INVENTORY MANAGEMENT

What are inventories?

Inventories are any quantities of items or economic resources being held in storage for some future use. Examples include:
- raw materials in a factory
- semi-finished goods in a multi-stage production process
- finished goods in a warehouse or retail outlet

Inventories are a non-earning asset. There are costs associated with holding them: storage costs and opportunity costs

Reasons for holding inventories

a) in a certain world
- fixed order costs may make it cheaper to buy in bulk
- in order to decouple demand from supply either:
  (i) when demand is smooth but production is seasonal
  (e.g. agricultural produce such as fruit or vegetables)
  (ii) when supply is smooth but demand fluctuates
  (e.g. seasonal goods like fireworks, Halloween masks)

b) in an uncertain world
- to avoid the risk of 'stock-out' where demand cannot be met from existing stock. This leads to a cost in terms of customer ill-will or loss of custom

Some Important Concepts

Costs
There are three components of inventory cost to be considered
- Ordering or Set-up costs
- Holding (Carrying) costs
- Stock-out costs

Lead Time
This is the time between putting in an order and receipt of the stock. Lead time can be either known or unknown.

DETERMINISTIC INVENTORY MODELS

In these models both demand and lead-time are known with certainty. Therefore stock-outs will never occur because they can be avoided.

Model 1: Economic Order Quantity Model (EOQ)

In this model, inventory is ordered from an outside supplier.
Assumptions of EOQ model

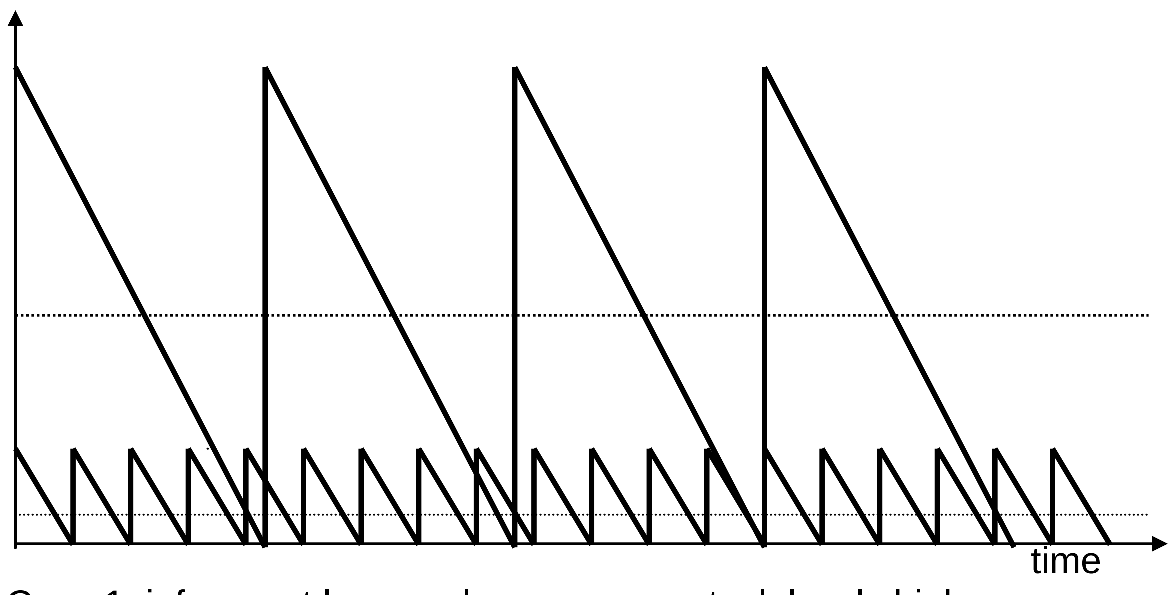
- Demand is constant
- Lead-time is constant
- Order cost is constant and independent of the quantity ordered

Given these assumptions, the quantity ordered will be constant.

