Common cycles in sectoral output in the UK*

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Abstract

This paper compares the cycles in UK sectoral output generated from both univariate and multivariate unobserved components models. Common trends and cycles are found among the sectors and it is found that these help to identify the cycles in the multivariate model.

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1 Introduction

There are many reasons why economists are interested in decomposing variables into trend and cycle components. Often, it is the cycle that is of primary interest and the trend is merely a nuisance that needs to be removed. Since Nelson and Plosser (1982), it has generally become accepted that economic variables possess unit roots which implies that the model for the trend needs to be stochastic rather than deterministic in order for the cycle to be stationary. No unique decomposion exists and two main approaches have been followed. One is the Beveridge-Nelson decomposition where the trend component is a random walk with drift. The other uses an unobserved components (UC) model where the trend component is a very flexible local level model, that includes, as special cases, both the random walk with drift model and the popular Hodrick-Prescott filter. Several univariate empirical studies have looked at aggregate output using one of the two approaches: Campbell and Mankiw (1987) estimated some simple ARIMA processes for US GNP while

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Harvey (1985), Watson (1986) and Clark (1987) used UC models to identify trend and cycle.[1]

A multivariate approach allows additional information from other variables to help identify the cycles in aggregate output. Some studies have made use of other aggregate variables. Kydland and Prescott (1988) used detrended US GNP and prices and found a negative relationship between the resulting cycles. For other examples see Clark (1989), Evans (1989), Blanchard and Quah (1989), King, Plosser, Stock and Watson (1991) and Evans and Reichlin (1994). Another source of information is disaggregated sectoral variables. The importance of sectoral information has been demonstrated by several authors. Long and Plosser (1983) showed that, in a multisectoral version of a real business cycle model, even when productivity shocks are independent across sectors, there will be comovement of activity measures in different sectors. Long and Plosser (1987) decomposed US output innovations into unobserved common factors or aggregate shocks and a set of independent disturbances unique to each sector. Their results suggested that common aggregate shocks, although significant, were less important than sector-specific shocks. Pesaran, Pierse and Lee (1993) and Lee, Pesaran and Pierse (1992) estimated multisectoral VAR models of output growth for the US and the UK respectively and investigated the effects of specific identified macroeconomic shocks and unidentified sectoral shocks on output persistence, finding that the latter were far more important than the former.

A multivariate analysis also introduces the possibility that trends and cycles may be common between variables. In fact, as demonstrated by Stock and Watson (1988b), if a set of n variables are cointegrated with r cointegrating vectors, then this implies that there are $n - r$ common trends between them. Engle and Kozicki (1993) introduced the more general concept of common features, defined as data features that are present in individual series but absent from a particular linear combination of those series. Cointegration is a common feature but another common feature of interest is common serial correlation patterns and this implies common cycles. Engle and Kozicki developed a test for the cofeature rank which is analogous to the Johansen (1988) test for the number of cointegrating vectors. If the cofeature rank is s, then this implies $n - s$ common features. Vahid and Engle (1993) showed how it was possible to use this approach to identify common trends and cycles.in the context of the multivariate Beveridge-Nelson Stock-Watson (BNSW) decomposition When the number of cointegrating vectors plus the number of common serial correlation features happens to sum to the number of variables, then this framework allows a very easy recovery of trend and

[1]Stock and Watson (1988a) is a good survey of this literature.

cycle components. Engle and Issler (1995) applied this approach to sectoral output for the US while Calcagnini (1995) looked at labour productivity for 6 different countries.

Common trends and cycles can also be introduced into the multivariate structural time series model of Harvey (1989) and Koopman et al. (1995). This approach constructs separate UC models of trend and cycle processes. Common trends or cycles imply a reduced rank for the covariance matrices of the corresponding components and this can be imposed on the model using factor loading matrices.

This paper applies the structural time series approach to a four sector model of quarterly output for the UK over the period 1980q1 to 1997q4. This can be compared with Engle and Issler (1995) who applied the Vahid and Engle approach to sectoral data for the US using 8 sectors and annual data. However, the primary focus of this paper is to examine the cycle component that comes out of the model and compare the results of univariate and multivariate analysis, the latter taking into account common trends and cycles. A plan of the paper is as follows: Section 2 sets out the model.. Section 3 examines the statistical properties of the data and tests for the number of common trends and cycles among the variables. Section 4 compares the cycles generated by univariate and multivariate models and compares both with a model of aggregate output. Section 5 presents the conclusions.

2 The Model

The model that we consider is an unobserved components structural time series model of the type described in Harvey (1985, 1989) and Koopman *et al.* (1995). The $n \times 1$ vector of variables $\mathbf{y}_t$ is decomposed into three components:

$$\mathbf{y}_t = \boldsymbol{\mu}_t + \boldsymbol{\psi}_t + \boldsymbol{\varepsilon}_t, \ \ t = 1, \cdots, T \tag{2.1}$$

where $\boldsymbol{\mu}_t$ is the trend component, $\boldsymbol{\psi}_t$.the cycle, and $\boldsymbol{\varepsilon}_t$ is an unexplained irregular component with covariance matrix $\boldsymbol{\Sigma}\varepsilon$. Separate unobserved components models are built for each of the first two components of (2.1) where the innovations in trend and cycle are constructed to be independent. The trend component is modelled by the local linear trend

