Macroeconomics
Lecture 6: The Traditional Keynesian Model

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1 Introduction

In the last five lectures we have explored various models of growth. These are all long run models in which output, the capital stock, consumption and employment (assumed equal to the labour force) all follow constant growth paths. In this lecture we turn to short run models in which output and consumption can vary cyclically and employment can be less than the labour force so that unemployment can exist. In the short run, the capital stock is less important than its rate of change, the investment rate. We will also introduce a government sector that imposes taxes and makes expenditure.

2 Short run fluctuations

In long run growth models like those we have studied in previous lectures, economic variables grow smoothly. In the real world things aren't like that. Figure 1 from Romer (2012) graphs the logarithm of output per head from 1947 to 2009. This does not follow a straight line path (consistent with a constant rate of growth) and though generally upward sloping has peaks and troughs around the general trend with several periods in which output per capita was falling (including 2008 at the very end of the graph). This short run fluctuation around a long run growth trend is known as the *business cycle*.

John Maynard Keynes published his *General Theory of Employment, Interest and Money* in 1936, (Keynes (1936)), while the world was still in the grip of the *Great Depression* following the *Wall Street crash* of October 24, 1929. In 1932, the rate of unemployment in the USA peaked at 23% and in the UK at over 14%. Keynes's *General Theory* was very much a book of its time. It promised both a theoretical explanation for how involuntary unemployment could persist in an economy and also a set of practical prescriptions (specifically government fiscal and monetary policy) through which it could be reduced.

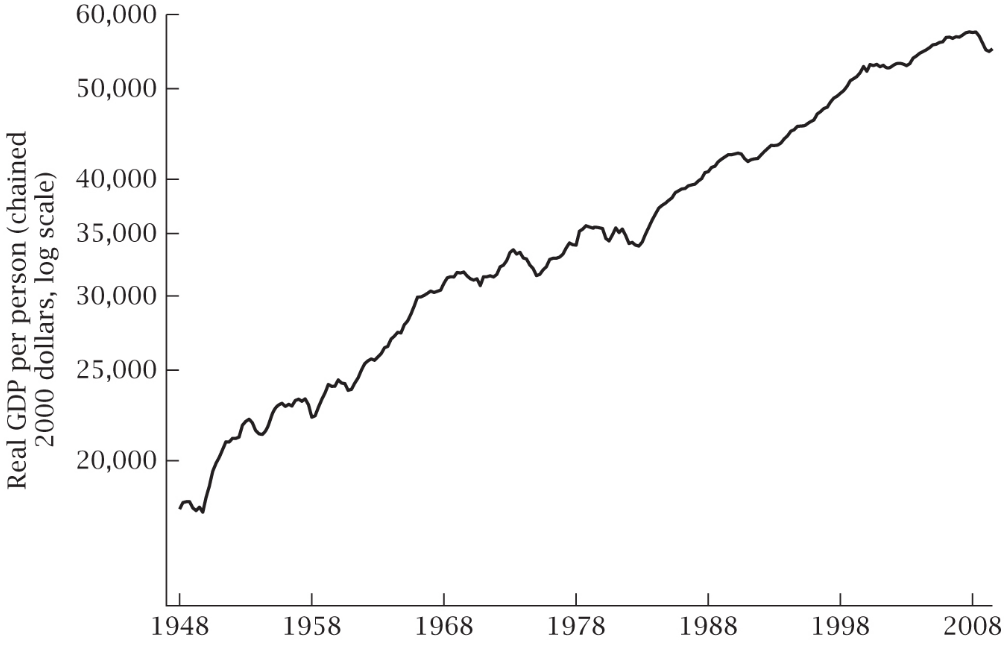

FIGURE 5.1 U.S. real GDP per person, 1947:1–2009:3

Figure 1: [Figure 5.1 in Romer (2012)]

It is difficult to overestimate the initial impact of the *General Theory* on economics. It virtually created the subject of macroeconomics and the Keynesian model was taught in Universities for decades as the main model for explaining macroeconomic phenomena. In the post-war period, many Western governments pursued Keynesian policies in an attempt to smooth out fluctuations in output and to maintain full employment.

However, the success of these policies was questionable as can be seen in Figure 1. Also, on a theoretical front, there was a revaluation of Keynes. Keynes's arguments in the *General Theory* were mainly verbal (the book contains very few equations) and what Keynes was saying seemed to contradict many of the precepts of neo-classical economics such as that markets always clear. The *neo-classical synthesis* was an attempt to reconcile an interpretation of Keynes's ideas with the standard neo-classical model. The pioneers of the *neo-classical synthesis* were John Hicks and Paul Samuelson (both early Nobel laureates in 1972 and 1970 respectively). Another key work in this literature is Patinkin (1956) which made the first attempt to reconcile Keynes with neo-classical economics in a micro-founded general equilibrium context. However, the resulting synthesis left Keynes's model just as a special case of the neo-classical model in which wages or prices are rigid or 'sticky' (slow to adjust), rather than being a more general model as Keynes himself claimed.

Some economists have argued that the *Keynesian model* as developed by Hicks and Samuelson is a mis-representation of what Keynes was actually saying. As is pointed out in Leijonhufvud (1967) and (1968), wage rigidity in the *General Theory* was a policy recommendation, not a behavioural assumption. Clower (1965) argues that Keynes's model describes disequilibrium transactions being taken in a dynamic context where quantity adjustments might be a better first response than price adjustments. Thus employers facing falling demand might first cut employment rather than reduce wages. Shackle (1967) argues that Keynes was writing about decision making in a world of great uncertainty where equilibrium is not relevant.