Total Inventory cost (TIC) comprises two components:

Ordering costs (OC) incurred each time stocks are reordered
Holding costs (HC) proportional to average stock holdings

TIC = OC + HC



Case 1: infrequent large orders: average stock levels high

⇒ Holding costs high but order costs low

Case 2: frequent small orders: average stock levels low

⇒ Order costs high but holding costs low

Cost Components 1: Ordering Costs

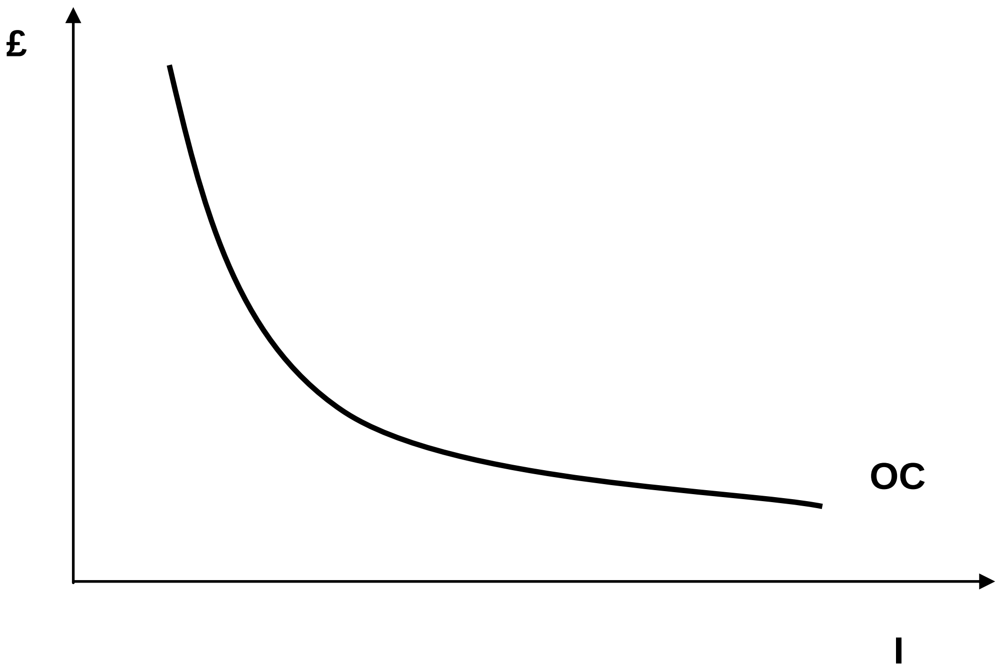
$$OC = C_o * D / I$$

where

	C_o	is the cost of placing each order
	I	is the size of inventory order placed
and	D	is (annual) demand for the inventory item

so that **D / I** is the number of inventory orders placed per year.

In this model, Order Costs are a nonlinear decreasing function of **I**.



Cost Components 2: Holding Costs

Holding costs will be either proportional to the average *quantity* of stock held or are proportional to the *value* of the stock held. This gives rise to two simple models:

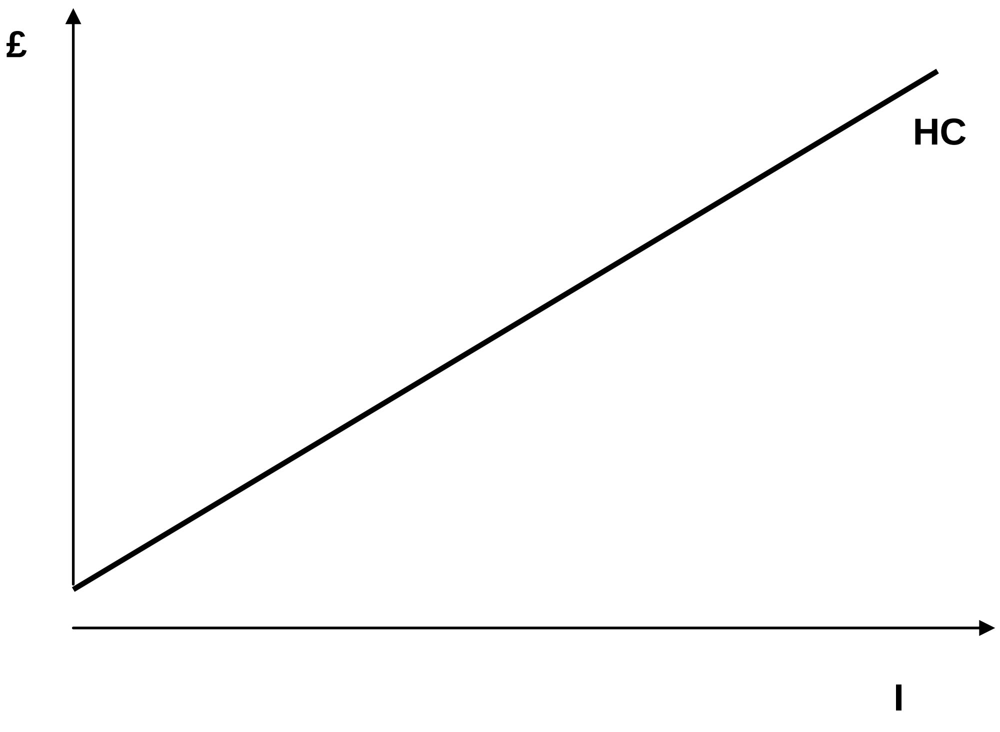
a) $$HC = C_h * I / 2$$

where C_h is the cost of holding each unit of inventory

b) $$HC = C_p * P * I / 2$$

where C_p is the cost as a proportion of the inventory value and **P** is the purchase price of the inventory

Both models imply that holding costs are an increasing linear function of **I**.



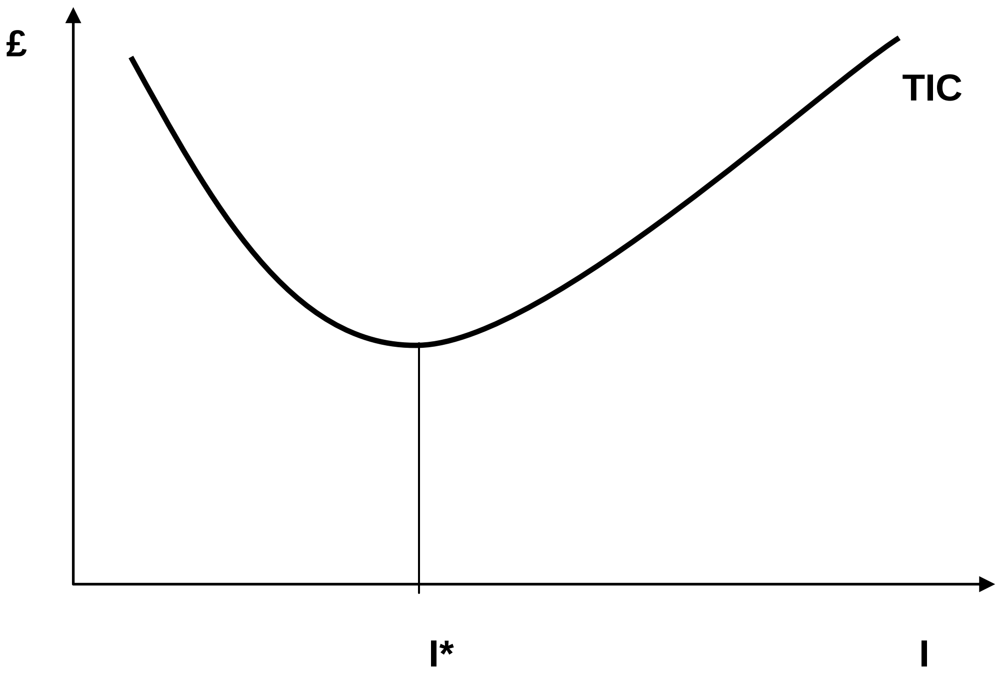
Total Inventory Costs

TIC = OC + HC

$= C_o * D / I + C_h * I / 2$ **(holding costs model a)**

or $= C_o * D / I + C_p * P * I / 2$ **(holding costs model b)**

Total costs are nonlinear in **I** with a minimum at **I***



Minimising TIC for case a) we have

$$\frac{\partial TIC}{\partial I} = - C_o * \frac{D}{I^2} + \frac{C_h}{2} = 0$$

implying an optimal order size quantity **I*** given by

$$I^* = \sqrt{((2 * C_o * D) / C_h)}$$

where the optimal number of orders per year is given by **D / I*.**

Similarly for case b)

$$I^* = \sqrt{((2 * C_o * D) / (C_p * P))}$$

Model 2: Production Lot Size Model (PLS)

In this model, inventory is produced internally by the firm. However, the rate of production is higher than the use rate so that continuous production would lead inventory levels to grow indefinitely. Instead, the firm has a production 'lot' or quota. It continues to produce until this quota is reached. Then it suspends production until it needs to start up again. Each time that production is restarted, there is a fixed set-up cost.

