form*, the three current endogenous variables C_t , I_t and W_t need to be jointly forecast together. Then, having solved for the forecast of these variables, the remaining variables Y_t , Π_t and K_t can simply be forecast from their identities.

2.2 The reduced form

The information set at the *start* of period t for forecasting the endogenous variables at time t consists of lagged values of the endogenous variables plus lagged and current values (assumed to be known in advance) of the exogenous variables. In the case of Klein's Model 1, the information set is

$$\Omega_{t-1} = \{1, Y_{t-1}, \Pi_{t-1}, K_{t-1}, G_t, T_t, S_t, t\}.$$

These eight variables are called the *predetermined variables* of the system. In a linear model like (2) it is always possible to write the current endogenous variables in terms of just the predetermined variables. This is known as the *reduced form* of the model. In the case of Klein's Model 1 the reduced form of the behavioural equations is defined by

$$\begin{aligned} C_t &= \hat{\pi}_{10} + \hat{\pi}_{11}Y_{t-1} + \hat{\pi}_{12}\Pi_{t-1} + \hat{\pi}_{13}K_{t-1} + \hat{\pi}_{14}G_t + \hat{\pi}_{15}T_t + \hat{\pi}_{16}S_t + \hat{\pi}_{17}t + v_{1t} \\ I_t &= \hat{\pi}_{20} + \hat{\pi}_{21}Y_{t-1} + \hat{\pi}_{22}\Pi_{t-1} + \hat{\pi}_{23}K_{t-1} + \hat{\pi}_{24}G_t + \hat{\pi}_{25}T_t + \hat{\pi}_{26}S_t + \hat{\pi}_{27}t + v_{2t} \\ W_t &= \hat{\pi}_{30} + \hat{\pi}_{31}Y_{t-1} + \hat{\pi}_{32}\Pi_{t-1} + \hat{\pi}_{33}K_{t-1} + \hat{\pi}_{34}G_t + \hat{\pi}_{35}T_t + \hat{\pi}_{36}S_t + \hat{\pi}_{37}t + v_{3t} \end{aligned}$$

where the $\hat{\pi}_{ij}$ parameter estimates are (complicated) linear functions of the structural parameter estimates \hat{a}_i , \hat{b}_i and \hat{c}_i and the v_{1t} , v_{2t} and v_{3t} are *reduced form residuals* which are linear functions of the original single equation residuals e_{1t} , e_{2t} and e_{3t} . The reduced form residuals are also sometimes known as *system residuals*. Note that once the model has been transformed into reduced form, one-step ahead forecasts for the endogenous variables C_t , I_t and W_t can be obtained separately and the other variables Y_t , Π_t and K_t can then be forecast using their identities.

3 Non-linear models

Klein Model 1 is a *linear* model since all the equations in the model are linear functions of the model variables. In practice most useful economic models are not linear. In particular, many behavioural equations in macroeconomic models are assumed to be log-linear since this implies constant elasticities and constant growth rates. Suppose that we were to replace the linear consumption function in (1) with the log-linear function

$$\log C_t = \hat{\alpha}_0 + \hat{\alpha}_1 \log \Pi_t + \hat{\alpha}_2 \log \Pi_{t-1} + \hat{\alpha}_3 \log(W_t + S_t) + e_{1t}. \quad (3)$$

The consequence of this simple change is that the model becomes a combination of log-linear behavioural equations and linear identities and ceases to be linear. Although this creates no problems for estimation, with a non-linear model the

reduced form cannot be found explicitly and in order to forecast we need to use iterative numeric methods to solve the model.