$$\boldsymbol{\mu}_t = \boldsymbol{\mu}_{t-1} + \boldsymbol{\beta}_{t-1} + \boldsymbol{\eta}_t \tag{2.2}$$

$$\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} + \boldsymbol{\xi}_t \tag{2.3}$$

where $\boldsymbol{\mu}_t$ and $\boldsymbol{\beta}_t$ are $n \times 1$ vectors representing the level and slope of the trend respectively, and $\boldsymbol{\eta}_t$ and $\boldsymbol{\xi}_t$ are independent error processes with covariance matrices $\boldsymbol{\Sigma}_\eta$ and $\boldsymbol{\Sigma}_\xi$. In the general case the trend process is second order integrated, $(I(2))$. The slope parameter $\boldsymbol{\beta}_t$ allows this trend to change smoothly but in the special case where $\boldsymbol{\Sigma}_\xi$ is zero, the trend reduces to a random walk with drift term $\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1} = \overline{\boldsymbol{\beta}}$. If, in addition, $\boldsymbol{\Sigma}_\eta$ is zero, then the trend becomes deterministic.

One other case of interest is where $\boldsymbol{\Sigma}_\eta$ is zero but $\boldsymbol{\Sigma}_\xi$ is non-zero in which case the trend can be rewritten as

$$\Delta^2 \boldsymbol{\mu}_t = \boldsymbol{\xi}_{t-1}. \tag{2.4}$$

In this model the trend component evolves smoothly over time and so this is known as the *smooth trend* model. The Hodrick-Prescott filter is a special case of the smooth trend model, for the univariate case, where the degree of smoothness, determined by the ratio of the variance of the trend component σ_ξ^2 to the variance of the irregular component σ_ε^2 , is fixed in advance.

The cycle component of the model is trigonometric in form and consists of one or more cycles defined by the pair of equations:

$$\begin{bmatrix} \boldsymbol{\psi}_t \\ \overline{\boldsymbol{\psi}}_t \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda \mathbf{I}_n & \sin\lambda \mathbf{I}_n \\ -\sin\lambda \mathbf{I}_n & \cos\lambda \mathbf{I}_n \end{bmatrix} \begin{matrix} \boldsymbol{\psi}_{t-1} \\ \overline{\boldsymbol{\psi}}_{t-1} \end{matrix} + \begin{bmatrix} \boldsymbol{\omega}_t \\ \overline{\boldsymbol{\omega}}_t \end{bmatrix} \tag{2.5}$$

where $\boldsymbol{\psi}_t$ and $\overline{\boldsymbol{\psi}}_t$ are $n \times 1$ vectors and $\boldsymbol{\omega}_t$ and $\overline{\boldsymbol{\omega}}_t$ are vector error processes independent of $\boldsymbol{\varepsilon}_t$, $\boldsymbol{\eta}_t$ and $\boldsymbol{\xi}_t$ and with the same covariance matrix $\boldsymbol{\Sigma}\omega = \Sigma_{\overline{\omega}}$, $\mathbf{I}_n$ is the identity matrix of dimension n and the vector process $\overline{\boldsymbol{\psi}}_t$ appears by construction. The scalar parameters ρ and λ (which satisfy the restrictions $0 < \rho < 1$ and $0 < \lambda < \pi$) represent the cycle damping factor and frequency respectively. In the univariate case ($n = 1$), equation (2.5) corresponds to a restricted $ARMA(2,1)$ process where the two autoregressive roots form a complex conjugate pair.[2]

When there is more than one variable, it can be seen from (2.5) that the cycle damping factor and frequency ρ and λ are imposed to be the same for each variable. In the terminology of Koopman *et al.* (1995), this is the assumption of *similar* cycles, and it implies that the cycles for different variables have the same time series properties - the same autocovariance function and spectrum. This is a strong assumption and imposes some restriction on the model, although it does have the advantage of limiting the number of parameters that have to be estimated.

[2]It is possible to model more complex dynamics by allowing several cycles of different frequencies, thus allowing a more general specification for the cycle. This is not pursued further here.

2.1 Common trends and cycles

Suppose that the $\mathbf{y}_t$ process in (2.1) is cointegrated, having r cointegrating vectors given by the $(n \times r)$ matrix $\boldsymbol{\alpha}$. Then, by the definition of cointegration, the vector $\boldsymbol{\alpha}'\mathbf{y}_t$ will be *stationary.* (See Engle and Granger (1987)). Furthermore, Stock and Watson (1988b) showed that this implies that the n variables share $n-r$ common trends. Engle and Kozicki (1993) generalised this concept to that of common features, which are data features that are present in individual series but absent from a linear combination of those series. In particular Vahid and Engle (1993) looked at the feature of common serial correlation among variables which, if it exists, implies common cycles. They used a test developed by Engle and Kozicki to test the cofeature rank. This test is based on canonical correlation analysis along the lines of the Johansen (1988) test for the number of cointegrating vectors. If the serial correlation cofeature rank is s, then this implies $n-s$ common cycles.