Assumptions of PLS model

- Demand is constant
- Lead-time is constant
- Set-up cost is constant and independent of the size of lot.

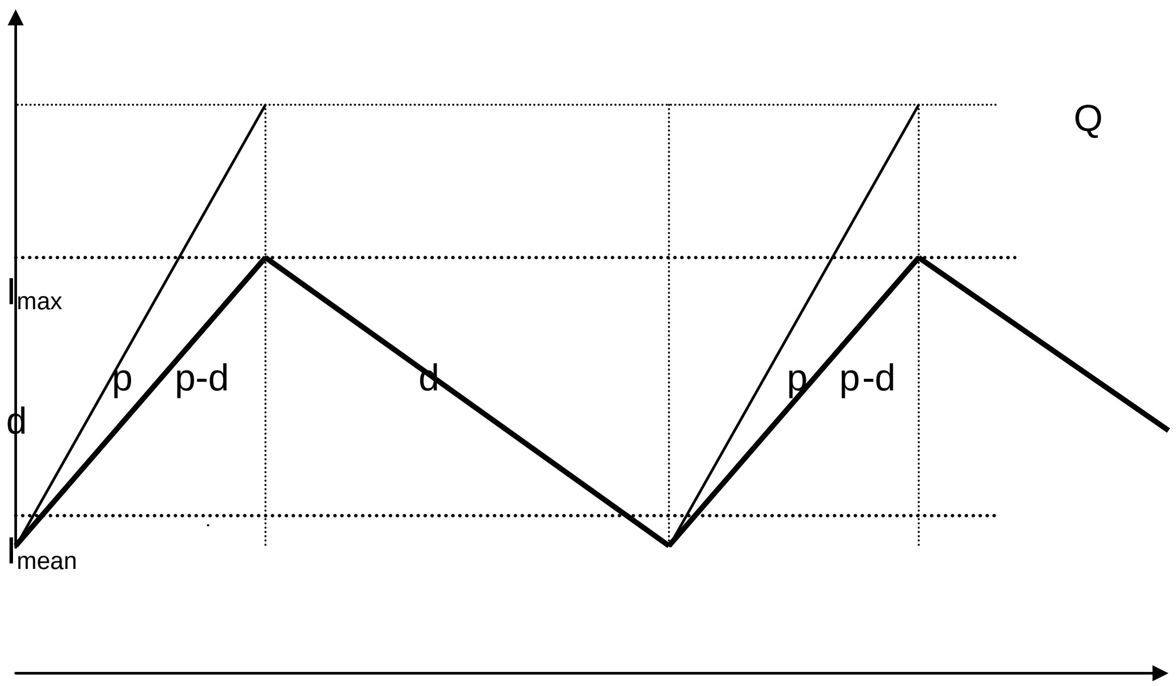
Given these assumptions, the production lot size will be constant.

Total Inventory cost (TIC) comprises two components:

Set-up costs (SC) incurred each time production is restarted
Holding costs (HC) proportional to average stock holdings

TIC = SC + HC

Inventory



{ production } { production }
time

p is the *rate* of production and d is the *rate* of demand

Cost Components

Set-up Costs

$$SC = C_s * D / Q$$

where

	C_s	is the cost of setting up each production run
	Q	is the size of the production run
and	D	is (annual) demand for the inventory item

so that **D / Q** is the number of production runs per year.

Holding Costs

The maximum inventory level I_{max} is given by:

$$I_{max} = Q (1 – d / p)$$

so (if holding costs are proportional to the average inventory level)

$$HC = 0.5 * C_h * Q (1 – d / p)$$

where C_h is the unit cost of holding inventory.