In order to forecast the set of variables $\{C_{t+1}, I_{t+1}, W_{t+1}, Y_{t+1}, \Pi_{t+1}, K_{t+1}\}$ using the information set $\mathbf{\Omega}_t = \{1, Y_t, \Pi_t, K_t, G_{t+1}, T_{t+1}, S_{t+1}, t\}$, we need to solve the set of equations

$$\begin{aligned}
\log \widehat{C}_{t+1,t} &= \widehat{\alpha}_0 + \widehat{\alpha}_1 \log \widehat{\Pi}_{t+1,t} + \widehat{\alpha}_2 \log \Pi_t + \widehat{\alpha}_3 \log(\widehat{W}_{t+1,t} + S_{t+1}) \\
\widehat{I}_{t+1,t} &= \widehat{b}_0 + \widehat{b}_1 \widehat{\Pi}_{t+1,t} + \widehat{b}_2 \Pi_t + \widehat{b}_3 K_t \\
\widehat{W}_{t+1,t} &= \widehat{c}_0 + \widehat{c}_1 \widehat{Y}_{t+1,t} + \widehat{c}_2 Y_t + \widehat{c}_3 (t + 1) \\
\widehat{Y}_{t+1,t} &= \widehat{C}_{t+1,t} + \widehat{I}_{t+1,t} + G_{t+1} \\
\widehat{\Pi}_{t+1,t} &= \widehat{Y}_{t+1,t} - \widehat{W}_{t+1,t} - T_{t+1} \\
\widehat{K}_{t+1,t} &= K_t + \widehat{I}_{t+1,t}
\end{aligned} \tag{4}$$

where we note that the equation residuals in period $t + 1$ have been set to their expected value of zero.

A simple iterative algorithm for solving this system of equations is the *Gauss-Seidel* algorithm in which the forecasts are initialised to some value and then the equations are evaluated repeatedly, updating the forecasts each time until the changes from the previous iteration are smaller than some threshold value. Although this algorithm is not guaranteed to converge, in practice it works satisfactorily in many cases. A more powerful algorithm is *Newton's method*, which uses additional information from the derivatives of the equations with respect to the model variables in order to improve convergence.

3.1 Types of forecast error

The forecast errors from the solution of the system of equations (4) are called *system forecast errors* as opposed to the *single equation forecast errors*. The one-step ahead system forecast error is based on a forecast for each variable in period $t + 1$ that is conditional on forecasts of all the other endogenous model variables in the same period. Consider a forecast of the variable K_{t+1} in (4). The *one-step ahead system forecast error* for this variable is given by

$$e_{t+1,t}^s = K_{t+1} - (\widehat{K}_{t+1} | K_t, \widehat{I}_{t+1,t}). \tag{5}$$

This forecast error will in general be non-zero because it includes the error made in forecasting I_{t+1} . By contrast the *one-step ahead single equation forecast error* is based on a forecast in period $t + 1$ that is conditional on *actual* values of all the other endogenous variables in $t + 1$ as in

$$e_{t+1,t} = K_{t+1} - (\widehat{K}_{t+1} | K_t, I_{t+1}). \tag{6}$$

This latter forecast error will be zero if the capital equation is an identity that is assumed to hold in the data in period $t + 1$.

Multi-step forecasts and forecast errors can be generated dynamically. For example, the *two-period ahead system forecast error* is the error from a forecast for the period $t + 2$ made using one-step ahead forecasts for variables dated at period $t + 1$ and two-step ahead forecasts for variables dated at period $t + 2$ as in

$$e_{t+2,t}^s = K_{t+2} - (\widehat{K}_{t+2} | \widehat{K}_{t+1,t}, \widehat{I}_{t+2,t}). \quad (7)$$

Similarly, the *two-step ahead single equation forecast error* is defined by

$$e_{t+2,t} = K_{t+2} - (\widehat{K}_{t+2} | \widehat{K}_{t+1,t}, I_{t+2}). \quad (8)$$

This forecast is based on the actual value of I_{t+2} because this is still a single equation forecast but the lagged value of the dependent variable, K_{t+1} needs to be forecast dynamically from the one-step ahead forecast in (6). Even so, in this particular case the forecast error will still be zero if the one-step ahead forecast error was zero and if the identity continues to hold in period $t + 2$.

Generally, system forecast errors will be expected to be larger than single equation errors since they include the errors made in forecasting all the variables in the model. As usual, the more steps ahead is the forecast, the larger the forecast errors are likely to be since errors will tend to cumulate dynamically.