Despite the question of whether or not the *Keynesian model* accurately represents the ideas of Keynes, this model has become the standard model for discussing short-run fluctuations in macroeconomics and is therefore the model we discuss in this lecture. A note on David Romer's *Advanced Macroeconomics* textbook is in order here. In the third edition, Romer (2006), Chapter 5, 'Traditional Keynesian Theories of Fluctuations', gives a terse account of the standard Keynesian model while Chapter 6 develops some micro-founded models with Keynesian features. In the fourth edition, Romer (2012), these two Chapters have been replaced by a single Chapter 6 on 'Nominal Rigidity' which is micro-founded from the start and 'Keynes's model' is relegated to a single special case of this model with nominal wage rigidity. Thus if you are using Romer's textbook, you will need to refer to the Third edition for the model in this lecture. Otherwise, take a look at one of the other textbooks recommended on the reading list. The micro-founded neo-Keynesian models will be the topic of next week's lecture.

3 The IS Curve

As in previous lectures we continue to assume a closed economy with no exports or imports. However, we now introduce a government sector. The income identity thus becomes

$$Y = C + I + G \tag{3.1}$$

where Y is income (or output), C is consumption by the private sector, I is investment by the private sector and G is consumption plus investment by the public sector (government). The government also raises taxes so that its budget surplus is

$$T - G \tag{3.2}$$

where T is net tax receipts (taxes minus subsidies). We will assume that government expenditure G and tax receipts T are determined exogenously by government fiscal policy.

Household consumption is determined by the consumption function

$$C = C(Y - T, r) \tag{3.3}$$

where $Y - T$ is disposable income (income less taxes) and r is the real interest rate defined by

$$r = i - \pi \tag{3.4}$$

where i is the nominal rate of interest and π the rate of price inflation. The derivatives of the consumption function are assumed to have the following properties:

$$0 < \frac{\partial C}{\partial (Y - T)} < 1, \qquad \text{and} \qquad \frac{\partial C}{\partial r} < 0. \tag{3.5}$$

The first of these derivatives was called by Keynes the *marginal propensity to consume*. In fact Keynes did not consider the dependence of consumption on the interest rate but this can be allowed without changing the properties of the model. Private savings are defined as

$$S = Y - T - C \tag{3.6}$$

so that

$$\frac{\partial S}{\partial (Y - T)} = 1 - \frac{\partial C}{\partial (Y - T)} > 0. \tag{3.7}$$

Although the Keynesian consumption function is *ad hoc* and not micro-founded, it is consistent with micro-founded theories of consumption such as the overlapping generations model from last week's lecture. In that model savings depends positively on income and also depends on the interest rate though the sign of the latter derivative is ambiguous. Here the assumption of a negative relationship between consumption and the interest rate simply reflects that increasing the interest rate makes saving relatively more attractive.

In the Keynesian model, investment depends negatively on the interest rate

$$I = I(r) \tag{3.8}$$

with

$$\frac{\partial I}{\partial r} < 0. \tag{3.9}$$

The rationale for the negative derivative is that the interest rate represents the cost of borrowing money for investment or, alternatively, the opportunity cost of investing rather than saving. One important difference from earlier models is that here investment decisions are thought of as being made by a different group of people, *entrepreneurs*, to the households who choose how much of their income to save. As a result, planned savings may be different from planned investment. However, in equilibrium we must have $S = I$ so that realised savings must be equal to realised investment. The two are reconciled through changes in income Y.

Substituting (3.3) and (3.8) into (3.1) gives

$$Y = C(Y - T, r) + I(r) + G. \tag{3.10}$$

Differentiating this equation with respect to r gives

$$\frac{\partial Y}{\partial r} = \frac{\partial C}{\partial r} + \frac{\partial I}{\partial r} < 0 \tag{3.11}$$

so, in equilibrium where $I = S$, there is a negative relationship between Y and r. This relationship can be represented by the IS curve in Figure 2 which shows the

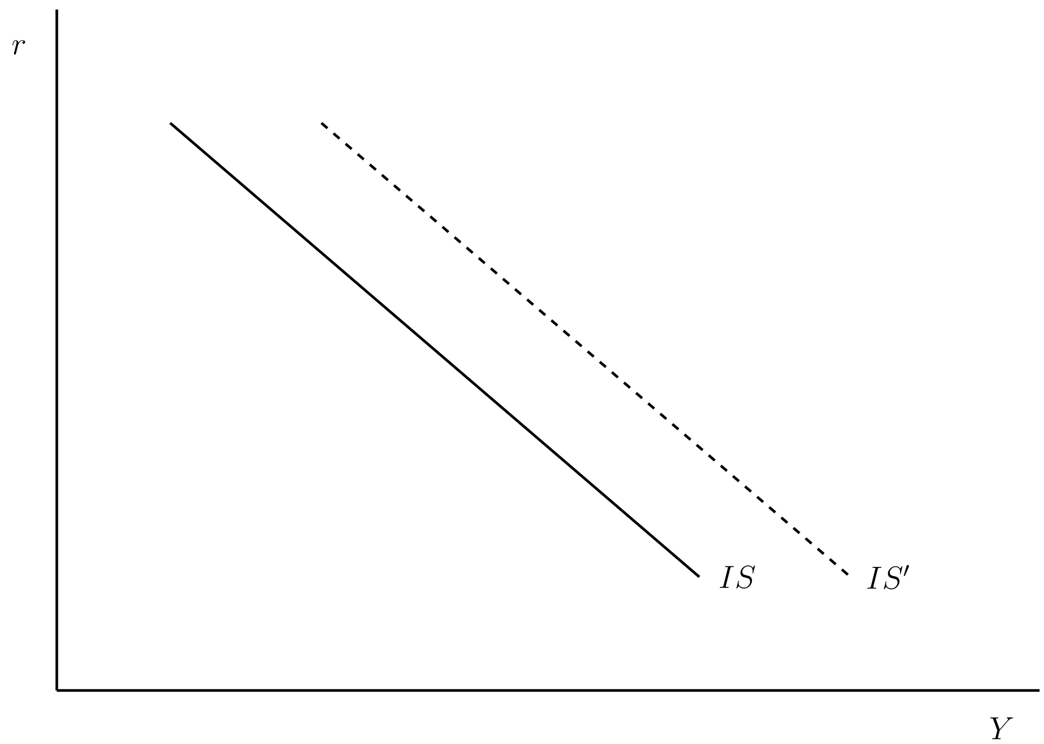

Figure 2: The IS curve

different combinations of Y and r for which $I = S$.