Total Inventory Costs

$$TIC = SC + HC$$

$$= C_s * D / Q + 0.5 * C_h * Q (1 – d / p)$$

Minimising TIC we have

$$\frac{\partial TIC}{\partial Q} = - C_s * \frac{D}{Q^2} + C_h \frac{(1 – d / p)}{2} = 0$$

implying an optimal production lot size **Q*** of

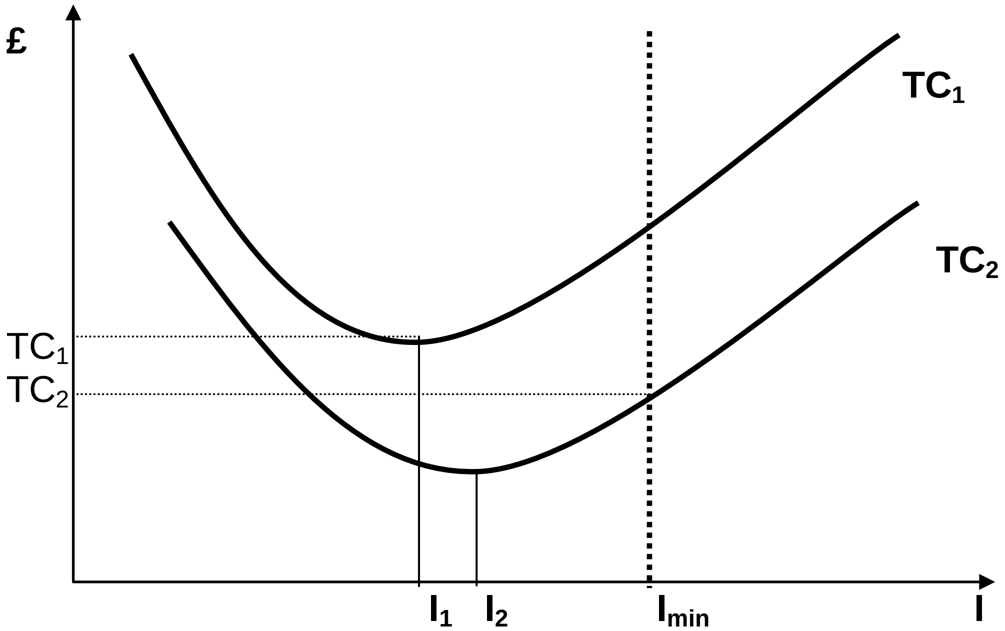
$$Q^* = \sqrt{((2 * C_s * D) / (C_h * (1 – d / p)))}.$$

The Economic Order Quantity Model with Quantity Discounts

Sometimes the supplier of inventory will offer the purchaser a quantity discount. This is a discounted price on the inventory available only for orders above some minimum threshold quantity. The supplier may offer a whole schedule of such discounts with minimum qualifying orders.

In this case we need to consider the total cost **TC** comprising the total inventory cost plus the purchase cost **D * P**

$$TC = TIC + D * P$$



In the diagram the lower total cost curve $\mathbf{TC_2}$ reflects the quantity discount which is only available for a minimum inventory order of $\mathbf{I_{min}}$.