Figure 1 shows three sets of forecast errors for consumption, C_t , from Klein's Model 1: single equation forecast errors (solid line), one-step ahead system forecast errors (dotted line) and dynamic system forecast errors (broken line). It can be seen that the single equation forecast errors are the smallest. These represent the errors in forecasting the consumption equation only, taking all other model variables as known. (These are also dynamic single equation forecast errors in this case since there are no lags of consumption in the consumption equation). The one-step ahead system forecast errors are larger since they include the effect of the interaction between the errors made in forecasting each of the model variables. The dynamic system forecast errors are the largest. These show the effect of forecasting the model dynamically from 1921 to 1941. For 1921, this error is a one-step ahead system error, for 1922 this is a two-step ahead system error and so on so that the forecast error in 1941 is from a twenty-one-step ahead forecast. Generally, these dynamic forecast errors will tend to cumulate although in this model it happens that by 1941 the forecasts have come back on track so that the last two errors are small. One possible reason for this is that the forecast period here is also the estimation period so that the parameter estimates incorporate information about the observations up to 1941. In a 'genuine' forecast beyond the end of the estimation period, this would be less likely to happen.

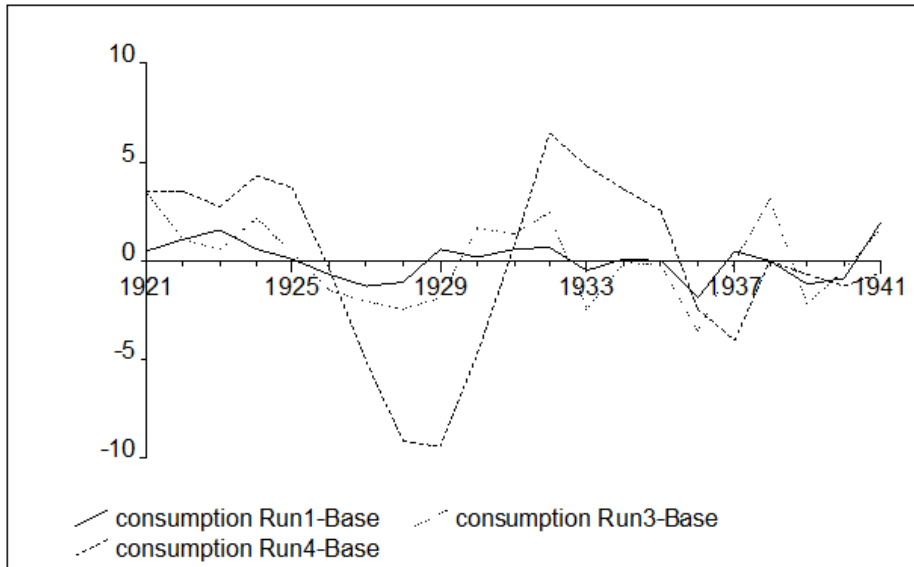


Figure 1: Consumption forecast residuals for Klein Model 1: single equation, one-step system and dynamic system residuals

4 Hypothetical forecasts: policy simulations

Some of the exogenous variables in Klein's Model 1 are policy variables that are either under the direct control of the government, such as government expenditure G_t and the government wage bill S_t , or are indirectly controllable such as indirect taxes T_t . It is possible to do forecasts based on different assumptions about these government policy variables and compare these with a base forecast. These type of *hypothetical forecasts* are generally known as 'scenarios' or *policy simulations*. In a forecast over the future, it would normally be assumed in the base forecast that the government continues to pursue policies unchanged from the past. With a forecast over a historical period as we have for Klein Model 1, the base forecast can simply be the historical policy actually pursued by the government and the policy simulations forecast what would have happened if the government had pursued an alternative policy.

What would have been the effect if the US government had pursued a Keynesian policy in the 1930s and increased its expenditure in order to offset the effect of the Great Depression of the 1930s? This question can be answered in Klein's Model 1 by performing a model simulation in which government expenditure G_t is