How does the *IS curve* respond to an increase in government expenditure G while holding T constant? Differentiating equation (3.10) with respect to G gives

$$\frac{dY}{dG} = \frac{\partial C}{\partial (Y-T)} \left(\frac{dY}{dG} - \frac{dT}{dG} \right) + \frac{dG}{dG} \tag{3.12}$$

$$= \frac{\partial C}{\partial (Y-T)} \frac{dY}{dG} + 1 \tag{3.13}$$

$$= \frac{1}{1 - \partial C / \partial (Y-T)} > 1. \tag{3.14}$$

This is known as the *Keynesian multiplier* for G. It shows that a unit increase in government expenditure G increases output Y by more than unity. For example if $\partial C/\partial(Y-T) = 0.9$, then the government expenditure multiplier $\partial Y/\partial G = 1/0.1 = 10$. An increase in G shifts the *IS* curve in Figure 2 rightwards from IS to IS', meaning that Y is higher for given r.

Consider a change in taxes T holding G constant. Differentiating equation (3.10) with respect to T gives

$$\frac{dY}{dT} = \frac{\partial C}{\partial (Y-T)} \left(\frac{dY}{dT} - \frac{dT}{dT} \right) + \frac{dG}{dT} \tag{3.15}$$

$$= \frac{\partial C}{\partial (Y-T)} \left(\frac{dY}{dT} - 1 \right) + 0 \tag{3.16}$$

$$= -\frac{\partial C / \partial (Y-T)}{1 - \partial C / \partial (Y-T)} < 0. \tag{3.17}$$

This is the *Keynesian multiplier* for T. An increase in taxes T causes the *IS* curve in Figure 2 to shift to the left. However, the leftward shift implied by (3.17) is smaller than the rightward shift implied by (3.14) since

$$\frac{1}{1 - \partial C / \partial (Y-T)} - \frac{\partial C / \partial (Y-T)}{1 - \partial C / \partial (Y-T)} = 1. \tag{3.18}$$

This can be verified by considering changing both G and T together so as to keep the government budget surplus T-G constant. To do this, differentiate (3.10) again with respect to G but this time imposing $dT/dG = 1$. This gives

$$\frac{dY}{dG} = \frac{\partial C}{\partial (Y-T)} \left(\frac{dY}{dG} - \frac{dT}{dG} \right) + \frac{dG}{dG} \tag{3.19}$$

$$= \frac{\partial C}{\partial (Y-T)} \left(\frac{dY}{dG} - 1 \right) + 1 \tag{3.20}$$

$$= \frac{1 - \partial C / \partial (Y-T)}{1 - \partial C / \partial (Y-T)} = 1. \tag{3.21}$$

4 The LM Curve

We now turn to consider the money market. The demand for money is

$$L = L(Y, i) \tag{4.1}$$

where L is real money balances $L = M/P$ where M is the quantity of money and P the price level and i is the nominal rate of interest defined by (3.4) or

$$i = r + \pi. \tag{4.2}$$

It is assumed that

$$\frac{\partial L}{\partial Y} > 0, \qquad \text{and} \qquad \frac{\partial L}{\partial i} < 0. \tag{4.3}$$

The demand for money depends positively on income because money is needed to make transactions. It depends negatively on the nominal rate of interest because this is the opportunity cost of holding money rather than interest bearing assets.

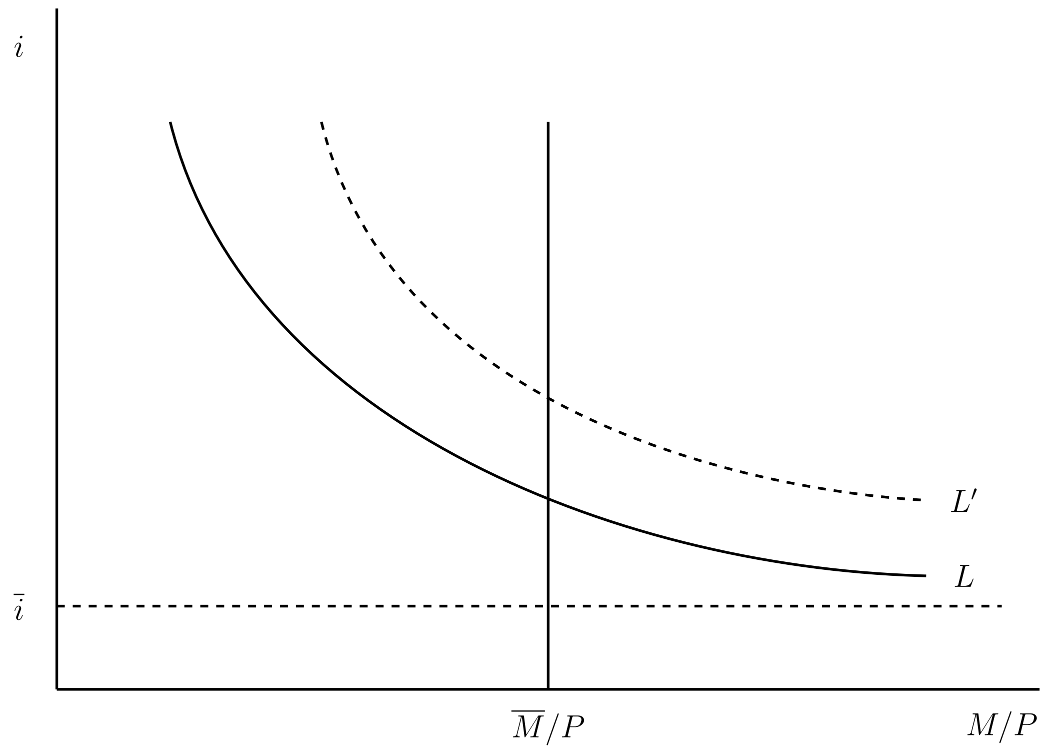

Figure 3: Equilibrium in the money market: $L = \overline{M}/P$

Money is assumed to be supplied inelastically by a monetary authority that chooses to supply a fixed amount of nominal money independent of the interest rate. Equilbrium in the money market is achieved when the demand and supply of real money balances are equal or

