Macroeconomics
Lecture 3: Growth with Land and Natural Resources

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1 Introduction

The Solow-Swan growth model considers output to depend on just two factor inputs, capital K and labour L, which are assumed to be able to grow indefinitely, labour through population growth (and in efficiency through technical progress) and capital through investment funded by savings. In this lecture we consider the effect of adding two further factors of production to the model: land which is in fixed supply, and natural resources, the supply of which we will asssume to be declining as they become exhausted. Perhaps surprisingly, it will turn out that the conclusions of the model remain largely unchanged, provided that the rate of technical progress is large enough.

2 Apocalyptic Visions

2.1 Malthus

Thomas Malthus's *Essay on the Principle of Population*, first published anonymously in 1798 (Malthus (1798)) presented an apocalyptic vision of the future. In it Malthus theorised that while population increases geometrically (exponential growth in modern terminology), agricultural production, which depends on a fixed supply of land, can increase at best only arithmetically (linear growth in modern terminology). The outcome, Malthus prophesied, would be that output per head would fall until it reached the subsistence level, beyond which the population would begin to decline due to starvation. Eventually, when the population had fallen enough to raise output per head above the subsistence level, then the birth rate would again begin to outstrip the death rate and the vicious cycle would begin again. Malthus's prescription to avoid this catastrophe and break the vicious circle was to advocate self-imposed moral constraints to curb population growth (abstinence or delayed marriage).

One interesting aspect of Malthus's theory is that it suggests that population growth is *endogenous*, the death rate depending on output per head. Obviously, we know that Malthus's bleak prophesy has failed to come true (so far at any rate!). There are several reasons for this. The most important is that technological innovations have continuously increased the efficiency of agricultural production. In fact the start of this process, the *British Agricultural Revolution*, was already underway by the time Malthus was writing (Jethro Tull's seed drill was invented in 1701 and early iron ploughs began to appear in the 1730s), and this was followed by the *Industrial Revolution* of the 19th century and the *Green Revolution* that began in the mid-20th century (Figure 1 shows the increase in wheat yields in developing countries between 1950 and 2004). All these revolutions have meant

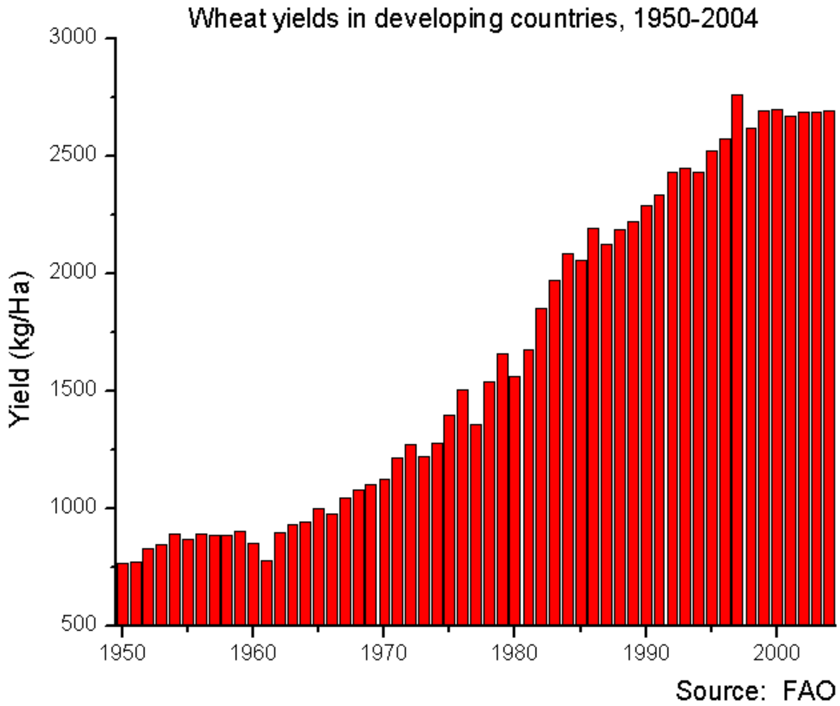

Figure 1: Reproduced from Wikipedia

that agricultural production has continued to increase despite the fixed (or falling) stock of agricultural land. In addition, although improvements in medicine have led to falling death rates, in the developed world at least, declining fertility and the increased use of contraception have meant that birth rates have also been falling so that population growth has been checked.