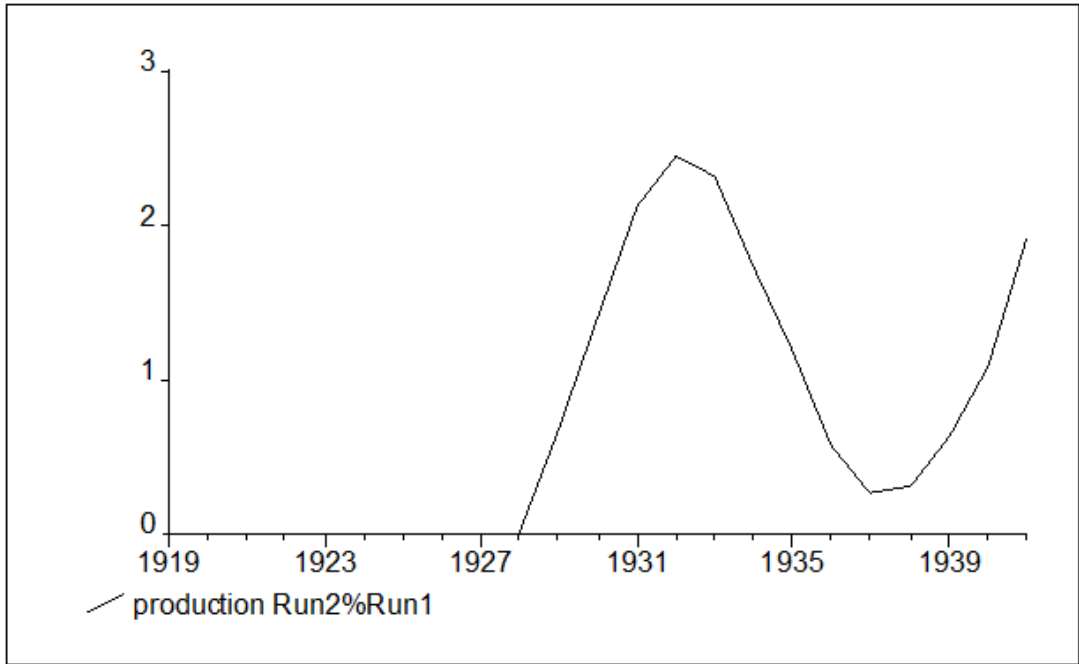


Figure 2: Government expenditure simulation: percent changes in production from base

increased by 5% relative to its historical values from the year 1929 (the year of the Wall Street crash that led to the Great Depression) until 1941 (the year that the USA entered the Second World War). Figure 2 shows the result of this simulation on the production variable Y_t . The results are reported as the percentage changes

$$100 \frac{\hat{y}_{t+h,t}^{sim} - \hat{y}_{t+h,t}^{base}}{\hat{y}_{t+h,t}^{base}}$$

from a base run forecast $\hat{y}_{t+h,t}^{base}$ in which G_t is at its historical values. Note that production relative to base increases by around 0.7% in the first year, rising to 2.4% in 1932. However, after this the effect begins to wear off and falls to about 0.27% in 1937 before rising again to 1.9% by the end of the period. This cyclical pattern is due to the dynamics in the model in which taxes affect current profits and then both current and lagged profits affect consumption and investment.

Simulations can also be used to examine the effect of changes to the endogenous variables. Since the endogenous variables are determined by equations, changes to these variables can be implemented as shocks to the endogenous equations. Two different types of shock can be distinguished: deterministic shocks and stochastic