$$L = L(Y, i) = \frac{\overline{M}}{P} \tag{4.4}$$

where $\overline{M}$ represents the exogenous supply of nominal money. This is illustrated in Figure 3. An increase in Y leads to an upward shift in the demand for money curve L and with the supply of real balances fixed at $\overline{M}/P$ this leads to an increase in the nominal interest rate i and, if prices P are constant, in the real interest rate r. The curvature in the money demand curve L as depicted in Figure 3 illustrates the *liquidity trap*, a hypothesis of Keynes that there is a minimum nominal interest rate $\bar{i} > 0$ at which no-one is prepared to hold interest bearing assets and so the demand for money is infinite. Formally,

$$\lim_{i \to \bar{i}} \frac{\partial L}{\partial i} = -\infty. \tag{4.5}$$

This hypothesis implies that when interest rates are very low, then increases in the money supply will no longer lead to a fall in the rate of interest.

Equilibrium in the money market can be represented in r-Y space as the *LM curve* illustrated in Figure 4. An increase in the supply of money $\overline{M}$ causes a downward shift in the *LM curve* from LM to LM' as the real interest rate that clears the money market falls for a given level of Y. However, at very low rates of interest the *LM curve* can be almost flat so that changes in the supply of money may have little effect on the interest rate. This is a result of Keynes's *liquidity trap* and, in these circumstances, Keynes argued that monetary policy might be ineffective in stimulating output.

5 IS-LM Analysis

Putting together the *IS curve* (3.10), which defines the equilibrium relationship between investment and savings and the *LM curve* (4.4), which defines equilibrium in the money market results in the *IS-LM* diagram in Figure 5 which jointly determines equilibrium r and Y. The *IS-LM* diagram was not invented by Keynes but by John Hicks in an expository paper on Keynes's General Theory, Hicks (1937). It is an important tool for looking at some aspects of Keynes's model but is not a full description of Keynes's theory. In particular the *LM curve* treats prices as fixed while in the full model changing prices will affect the real interest rate. Also the *IS-LM* diagram does not include the labour market which determines the level of employment, perhaps the key variable in the *General Theory of Employment, Interest and Money.*

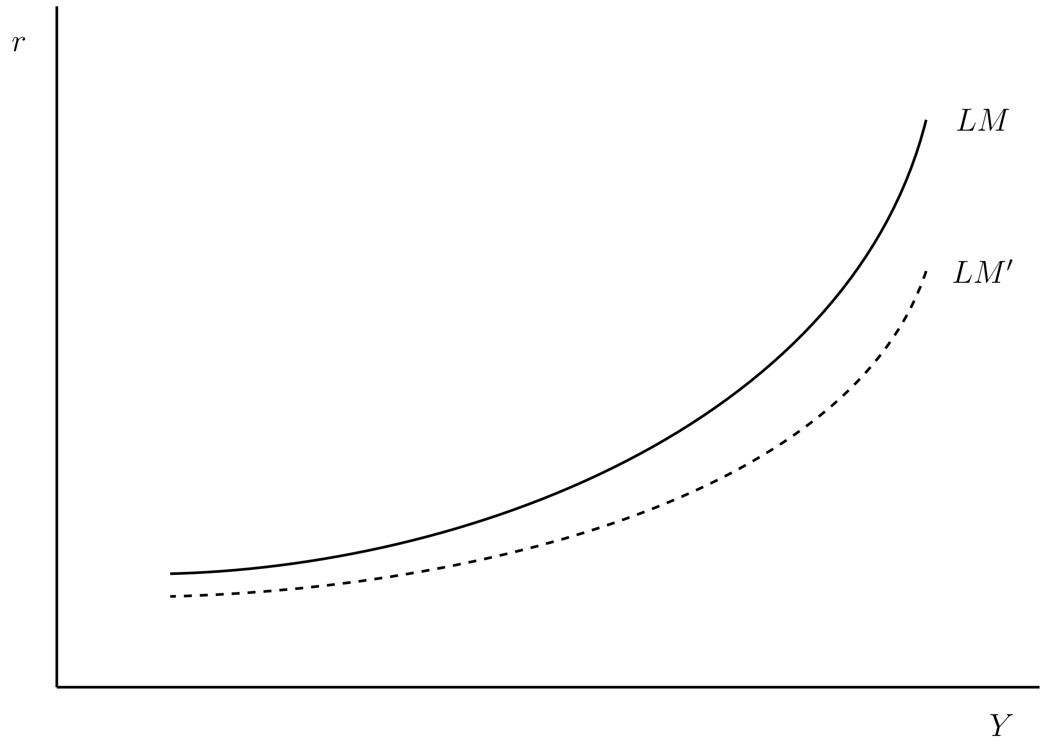

Figure 4: The LM curve

Consider again an increase in government expenditure G while holding T constant. This shifts the *IS curve* to the right. In Figure 5, this results in an increase in output Y from Y_1 to Y_1' which is the Keynesian multiplier. However, the diagram shows that the new equilibrium output is at Y_2 which is less then Y_1'. This is because the increase in income means that the demand for money is higher which causes an increase in interest rates (assuming that the supply of money does not change).