2.2 The Limits To Growth

In the 1970s, a new variant of Malthusian theory became popular, this time focusing not on land like Malthus, but on declining stocks of natural resources such as oil and rare metals. In 1972, The Club of Rome published an influential book called *The Limits To Growth*, (Meadows *et al.* (1972)). Using a computer model based on Jay Wright Forrester's *system dynamics* methodology, (Forrester (1971)), they extrapolated current consumption rates of these resources (or consumption growth rates) and computed how long current reserves would last. Table 1 (reproduced from Meadows *et al.* (1972)), shows the results for four key resources: chromium, gold, iron and oil (petroleum). Based on reserves known in 1972, their model predicted that, at current consumption levels, oil would run out in 31 years and, if consumption were to continue to grow at the current rate, it would run out in only 20 years (*i.e.* by 1992!). Even assuming that actual reserves were five times known reserves, at the current growth of consumption, oil would still run out in 50 years.

		Years remaining		
Resource	*Consumption growth*	*Static*	*Exponential*	*5 × reserves*
Chromium	2.6%	420	95	154
Gold	4.1%	11	9	29
Iron	1.8%	240	93	173
Petroleum	3.9%	31	20	50

Table 1: Reproduced from Meadows *et al.* (1972)

In addition to declining stocks of natural resources, *The Limits To Growth* also modelled environmental pollution, a by-product of industrial production which the model assumed would also grow exponentially, while the ability of the planet to absorb pollution was assumed to be constant.

On its publication, *The Limits To Growth* was roundly criticised by economists (including Robert Solow). One criticism was that the model does not allow for the effect of the price mechanism. As resources become more scarce, rising prices should both depress demand (to an extent depending on the demand elasticity) and increase supply (as more expensive production processes become profitable).

3 The Solow Model with Land and Natural Resources

Nordhaus (1992) extended the Solow-Swan model to allow for both land and natural resources as additional factors of production. The production function becomes

$$Y(t) = F(K(t), A(t)L(t), T(t), R(t)) \tag{3.1}$$

where T is the stock of land and R is the flow of natural resources used in production. We assume that the stock of land is fixed ao that

$$\frac{\dot{T}}{T} = 0 \tag{3.2}$$

while the flow natural resources available to production is declining over time as the stock becomes exhausted so that

$$\frac{\dot{R}}{R} = -\rho. \tag{3.3}$$

where $\rho > 0$. As before, we assume that technical progress and population grow at constant rates given by

$$\frac{\dot{A}}{A} = g. \tag{3.4}$$

and

$$\frac{\dot{L}}{L} = n. \tag{3.5}$$

Nordhaus assumes a constant returns to scale Cobb-Douglas production function of the form

$$Y = K^{\alpha} T^{\beta} R^{\gamma} (AL)^{1-\alpha-\beta-\gamma} \tag{3.6}$$

where $\alpha > 0$, $\beta > 0$, $\gamma > 0$ and $\alpha + \beta + \gamma < 1$.

Taking logarithms of (3.6) we have

$$\log Y = \alpha \log K + \beta \log T + \gamma \log R + (1 - \alpha - \beta - \gamma) \log (AL) \tag{3.7}$$

and differentiating with respect to t,

$$\frac{d \log Y}{dt} = \alpha \frac{d \log K}{dt} + \beta \frac{d \log T}{dt} + \gamma \frac{d \log R}{dt} + (1 - \alpha - \beta - \gamma) \frac{d \log (AL)}{dt} \tag{3.8}$$

or

$$\frac{\dot{Y}}{Y} = \alpha \frac{\dot{K}}{K} + \beta \frac{\dot{T}}{T} + \gamma \frac{\dot{R}}{R} + (1 - \alpha - \beta - \gamma) \frac{\dot{(AL)}}{AL} \tag{3.9}$$

where we make use of the result that

$$\frac{d\log x}{dt} = \frac{\dot{x}}{x}.$$

Substituting (3.2), (3.3), (3.4) and (3.5) into (3.9), gives

$$\frac{\dot{Y}}{Y} = \alpha\frac{\dot{K}}{K} - \gamma\rho + (1-\alpha-\beta-\gamma)(g+n). \tag{3.10}$$