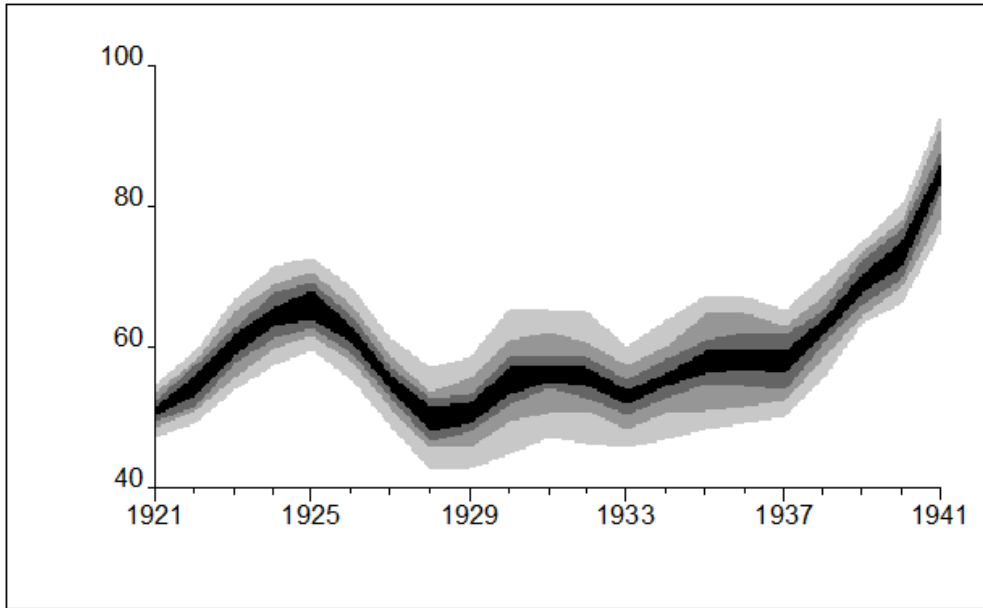


Figure 3: 10% quantiles of the forecast distribution of Y_t in Klein Model 1

shocks. Deterministic shocks can be used to explore the effect on the model of external events not captured in the model equations. Such external events would exhibit themselves as shocks to the model equations. For example, we could consider simulating the impact of the Wall Street crash of 1929 as a negative shock to the investment equation in Klein's Model 1. This could either be thought of as a single shock in the year 1929 or as a sustained shock lasting several years. Such a shock could either be additive (reducing I_t by \$10m *p.a.*) or multiplicative (reducing I_t by 10%).

Stochastic shocks can be used to derive interval and density forecasts through *stochastic simulation*. This involves solving a model repeatedly, each time adding a new drawing of pseudo-random shocks to the equations based on the distribution of the equation disturbances such as ε_{1t} , ε_{2t} and ε_{3t} in (1). Each model solution is called a *replication*. Generally the normal distribution is assumed for the shocks although in principle any distribution could be used. Percentiles of the distribution of the model forecasts can then be estimated from the distribution of the replications. The accuracy of these estimates can be increased simply by increasing the number of replications. Figure 3 shows deciles of the forecast distribution of production, Y_t in Klein's Model 1, estimated by stochastic simu-

lation using 100 replications. The bands in the graph can also be used to define interval forecasts, for example the outer bands correspond to a 90% confidence interval around the central point forecast. Since this model is linear, the intervals are symmetric. In a non-linear model, the intervals would be asymmetric and in general could only be calculated by stochastic simulation.

5 Optimal control

In the previous section we asked the question of Klein’s Model 1: what would have been the effect if the US government had increased its expenditure by 10% in the 1930s? This question we can answer by standard simulation techniques. Another question we could ask of the model is this: what would US government expenditure have needed to have been in order to offset the effects of the Great Depression and ensure that production grew over the entire period by 5%? This question cannot be answered by standard simulation techniques because the government expenditure variable G_t is *exogenous* and so is not determined in the model whereas the production variable Y_t is *endogenous* to the model and so cannot be fixed to grow at 5%.

To answer this sort of question of a model we need to use the techniques of *optimal control*. In optimal control it is assumed that there is a policy maker with targets that it wishes to achieve and policy variables that are under its control. The *target variables* are endogenous and so are determined by the model and the *control variables* are exogenous variables that the policy maker can set at will in order to achieve the targets. The targets of the policy maker can be defined in terms of a loss function to be minimised subject to the model. For example the government expenditure question posed above could be formulated in terms of the loss function

$$\min_{G_1 \dots G_T} \sum_{t=1}^T w_t (Y_t - Y_t^*)^2 \quad (9)$$

where the minimisation is subject to the model (1). Here Y_t^* are the target values of Y_t corresponding to a 5% annual growth rate and the loss function aims to minimise the sum of the weighted squared deviations of Y_t from Y_t^* . The parameters w_t are weights that in principle can be time varying. The solution to this problem is a set of optimal values for the control variables: G_1^*, \dots, G_T^* . The problem can be solved by numerical minimisation of the function (9) where each function evaluation involves solving the model for periods $1, \dots, T$ with guesses of G_1^*, \dots, G_T^* until the minimum is found.