This can be seen by differentiating the *IS* equation (3.10) and the *LM* equation (4.4) with respect to G but this time taking into account that G can affect i via the *LM* equation. First we rewrite (3.10) and (4.4) using (3.4) as

$$Y = C(Y - T, i - \pi) + I(i - \pi) + G \tag{5.1}$$

and

$$L(Y, i) = \frac{\overline{M}}{P}. \tag{5.2}$$

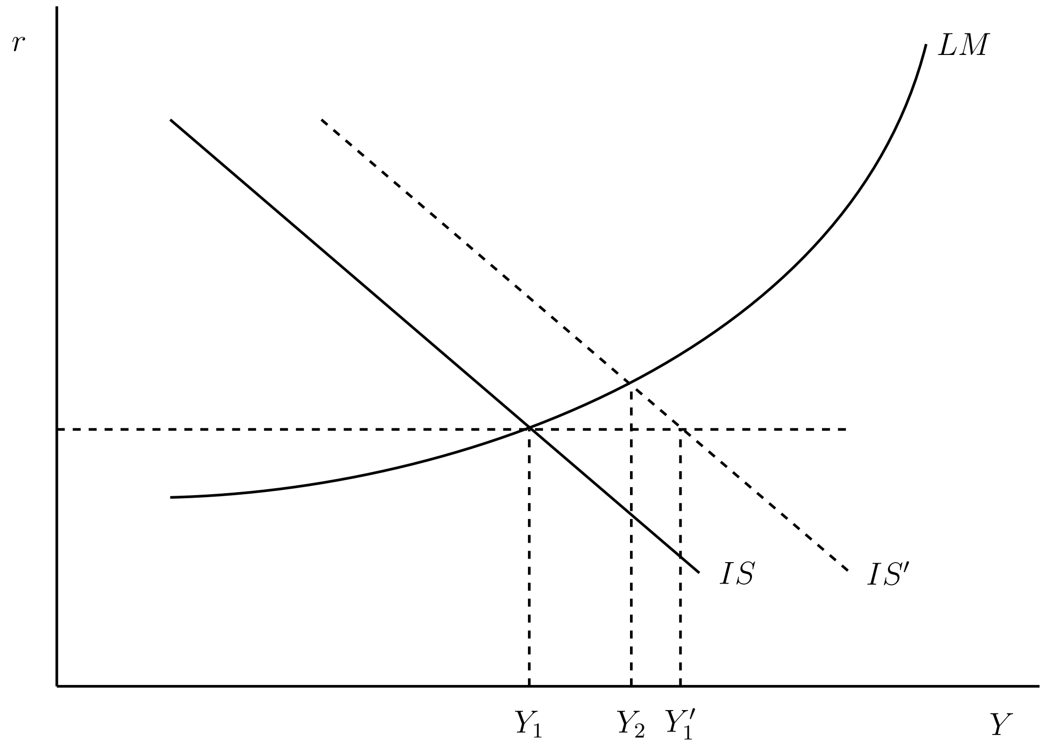

Figure 5: The IS-LM diagram

Differentiating with respect to G treating T, $\overline{M}$, P and π as fixed

$$\frac{dY}{dG} = \frac{\partial C}{\partial(Y-T)}\left(\frac{dY}{dG} - \frac{dT}{dG}\right) + \frac{\partial C}{\partial(i-\pi)}\left(\frac{di}{dG} - \frac{d\pi}{dG}\right) \tag{5.3}$$

$$+ \frac{\partial I}{\partial(i-\pi)}\left(\frac{di}{dG} - \frac{d\pi}{dG}\right) + \frac{dG}{dG} \tag{5.4}$$

$$= \frac{\partial C}{\partial(Y-T)}\frac{dY}{dG} + \left(\frac{\partial C}{\partial(i-\pi)} + \frac{\partial I}{\partial(i-\pi)}\right)\frac{di}{dG} + 1 \tag{5.5}$$

and

$$\frac{\partial L}{\partial Y}\frac{dY}{dG} + \frac{\partial L}{\partial i}\frac{di}{dG} = \frac{d(\overline{M}/P)}{dG} = 0. \tag{5.6}$$

Rearranging (5.6) gives

$$\frac{di}{dG} = -\left(\frac{\partial L/\partial Y}{\partial L/\partial i}\right)\frac{dY}{dG} \tag{5.7}$$

where the term in brackets is negative and is related to the slope of the *LM curve*.

Substituting into (5.5),

$$\frac{dY}{dG} = \frac{\partial C}{\partial(Y-T)}\frac{dY}{dG} - \left(\frac{\partial C}{\partial(i-\pi)} + \frac{\partial I}{\partial(i-\pi)}\right)\left(\frac{\partial L/\partial Y}{\partial L/\partial i}\right)\frac{dY}{dG} + 1 \tag{5.8}$$

$$= \left[1 - \frac{\partial C}{\partial(Y-T)} + \left(\frac{\partial C}{\partial(i-\pi)} + \frac{\partial I}{\partial(i-\pi)}\right)\left(\frac{\partial L/\partial Y}{\partial L/\partial i}\right)\right]^{-1} \tag{5.9}$$

Comparing this result with the simple government expenditure multiplier (3.14)

$$\frac{dY}{dG} = \frac{1}{1 - \partial C/\partial(Y-T)} \tag{5.10}$$

it can be verified that taking into account the effect of the LM equation reduces the multiplier since the extra term in the denominator in (5.9)

$$\left(\frac{\partial C}{\partial(i-\pi)} + \frac{\partial I}{\partial(i-\pi)}\right)\left(\frac{\partial L/\partial Y}{\partial L/\partial i}\right) \tag{5.11}$$

is the product of two negative terms and so is positive.