For a balanced growth path, we require that capital and output are growing at the same rate. To see this, recall the equation for the evolution of capital:

$$\dot{K} = sY - \delta K. \tag{3.11}$$

Dividing by K gives

$$\frac{\dot{K}}{K} = s\frac{Y}{K} - \delta \tag{3.12}$$

so for the capital growth rate to be constant, the capital output ratio K/Y must be constant which implies that

$$\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K}. \tag{3.13}$$

Using this result in (3.10), the balanced growth path for output Y is given by

$$\frac{\dot{Y}}{Y} = \frac{(1-\alpha-\beta-\gamma)(g+n) - \gamma\rho}{1-\alpha}. \tag{3.14}$$

For this growth rate to be positive requires that

$$(1-\alpha-\beta-\gamma)(g+n) > \gamma\rho. \tag{3.15}$$

If the rate of natural resource depletion ρ is too high, positive output growth will not be possible.

The growth path for output per capita $y = Y/L$ is given by

$$\frac{\dot{y}}{y} = \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = \frac{(1-\alpha-\beta-\gamma)(g+n) - \gamma\rho}{1-\alpha} - n \tag{3.16}$$

$$= \frac{(1-\alpha-\beta-\gamma)g - (\beta+\gamma)n - \gamma\rho}{1-\alpha} \tag{3.17}$$

so that per capita growth will ony be positive if the rate of technical progress g outweighs the rate of population growth n and the rate of depletion of natural resources ρ.

To assess the extent of the *drag* on growth caused by a fixed stock of land and diminishing natural resources, Nordhaus compares (3.17) to the per capita growth rate that would result if T and R were growing at the rate of population growth n. In this base case, (3.10) would be replaced by

$$\frac{\dot{Y}}{Y} = \alpha\frac{\dot{K}}{K} + \beta n + \gamma n + (1-\alpha-\beta-\gamma)(g+n) \tag{3.18}$$

or, using (3.13),

$$\frac{\dot{Y}}{Y} = \frac{(1-\alpha-\beta-\gamma)g + (1-\alpha)n}{1-\alpha}. \tag{3.19}$$

and

$$\frac{\dot{y}}{y} = \frac{(1-\alpha-\beta-\gamma)}{1-\alpha}g. \tag{3.20}$$

The *drag* to growth caused by finite resources is the difference between (3.17) and (3.20) which is

$$\frac{(\beta+\gamma)n + \gamma\rho}{1-\alpha}. \tag{3.21}$$

Nordhaus's model assumes a Cobb-Douglas production function wich has a constant elasticity of substitution of unity between factors. This means that a given percentage increase in technical progress A has the same effect on output regardless on how small land per head T/L and natural resources per head R/L become as they decline. What would happen if the elasticity of factor substitution were less than unity? In this case, as the declining inputs become more scarce, the share of income going to them would rise over time and the *growth drag* would be increasing over time. In the limit, the share of income going to the slowest growing factor, natural resources R, which is γ, would approach 1 while α and β would approach 0. From (3.21), it can be seen that the *growth drag* would approach $n + \rho$ and from (3.17), output per capita would end up falling at the rate $n + \rho$. However, historically there is no evidence of shares of income going to land and natural resources increasing over time, in fact the reverse is the case. This would suggest that the elasticity of factor substitution is actually greater than one which implies that the economy can always compensate for declining resources by substituting away from them.

4 Conclusions

Nordhaus shows that per capita growth is still possible in the Solow-Swan model even with fixed land and declining natural resources, but only if the rate of technical progress g is large enough to outweigh the drag caused by finite resources.

Historically, since per capita income has continued to grow around the world over the centuries and the Domesday scenarios envisaged by Malthus and *The Limits To Growth* have failed to materialise, it seems that this condition must always have held up to now. However, the rate of technical progress is just exogenously given in this model so there is no explanation for why it is what it is or whether it can be relied upon to continue at the same rate. In the next lecture, we look at endogenous growth models which endeavour to account for why technical progress takes place.

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