In the context of the unobserved components model (2.1)-(2.5), common trends can arise either through common level components $\boldsymbol{\mu}_t$ or common slopes $\boldsymbol{\beta}_t$ or both. For the level components, the existence of common trends implies that

$$\boldsymbol{\mu}_t = \boldsymbol{\Theta}\mu \; \widetilde{\boldsymbol{\mu}}_t + \boldsymbol{\mu}_0 \tag{2.6}$$

where $\widetilde{\boldsymbol{\mu}}_t$ is the $(n-r \times 1)$ vector of *common levels*, $\boldsymbol{\mu}_0$ is a vector of fixed values and $\boldsymbol{\Theta}\mu$ is an $(n \times n-r)$ matrix of coefficients known as a factor loading matrix that satisfies the restriction that $\boldsymbol{\alpha}'\boldsymbol{\Theta}\mu = \mathbf{0}$. The variance covariance matrix $\boldsymbol{\Sigma}\eta$ will also be singular. For the slope components, common trends implies that

$$\boldsymbol{\beta}_t = \boldsymbol{\Theta}\beta \; \widetilde{\boldsymbol{\beta}}_t + \boldsymbol{\beta}_0 \tag{2.7}$$

where $\widetilde{\boldsymbol{\beta}}_t$ is the $(n-r \times 1)$ vector of *common slopes*, $\boldsymbol{\beta}_0$ is a vector of fixed values and $\boldsymbol{\Theta}\beta$ is an $(n \times n-r)$ factor loading matrix.

Similarly, the existence of common cycles in (2.5) implies that

$$\boldsymbol{\psi}_t = \boldsymbol{\Theta}\psi\widetilde{\boldsymbol{\psi}}_t \tag{2.8}$$

where $\widetilde{\boldsymbol{\psi}}_t$ is the $(n-s \times 1)$ vector of *common cycles* and the $(n \times n-s)$ factor loading matrix $\boldsymbol{\Theta}\psi$ satisfies the restriction $\widetilde{\boldsymbol{\alpha}}'\boldsymbol{\Theta}\psi = \mathbf{0}$. Note that no vector of fixed values is needed here because the cycle has zero mean.

The system (2.1)-(2.5) can then be rewritten in terms of the common trends and cycles as

$$\begin{aligned} \mathbf{y}_t &= \boldsymbol{\Theta}_\mu \; \widetilde{\boldsymbol{\mu}}_t + \overline{\boldsymbol{\mu}}_0 + \boldsymbol{\Theta}_\psi \; \widetilde{\boldsymbol{\psi}}_t + \boldsymbol{\varepsilon}_t, \; t = 1, \cdots, T \\ \widetilde{\boldsymbol{\mu}}_t &= \widetilde{\boldsymbol{\mu}}_{t-1} + \overline{\boldsymbol{\Theta}}_\beta \; \widetilde{\boldsymbol{\beta}}_{t-1} + \widetilde{\boldsymbol{\eta}}_t \end{aligned} \tag{2.9}$$

$$\widetilde{\boldsymbol{\beta}}_t = \widetilde{\boldsymbol{\beta}}_{t-1} + \widetilde{\boldsymbol{\xi}}_t$$

$$\begin{bmatrix} \widetilde{\boldsymbol{\psi}}_t \\ \widetilde{\overline{\boldsymbol{\psi}}}_t \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda \mathbf{I}_n & \sin\lambda \mathbf{I}_n \\ -\sin\lambda \mathbf{I}_n & \cos\lambda \mathbf{I}_n \end{bmatrix} \begin{bmatrix} \widetilde{\boldsymbol{\psi}}_{t-1} \\ \widetilde{\overline{\boldsymbol{\psi}}}_{t-1} \end{bmatrix} + \begin{bmatrix} \widetilde{\boldsymbol{\omega}}_t \\ \widetilde{\overline{\boldsymbol{\omega}}}_t \end{bmatrix}$$

where the error processes $\widetilde{\boldsymbol{\mu}}_t$, $\widetilde{\boldsymbol{\xi}}_t$, $\widetilde{\boldsymbol{\omega}}_t$ and $\widetilde{\overline{\boldsymbol{\omega}}}_t$ are of dimensions $n - r \times 1$, $n - r \times 1$, $n - s \times 1$, and $n - s \times 1$ respectively with nonsingular covariance matrices $\boldsymbol{\Sigma}_{\widetilde{\eta}}$, $\boldsymbol{\Sigma}_{\widetilde{\xi}}$ and $\boldsymbol{\Sigma}_{\widetilde{\omega}} = \boldsymbol{\Sigma}_{\widetilde{\overline{\omega}}}$. In the smooth trend case where $\boldsymbol{\Sigma}_{\widetilde{\eta}}$ is zero, the second equation in (2.9) may be replaced with

$$\widetilde{\boldsymbol{\mu}}_t = \widetilde{\boldsymbol{\mu}}_{t-1} + \widetilde{\boldsymbol{\beta}}_{t-1}$$

where $\boldsymbol{\Theta}_\mu = \overline{\boldsymbol{\Theta}}_\beta$. This is the version of the unobserved components model with common trends and cycles that is used in the estimation in the results reported below.