6 Aggregate Demand

The IS and LM curves together determine *aggregate demand* Y but only conditional on prices P and inflation π remaining fixed. In fact the model implies a negative relationship between Y and P as can be seen by differentiating the IS equation (3.10) and the LM equation (4.4) with respect to P treating G , T and $\overline{M}$ as fixed. This model involves both the price level P and the inflation rate π which complicates the analysis. For simplicity we assume that $d\pi/dP = 0$.

Differentiating (3.10) and (4.4) with respect to P gives

$$\frac{dY}{dP} = \frac{\partial C}{\partial(Y-T)}\left(\frac{dY}{dP} - \frac{dT}{dP}\right) + \frac{\partial C}{\partial(i-\pi)}\left(\frac{di}{dP} - \frac{d\pi}{dP}\right) \tag{6.1}$$

$$+ \frac{\partial I}{\partial(i-\pi)}\left(\frac{di}{dP} - \frac{d\pi}{dP}\right) + \frac{dG}{dP} \tag{6.2}$$

$$= \frac{\partial C}{\partial(Y-T)}\frac{dY}{dP} + \left(\frac{\partial C}{\partial(i-\pi)} + \frac{\partial I}{\partial(i-\pi)}\right)\frac{di}{dP} \tag{6.3}$$

and

$$\frac{\partial L}{\partial Y}\frac{dY}{dP} + \frac{\partial L}{\partial i}\frac{di}{dP} = \frac{d(\overline{M}/P)}{dP} = -\frac{\overline{M}}{P^2}. \tag{6.4}$$

Rearranging (5.6) gives

$$\frac{di}{dP} = -\left(\frac{\partial L/\partial Y}{\partial L/\partial i}\right)\frac{dY}{dP} - \frac{\overline{M}/(P^2)}{\partial L/\partial i}. \tag{6.5}$$

Substituting into (6.3) gives

$$\frac{dY}{dP} = \frac{\partial C}{\partial(Y-T)}\,\frac{dY}{dP} - q\left[\left(\frac{\partial L/\partial Y}{\partial L/\partial i}\right)\frac{dY}{dP} + \frac{\overline{M}/(P^2)}{\partial L/\partial i}\right] \tag{6.6}$$

$$= -q\left(1 - \frac{\partial C}{\partial(Y-T)} + q\frac{\partial L/\partial Y}{\partial L/\partial i}\right)^{-1}\left[\frac{\overline{M}/(P^2)}{\partial L/\partial i}\right] \tag{6.7}$$

where

$$q = \frac{\partial C}{\partial(i-\pi)} + \frac{\partial I}{\partial(i-\pi)} < 0. \tag{6.8}$$

The term in round brackets is positive but the term in square brackets is negative so the overall derivative is negative and the aggregate demand curve is downward sloping in P-Y space as in Figure 6.

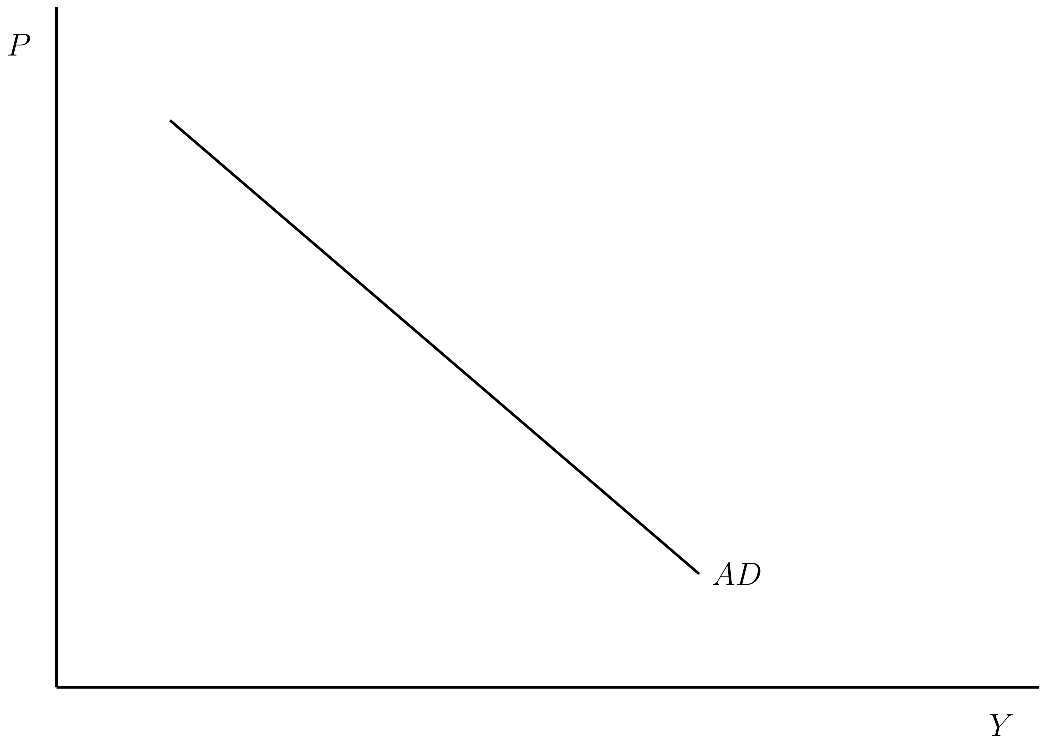

Figure 6: Aggregate Demand

7 The Labour Market and Aggregate Supply

The final market in Keynes's model is the labour market and this determines aggregate supply in the economy. The demand for labour derives from the aggregate production function

$$Y = F(K, N) \tag{7.1}$$

where now we use the notation N for employment. The real wage paid to workers is the marginal product of labour

$$\frac{W}{P} = \frac{\partial Y}{\partial N} \tag{7.2}$$

where W is money wages and P is the price level, but diminishing marginal product means that there is a negative relationship between employment N and the real wage W/P. The supply of labour is assumed to be an increasing function