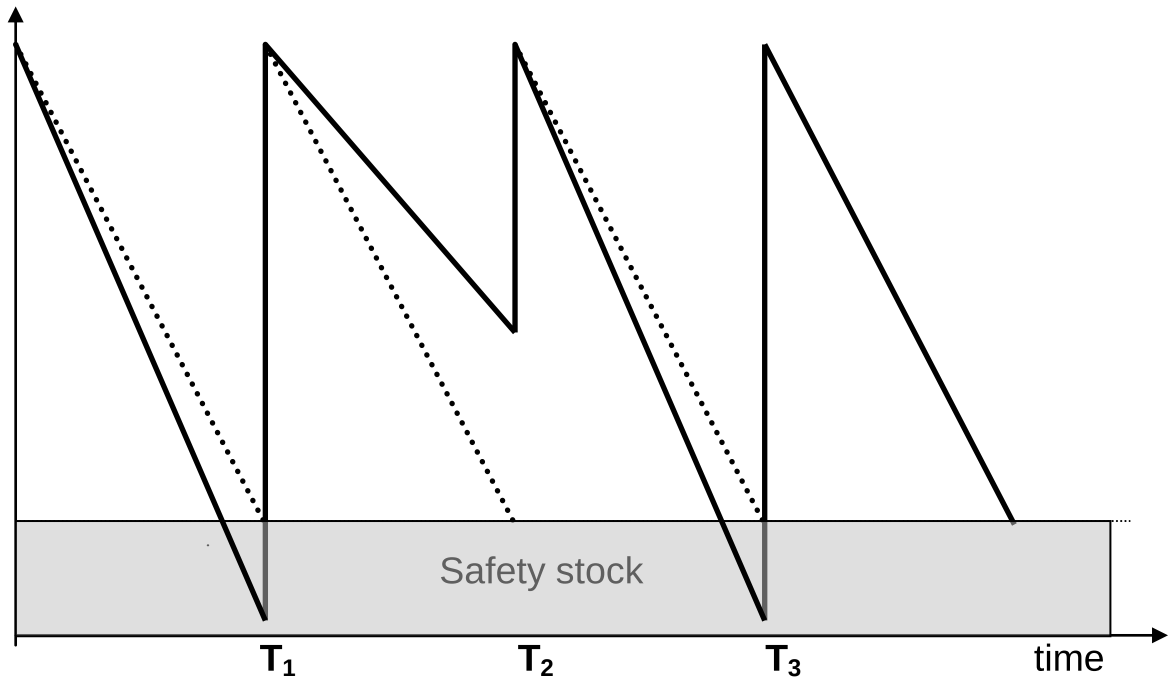
Ignoring the discount, the firm would choose to order $\mathbf{I_1}$ at a total cost of $\mathbf{TC_1}$.

At the discounted price the firm would ideally order $\mathbf{I_2}$ but this is below the minimum order to qualify for a discount and so is unavailable. The smallest order the firm could make to qualify for the discounted price would be $\mathbf{I_{min}}$ in which case total costs would be $\mathbf{TC_2}$. In this particular case, this happens to be lower than $\mathbf{TC_1}$ and so it pays the firm to take up the discount.

INVENTORY MODELS WITH UNCERTAINTY

Uncertainty about demand or lead-time leads to the possibility of stock-out when stocks run out too soon and demand cannot be met. With stock-out are associated costs of loss of goodwill and loss of custom.

Maintaining a buffer level of stocks or level of ‘safety stock’ can decrease the risk of stock-out.



In the diagram the dotted lines represent expected demand but the unbroken lines represent realised demand. Without safety stock, stock-out would have occurred at time $\mathbf{T_1}$ and $\mathbf{T_3}$.

With safety stock, stock-out will only occur when demand exceeds expected demand plus safety stock. With sufficiently high level of safety stock, this possibility can be made very unlikely. However, the higher the level of safety stock, the greater the inventory cost, so we need to determine the optimal level of safety stock to minimise expected total costs.

Determining the Optimal Level of Safety Stock

To determine the optimal level of safety stock, we need to know the likelihood of different levels of demand. We will assume that we can attach probabilities to different demand levels.

i	Demand	Probability
1	150	.12
2	200	.17
3	250	.44
4	300	.17
5	350	.06
6	400	.04

Expected demand (= average demand) = E(D) = $\sum_i D_i * p_i = 250$.

The amount of stock-out S_i depends on demand D_i and the level of safety stock, SS:

$$S_i = D_i - E(D) - SS \quad \text{(where } S_i = 0 \text{ when } D_i < E(D) + SS\text{)}.$$

The expected level of stock-out is thus given by $\sum_i S_i * p_i$

And expected stock-out costs **SOC** are given by

$$\mathbf{SOC = n\, C_{so} \sum_i S_i\, p_i}$$

where $\mathbf{C_{so}}$ is the cost per stock-out and n is the number of stock orders per year.

For example when SS = 50 for n=4 and $\mathbf{C_{so}}$=55

Demand	Prob	$\mathbf{S_i}$	$\mathbf{n\, C_{so} S_I}$
≤ 300	0.9	0	0
350	.06	50	11000
400	.04	100	22000

and expected stock-out cost = .06 * 11000 + .04 * 22000 = 1540.

The optimal level of safety stock is that which minimises total costs

$$\mathbf{TIC = OC + HC + SOC}.$$