Figure 4 plots the optimal values of G_t (dotted line) along with the actual historical values (solid line) over the solution period 1921–1941. It can be seen

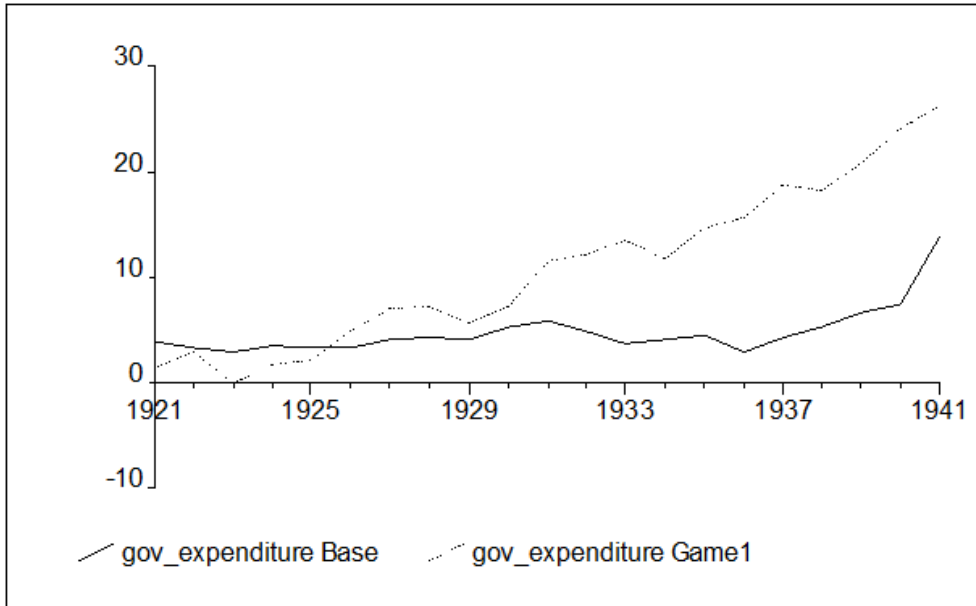


Figure 4: Optimal values of G_t necessary to achieve a 5% growth rate in Y_t

that from 1926 onwards, the level of government expenditure necessary to achieve the government's growth target is above the actual historical level, rising to approximately twice the historical expenditure by 1941. For the first five years of the solution period, however, the optimal level is actually less than the historical level. This is because, during this period, historical production growth was above the target rate of 5%.

6 Conclusions

Macroeconomic models provide a forecasting framework in which variables are jointly determined by sets of equations coming from economic theory. We have seen that we can use macroeconomic models to produce hypothetical forecasts or simulations to ask what would have happened if different policies had been pursued. Policy makers can use optimal control techniques to determine the setting of their policy instruments and achieve their targets. In a world in which agents in the economy form rational expectations of future government policy and will react today to changes expected in the future, optimal government policy becomes

more complicated. In this world there can be issues of *time inconsistency* where the government will have an incentive to change a previously announced optimal policy. Despite these complications, the techniques discussed in this lecture can still be applied. Whitley (1994) is a simple and readable introduction to macroeconomic forecasting that discusses many of these issues. *WinSolve* (Pierse (2003)) is a user-friendly software package designed for forecasting and simulation with non-linear macroeconomic models.

References

- [1] Klein, L.R. (1950), *Economic Fluctuations in the United States, 1921–1941*, Cowles Commission Monograph 11, New York
- [2] Klein, L.R., R.J. Ball, A. Hazlewood and P. Vandome (1962), *An Econometric Model of the United Kingdom*, Basil Blackwell, Oxford.
- [3] Pierse, R.G. (2003), ‘WinSolve Version 3: an introductory guide’, available at <http://www.econ.surrey.ac.uk/winsolve/>
- [4] Stone, J.R.N. and J.A.C. Brown (1962), *A Computable Model of Economic Growth*, Chapman and Hall, London.
- [5] Whitley, J.D. (1994), *A Course in Macroeconomic Modelling and Forecasting*, Harvester Wheatsheaf, Hemel Hempstead.