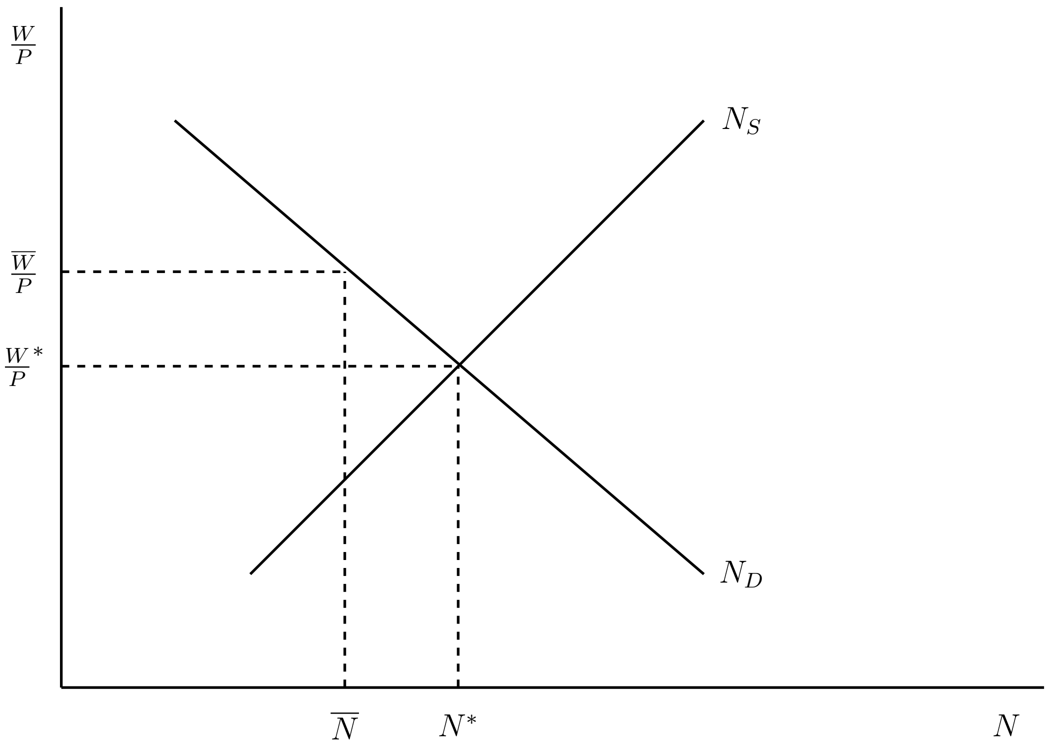

Figure 7: The Labour Market

of the real wage encapsulating the idea that the higher are wages, the more people will be willing to work. Labour market equilibrium is illustrated in Figure 7. Demand and supply are equal at employment N^* which denotes full employment.

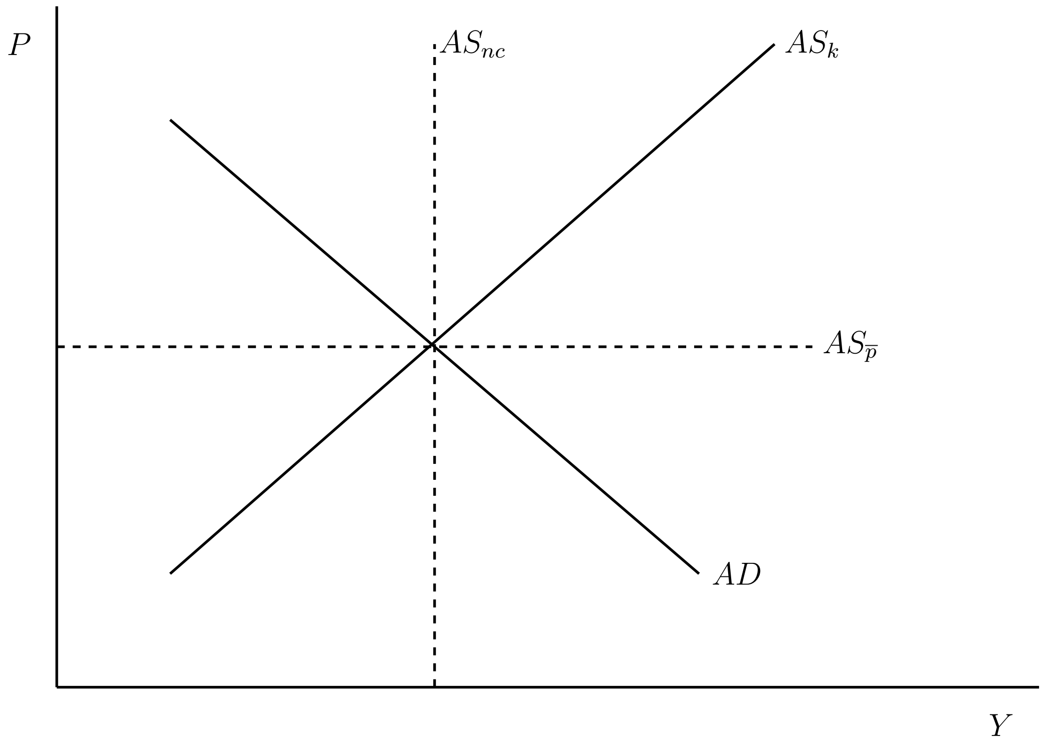

Figure 8: Alternative Aggregate Supply Functions

Suppose that the price level falls. In a neoclassical world where the nominal wage is flexible and defined by

$$W = P\frac{\partial Y}{\partial N} \tag{7.3}$$

this should have no effect on output. Since the marginal product of labour has not changed, the nominal wage will fall with the price level so that the real wage and the rate of employment are unchanged. The neoclassical aggregate supply function is vertical as depicted by the curve AS_{nc} in Figure 8.