The factor loading matrices $\boldsymbol{\Theta}_\mu$, $\overline{\boldsymbol{\Theta}}_\beta$ and $\boldsymbol{\Theta}_\psi$ are not uniquely defined unless some conditions are imposed to ensure identification. The standard identification restrictions impose lower triangularity so that $\Theta_{ij} = 0$ for $\forall j > i$ and $\Theta_{ii} = 1$ for $\forall i$, and the associated covariance matrices $\boldsymbol{\Sigma}_{\widetilde{\eta}}$, $\boldsymbol{\Sigma}_{\widetilde{\xi}}$ and $\boldsymbol{\Sigma}_{\widetilde{\omega}}$ are diagonal. The last condition ensures that the common trends (cycles) are all uncorrelated with each other and the first two conditions imply that only the first common trend (cycle) affects the first sector, the first two common trends (cycles) the second sector and so on for the first $n - r$ ($n - s$) of the n sectors. These restrictions merely ensure identification; once the model parameters have been estimated, the common trends and cycles can then be transformed by premultiplication by any orthogonal matrix. This is called *factor rotation* and may allow the transformed common factors to be given a more useful interpretation.

3 The Data

Using the unobserved components approach, in this section we construct trends and cycles for Gross Domestic Product (GDP) of the United Kingdom disaggregated into the following four sectors (listed in order of size with their 1990 weight in parentheses, totalling 1000): Services (631), Production (278), Construction (72) and Agriculture (19). These sectors represent the highest level breakdown of the UK Standard Industrial Classification. The data come from Table 2.8 of the Office for National Statistics (ONS) Economic Trends; the observations are quarterly seasonally adjusted indices at constant factor cost (1990=100) for the period 1980q1 to 1997q4 (72 observations).

The data are plotted in natural logarithms in Figures 1, where all the analysis uses the logarithmic transformation. The largest sector, Services, is the most smoothly trended series and the one that most closely resembles aggregate GDP (of which it forms 63%). Production is noticeably more volatile than Services but clearly exhibits a similar pattern, where clear fall in output from 1990 which only recovers the 1990 level in 1994. Construction is also volatile in comparison with Services, and despite having the general appearance of an upward trend, shows periods of sharp decline in 1990 which only recovers partially, remaining below the 1990 level until the end of the period. The smallest sector, Agriculture, is highly volatile and is clearly the most separate and distinct sector of the four sectors, with no obvious pattern comparable with the other three sectors.

The first step in a more formal analysis is to test the order of integration of the series through the application of augmented Dickey-Fuller tests (ADF). Table 1 reports the results of ADF tests, for a range of lag augmentation, for each sector both in levels and first differences. With the exception of production all sectors cannot reject a unit root in levels. In the case of the production sector the result is marginal, with the Akaike Information Criterion selecting the $ADF(1)$ statistic of -3.76 as the preferred choice, which when compared with a 95% critical value of -3.47 rejects the presence of a unit root. The DF and $ADF(4)$ cannot reject the null of a unit root where the 99% critical value is -3.80 (Figure 1 does suggest production to be non-stationary I(1) variable). In first differences, the unit root null hypothesis is clearly rejected in all sectors. In the subsequent analysis we model four series for the sectors sectors as being I(1), although we need to be aware of the potential ambiguity regarding the order of integration of the production sector.[3]

[3]It is worth noting the relatively small sample in this application and the well known limitation of the small sample properties of these test statistics.

Table 1: Augmented Dickey Fuller Unit Root Tests

(i) Levels

Sector	ADF(0)	ADF(1)	ADF(2)	ADF(3)	ADF(4)
Services	-1.20	-1.89	-2.35^a	-2.59	-2.79
Production	-3.13	-3.76^a	-3.68	-3.60	-3.41
Construction	-1.18	-1.23	-2.09^a	-2.20	-2.32
Agriculture	-2.93	-2.89	-2.90	-2.77	-2.88^a

(i) First Differences

Sector	ADF(0)	ADF(1)	ADF(2)	ADF(3)	ADF(4)
Services	-4.56	-2.91^a	-2.56	-2.35	-1.92
Production	-5.06^a	-4.58	-4.33	-4.26	-3.49
Construction	-8.13	-3.51^a	-3.17	-2.89	-2.81
Agriculture	-6.37	-4.82	-5.40	-6.34^a	-4.88

Notes: The statistics are computed using 72 observations for the period 1980q1-1997q4. When applied to the levels the ADF test statistics are computed using ADF regressions with an intercept, a linear time trend and s lagged first differences of the dependant variable, while for the first differences an intercept and s lagged first differences of the dependant variable are used. The 95% critical value with time trend is -3.47 and without is -2.90. The symbol "a" denotes the order of augmentation in the Dickey-Fuller regressions chosen using the Akaike Information Criterion.

Turning to examine the cointegrating properties of the data using the Johansen methodology, the first step is to identify the appropriate model. A likelihood ratio test on an unrestricted VAR suggested an optimal lag length of 3, which is consistent with the choice made using the Akaike Information Criterion (AIC), all remaining tests are contructed using this assumption. In all subsquent analysis we work with a VAR model with unrestricted intercepts and restricted trend coefficients. This allows the intercepts to be freely determined but restricts the trend coefficients such that the solution to the model in levels will not contain a quadratic constant. Table 2a reports the Johansen cointegration test statistics.

The test statistics strongly support the null hypothesis of one cointegrating vector. Using either the trace or the maximum eigenvalue statistic, we reject the null hypothesis that r (the number of cointegrating vectors) is equal to zero but cannot reject the null that $r = 1$, at the 5% level of significance. This suggests three separate common trends amoung the four sectors.

The possibility of common (synchronous) cycles was then investigated using the cofeature test described in Engle and Kozicki (1993). This is based on the canonical correlations of the first differences of the data with their lags. The value of the test statistic for the number of cofeature vectors are reported in Table 2b.At the 5 per cent level of significance, the test suggests two cofeature vectors implying two common cycles.

The conclusion of the data analysis is therefore that the over the our sample period the four sectors exhibit three common trends and two common cycles.

Table 2a: Cointegration Rank Statistics

		Trace			Max		
H_0	H_1	Statistic	95% cv	90% cv	Statistic	95% cv	90% cv
$r = 0$	$r = 1$	68.47	63.00	59.16	38.61	31.79	29.13
$r \leq 1$	$r = 2$	29.86	42.34	39.34	16.10	25.42	23.10
$r \leq 2$	$r = 3$	13.76	25.77	23.08	8.27	19.22	17.18
$r \leq 3$	$r = 4$	5.49	12.39	10.55	5.49	12.39	10.55

Notes: The statistics are computed using 72 observations for the period 1980q1-1997q4. The underlying VAR model is of order 3 and contains unrestricted intercepts and restricted trend coefficients. "Trace" and "Max" represent Johansen's log-likelihood-based trace and maximum eigenvalue statistics, respectively, and 'cv' stands for critical value of the tests, which are obtained from Pesaran, Shin and Smith (1997).

Table 2b: Cofeature (Common Cycle) Tests

H_0	H_1	Test Statistic	Degrees of Freedom	p-value
$s \geqq 1$	$s = 0$	5.14	6	0.526
$s \geqq 2$	$s \leqq 1$	20.33	14	0.120
$s \geqq 3$	$s \leqq 2$	52.11	24	0.001
$s = 4$	$s \leqq 3$	112.46	36	0.000

Notes: The statistics are computed using 72 observations for the period 1980q1-1997q4. The underlying VAR model is of order 3.

4 Constructing Sectoral Trends and Cycles

Here we describe the sets of sectoral trends and cycles which result from the estimation of the model outlined in Section 2. We conduct the analysis first in a univariate context, to both aggregate and disaggregate GDP, and then in a multivariate setting where, following the previous sections analysis, we impose three common trends and two common cycles on the four sector breakdown of GDP. [4] In all cases we adopt the smooth trend specification with a fixed level and a stochastic slope plus an irregular and a trigonometric cyclical component[5]. There is an argument which would suggest starting from the most general structural time series specification, which in this case would imply substituting the fixed level for stochastic level. However, in this particular data set, if one starts from a stochastic level and slope with a cycle then the tendency is for a confabulation of these elements, such that the surface of the likelihood function becomes very flat. By imposing a fixed level, the stochastic part of the trend is the slope and this allows for a clearer identification of the cycle.

Before we proceed it is useful to establish some reference points with respect to cyclical movements in aggregate economic activity in the UK. Unlike the US, which has the NBER dated reference cycle, the UK has no established set of dated cycles or turning points in aggregate output. Therefore, as an alternative, we cite a number of reference points which have been noted and constructed by Quah (1994) and Artis, Bladen-Hovell and Zhang (1994). The reference points are first using Quah (1994) who identified one peak during our sample period in 1990q1 and then using Artis, Bladen-Hovell and Zhang (1994), with identified peaks (P) and troughs (T) in months, 1981m5(T), 1984m1(P), 1984m8(T), 1989m4(P) and 1992m5(T). The synchronisation of these two sets of peaks is not exact but does give a reasonable indication of turning points.[6]

4.1 Univariate Trends and Cycles

Beginning with the aggregate GDP trend in Figure 2, plottted in conjunction with the level aggregate GDP data, it is very clear that the estimated trend fits the data extremely well. This degree of overfitting is a particular feature

[4]The calculations for the unobserved components model EV model were performed in Version 5.0 of the computer package Stamp (Koopman et al. (1995)) .

[5]It is worth noting that this specification in the univarite case is equivalent to the Hodrick-Prescott trend when $\sigma_\xi/\sigma_\varepsilon = 0.25$.

[6]The approximate dating of these turning points is also consistent with the timing of peaks in the now discontinued ONS coincident index of economic activity.

of the unobserved components approach, which when faced with a series like aggregate GDP (and in the sectoral breakdown the Service sector) which for our sample period is very smooth. As a result the second graph in Figure 2, which plots the aggregate GDP cycle, shows a cyclical component whose order of magnitude is very small, with standard deviation of 0.304 (details of period etc included here). The size of the fluctuations appear to be relatively large at the beginning of the period trend, but noticeably are reduced from 1986 onwards. However there is some conformity in the peaks and troughs which correspond approximately with the datings given in Quah (1994) and Artis, Bladen-Hovell and Zhang (1994). The periods or specific points of peak and trough suggested by the cycle in aggregate GDP are the following. Peaks we date at 1984q2, and 1985q3 and 1990q3, two of which, the first and last, are close to peaks identified above. The incidence of troughs is the same as those for peaks and occur on the following dates: 1981q1, 1984q4 and 1983q1. Again two of the three, the first two, correspond closely with timings of troughs suggested by above.