Suppose instead that nominal wages are rigid at the value $\overline{W}$. if the price level falls then real wages are increased above the marginal product of labour. Firms respond by decreasing employment and output until

$$\frac{\overline{W}}{P} = \frac{\partial Y}{\partial N} \tag{7.4}$$

at the new lower price level. The effect on the labour market is shown in Figure 7. When the price falls the real wage rises from W/P^* to $\overline{W}/P$. Since nominal wages are rigid, employment must fall from N^* to $\overline{N}$. In this case the aggregate supply function is upward sloping as depicted by the curve AS_k in Figure 8. The assumption that nominal wages are rigid is what is normally regarded as the standard assumption of the Keynesian model even though, as has been pointed out, Keynes never made it.

An alternative assumption that also leads to a non-vertical AS curve is the assumption that prices P are fixed at $\overline{P}$. In this case, fluctuations in aggregate demand cause firms to adjust aggregate supply through output and employment until

$$\frac{W}{\overline{P}} = \frac{\partial Y}{\partial N}. \tag{7.5}$$

The aggregate supply function in this case is horizontal as depicted by the curve $AS_{\overline{p}}$ in Figure 8.

8 Monetary Policy with an Interest Rate Target

The *LM curve* in the standard Keynesian model is based on the assumption that the monetary authority sets the money supply $\overline{M}$. An alternative and more realistic assumption is that the monetary authority sets interest rates to achieve a target related to output Y and the rate of inflation π. Writing this target in terms of the real interest rate, we have

$$r^* = r(Y, \pi) \tag{8.1}$$

with $\partial r/\partial Y > 0$ and $\partial r/\partial \pi > 0$. An equation such as (8.1) is known as a *Taylor rule* after Taylor (1993). Rules like this are explicitly followed by monetary authorities pursuing *inflation targetting* policies such as the Bank of England.

Combining (8.1) with the money demand function

$$L(Y, r + \pi) = \frac{M}{P} \tag{8.2}$$

gives

$$M = PL(Y, r(Y, \pi) + \pi) \tag{8.3}$$

where the money supply M is now an endogenous variable.

This variant of the standard *LM curve* defines an upward sloping relationship between r and Y known as the *MP curve*. It also defines a downward sloping aggregate demand curve in π-Y space as in Figure 9. This is technically easier to deal with than the standard AD function which depends on both P and π as it depends on inflation π only. The aggregate supply function can also be recast as an upward sloping function in π-Y space on the assumption of rigidity in either the nominal wage rate W or in inflation π.

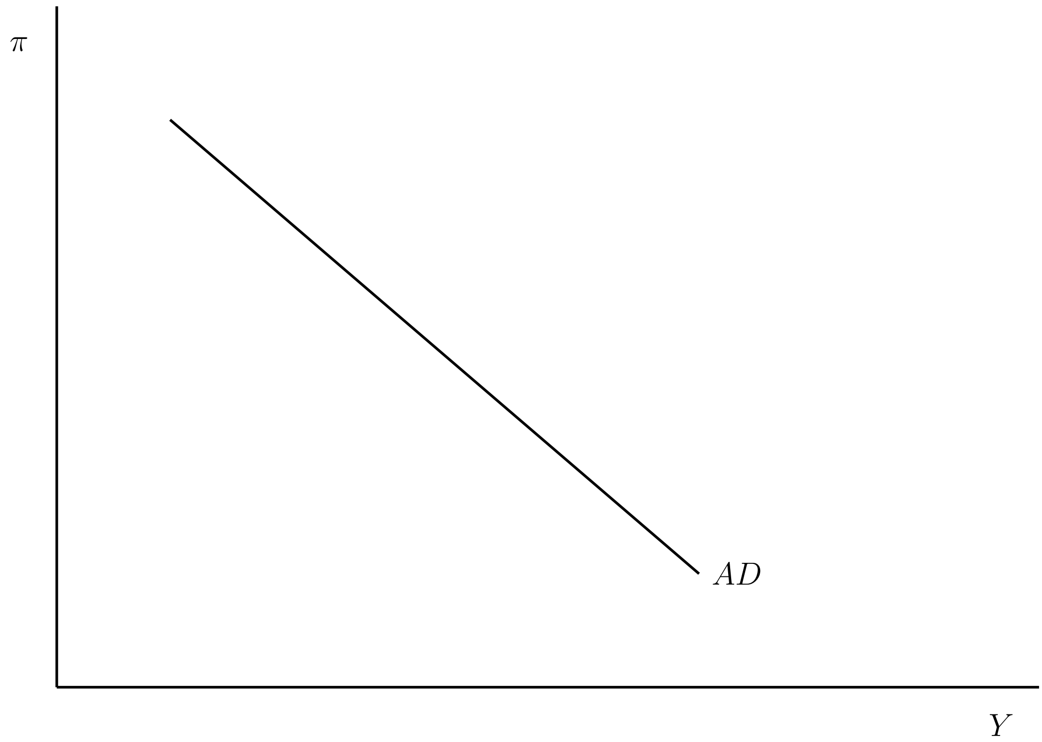

Figure 9: Aggregate Demand with an Interest Rate Target

9 Conclusions

In this lecture we have explored the standard version of the Keynesian model as expounded in most macroeconomic textbooks. This model is the outcome of the *neoclassical synthesis* that casts Keynes's model as a special case of the neoclassical model with rigid wages or prices. The Keynesian non-vertical AS curve is sometimes thought of as a short-run aggregate supply function while, in the long run, wages and prices will adjust so that the long run AS curve becomes vertical.

In next week's lecture we look at some new Keynesian models which are micro-founded but have Keynesian features.

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