The univariate cycles for the sectors have a varied interpretation where an important consideration is the degree of fit of the estimated trend term relative to the actual data. The trend term in the Service sector (not plotted), like the aggregate trend term fits the data very closely, where this is attributed to the very smooth nature of Service output data for this period. As a consequence the Service sector cycle, plotted along side the aggregate cycle in Figure 3, has a small size, with a standard deviation similar to the aggregate cycle of 0.2443. In the early period upto 1987 the Service sector cycle does not appear to be synchronous with the aggregate cycle but from this point onwards it exhibits close movement with the aggregate cycle. The contemporaneous correlation coefficient over the the whole sample is 0.51. The Production sectors exhibit close comovement with the aggregate cycle throughout the sample, with contemporaneous correlation coefficient of 0.85. The timing of the cycles peaks and troughs appear to be approximately the same, but Production shows greater volatility with a standard deviation of 0.85. The cycle in the Construction sector is difficult to compare with the aggregate as it appears to exhibit a sin-cosine pattern with virtually zero corelation with the aggregate, perhaps highlighting the limitation of the univariate estimation. Finally, the cycle in Agriculture appears to exhibit a very different cycle to that of the aggregate, with a weak negative correlation of -0.26 and a high standard deviation of 2.78. The economic meaning one might give to this cycle would require a specific comment on factors particular to the Agricultural sector.

The above results are consistent with some of the main qualitative features of business cycle time series which were highlighted by Lucas (1976):

namely that broadly defined sector do appear to move together, that is they have high conformity or high coherence and that the production (and prices) of agricultural goods (and natural resources) have lower than average conformity.

4.2 Multivariate Trends and Cycles

The univariate analysis does not incorporate the additional information potentially available by taking into account the correlations between the sectors. Also as we have seen above, both for the aggregate GDP and the Services sector there is an issue of overfitting the trend leading to a cycle with a small order of magnitude. In Construction the cycle follows a pure sine-cosine wave having little economic interpretation. In this section we therefore conduct a multivariate analysis analysis estimating the model in equation (2.9) imposing three common trends and two common cycles. Table 3 reports the results of the maximum likelihood estimation of the model, the coefficients of the factor loading matrices and the standard deviations of the error terms on trend, cycle and irregular components.

Table 3: Maximum likelihood estimation of UC Model

Table 3a: Common Trend Components

Sector	Θ_μ			μ_0	$\sigma(\xi)$	$\sigma(\varepsilon)$
Services	1	0	0	0	.00262	.00229
Production	1.23	1	0	0	.00814	.00063
Construction	2.13	0.65	1	0	.00767	.01062
Agriculture	-0.82	-1.53	1.05	.009	.01187	.00231

Table 3b: Common Cyclical Components

Sector	$\Theta\psi$		$\sigma(\omega)$
Services	1	0	.00009
Production	-72.13	1	.00692
Construction	37.90	37.12	.01008
Agriculture	-79.82	75.05	.02051

Notes: Estimation period: 1980Q1- 1997Q4; Model log-likelihood is 1205.93; Cycle Frequency= 0.316 Cycle damping factor = 0.829; Period of cycle: 4.97 years.$\Theta\mu$, $\Theta\psi$ are estimated coefficients of factor loading matrices of trend and cycle respectively μ_0 are estimated coefficients of the trend fixed values. $\sigma(\xi)$, $\sigma(\varepsilon)$ and $\sigma(\omega)$ are standard deviations of the error on the trend slope, irregular and cyclical components respectively.

The multivariate UC cycles are plotted in Figure 4, alongside the aggregate GDP cycle. Their common damping factor is 0.829 and the cycles have a period of 4.97 years, which corresponds quite well with a standard business cycle period. The overall features of these cycles is that they are (i) less correlated to the aggregate cycle measure than the univariate cycles (ii) correspond reasonably closely with the univariate cycles for Agriculture (see Figure 5) (iii) show an improved and interpreable fit in the construction sector but the feature of overfitting of the trend in the Service sector remains. In this instance the effect has been for cause a near non-existent cycle in the Service sector, whose standard error is 0.0157.

The cycle in the Production sector has greater volatility in the earlier period upto 1988, compared with the later period. Overall the volatility is higher in the multivariate case compared with both the univariate examples, increasing to 1.13. The correlation coefficient with the aggregate GDP cycle has fallen considerably from 0.8 to 0.23. The timing of the peaks and troughs also differs from those of the univariate analysis and corresponding less closely with out reference points. A peak occurs in 1985q2, later than the 1984m1 point identified in our reference cycle and a trough in 1983q3 which is between the trough reference points.

5 Conclusions

This paper has followed the unobserved components structural time series methodology for decomposing a set of variables into trend and cycle, allowing for possibility of common trends and common cycles. The approach was applied to a four sector disaggregation of output for the UK, where we found three common trends and two common cycles. The initial univariate results were unsatisfactory for both the Service and Construction sectors. The imposition of common factor restrictions within a multivariate analysis resulted in a cycle for Construction with a more plausible economic interpretation. However, the problem of a lack of role for the cycle in the Service sector remained.

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Figure 1: **Sectoral Output (in logarithms)**

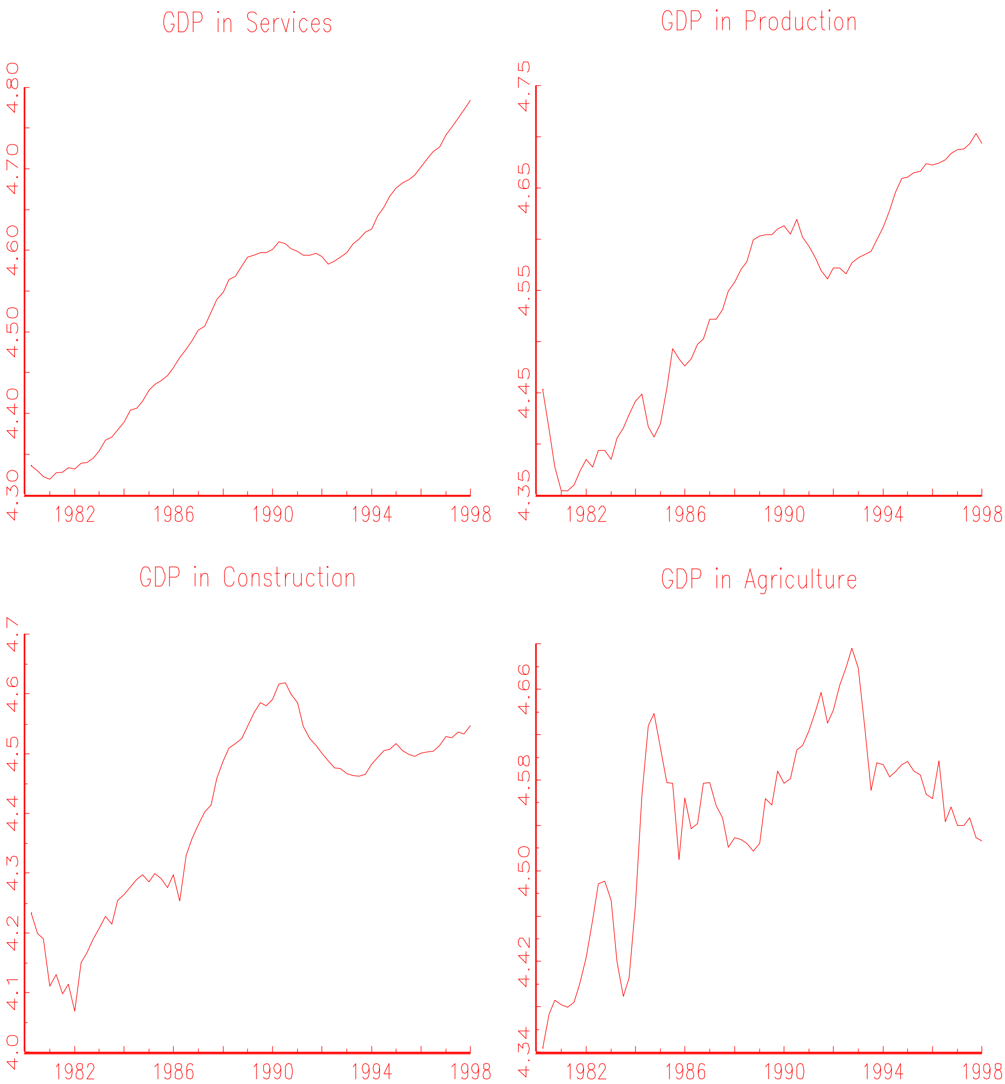

Figure 2: **Trend and Cycle in Aggregate GDP**

Figure 3: **Univariate Cycles in Sectoral and Aggregate GDP**

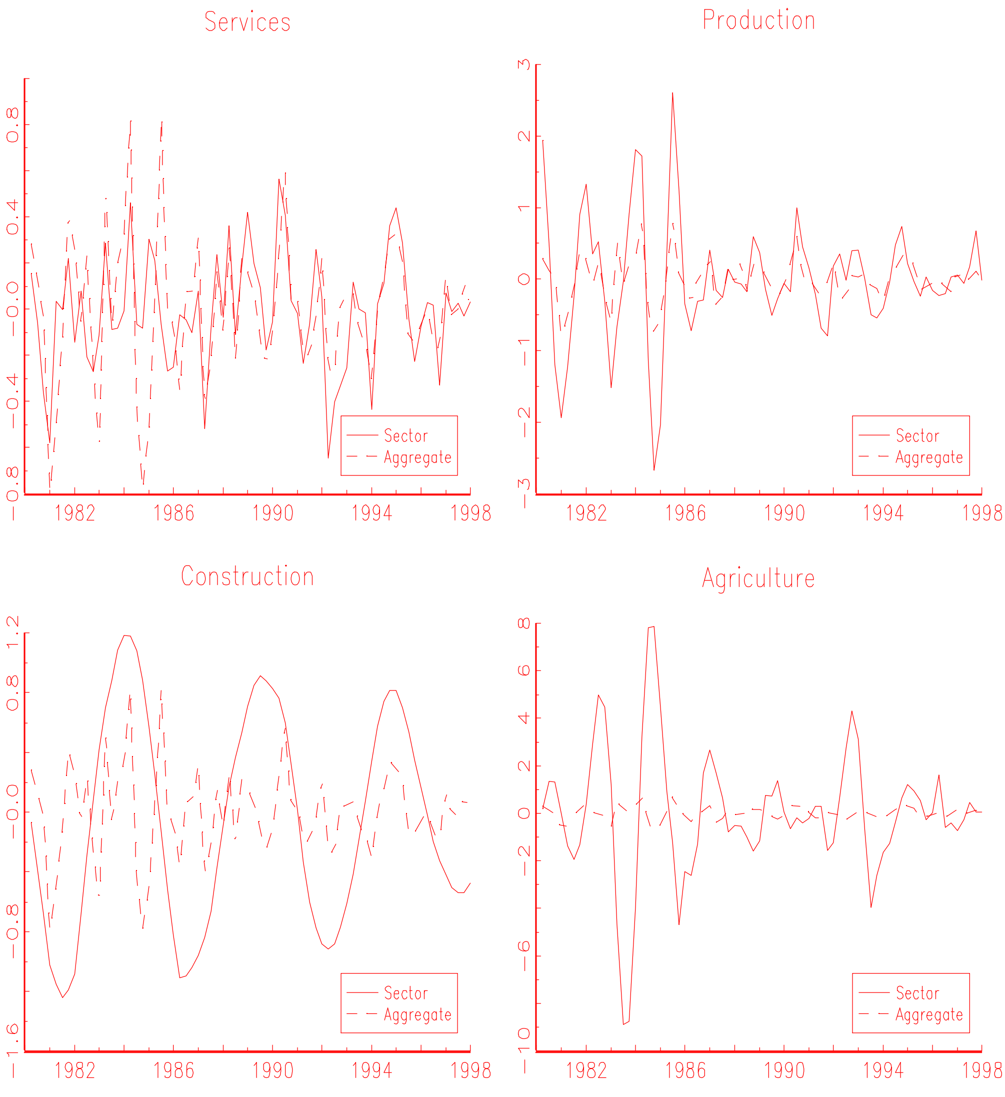

Figure 4: **Multivariate Cycles in Sectoral and Aggregate GDP**

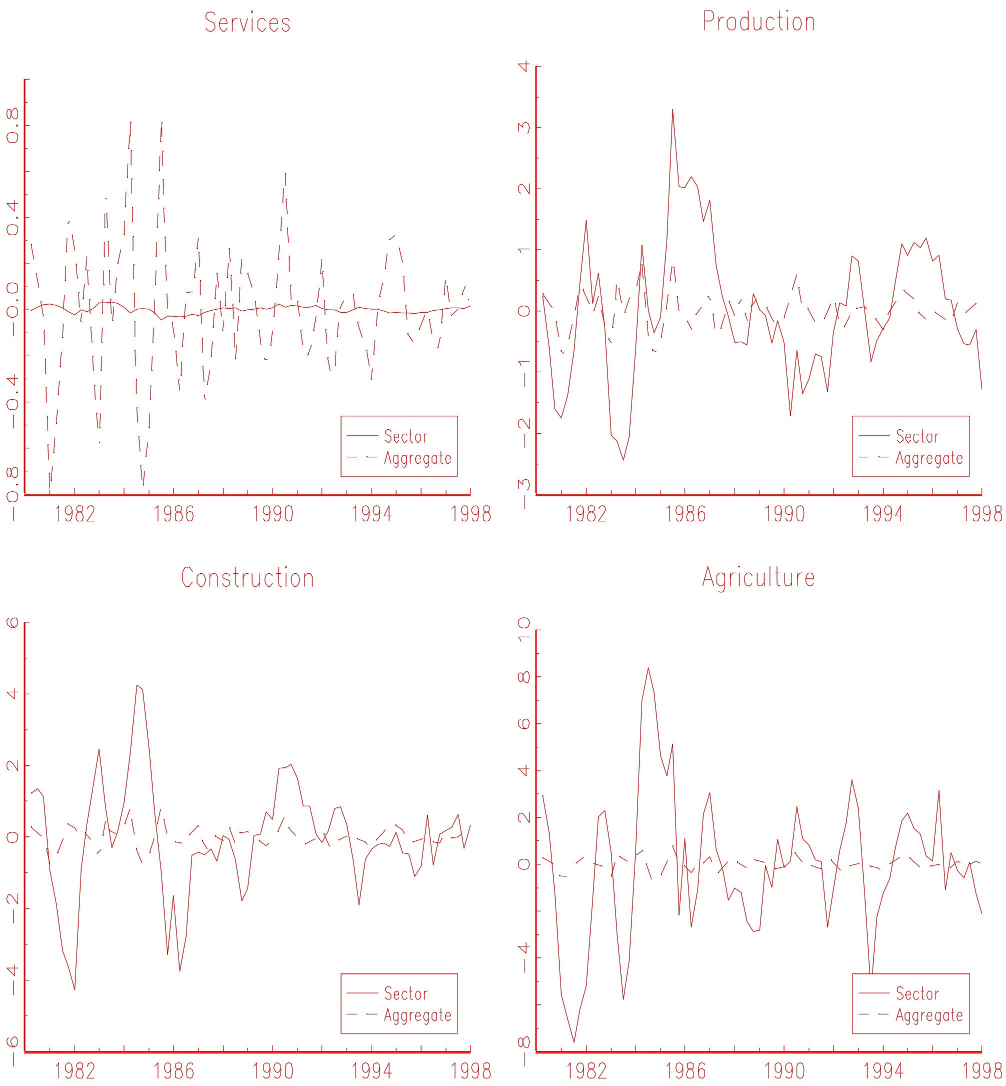

Figure 5: **Univariate versus Multivariate Sectoral Cycles**