Macroeconomics
Lecture 7: New Keynesian Models

Richard G. Pierse

1 Introduction

In last week's lecture we looked at the traditional Keynesian model. This model implies an upward sloping aggregate supply relationship between output and prices (or inflation) that flies in the face of the fundamental neo-classical precept that real variables like output should be independent of nominal variables like prices or inflation. In this lecture we look at the Phillips curve, an empirical relationship between wage inflation and unemployment that seemed to provide support for the upward sloping aggregate supply curve, but which dramatically broke down in the 1970s. The resolution of this puzzle led to a new focus on expectations formation in a world of uncertainty. We then take a look at a simple new Keynesian model of price setting in which we derive a new Keynesian version of the Phillips curve with properties similar to those of the classic Phillips curve. This version though is based on rational micro-founded behaviour in an uncertain world with imperfect competition where short-run stickiness of prices or wages can arise because of the costs associated with changing prices. Finally, we take a brief look at a complete new Keynesian model due to Clarida, Galí and Gertler (2000).

2 The Phillips Curve

The New Zealand born economist Bill Phillips published in 1958 an empirical study of the relationship between nominal wage inflation and the rate of unemployment in the UK between 1861 and 1957, Phillips (1958). What he found was a stable negative relationship that can be formalised as

$$\frac{\dot{W}}{W} = \alpha(\overline{u} - u) \tag{2.1}$$

where W is nominal wages and u is the rate of unemployment, $\overline{u}$ is the natural rate of unemployment (corresponding to full employment) and $\alpha > 0$ is a parameter. Other economists found similar relationships for other countries and also with price inflation $\pi = \dot{P}/P$ replacing wage inflation. Phillips found that this relationship, estimated over the period 1861-1913, could also explain the much later period 1948-1957. This stable relationship came to known as the *Phillips curve*. The Phillips curve implied a positive relationship between (wage) inflation and output and so provided empirical support for an upward sloping Keynesian aggregate supply curve. The Phillips curve trade-off between unemployment and

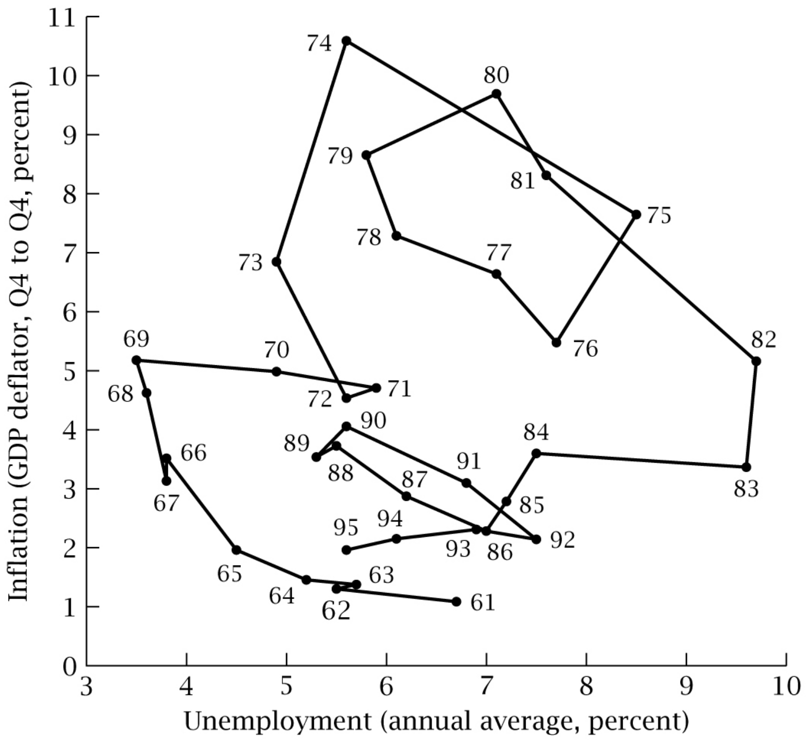

FIGURE 6.7 Unemployment and inflation in the United States, 1961–1995

Figure 1: [Figure 6.7 in Romer (2012)]

inflation continued to hold throughout the 1960s in the developed countries but then dramatically broke down in the 1970s as can be seen in Figure 1 for the USA reproduced from Romer (2012).

3 The Expectations-Augmented Phillips Curve

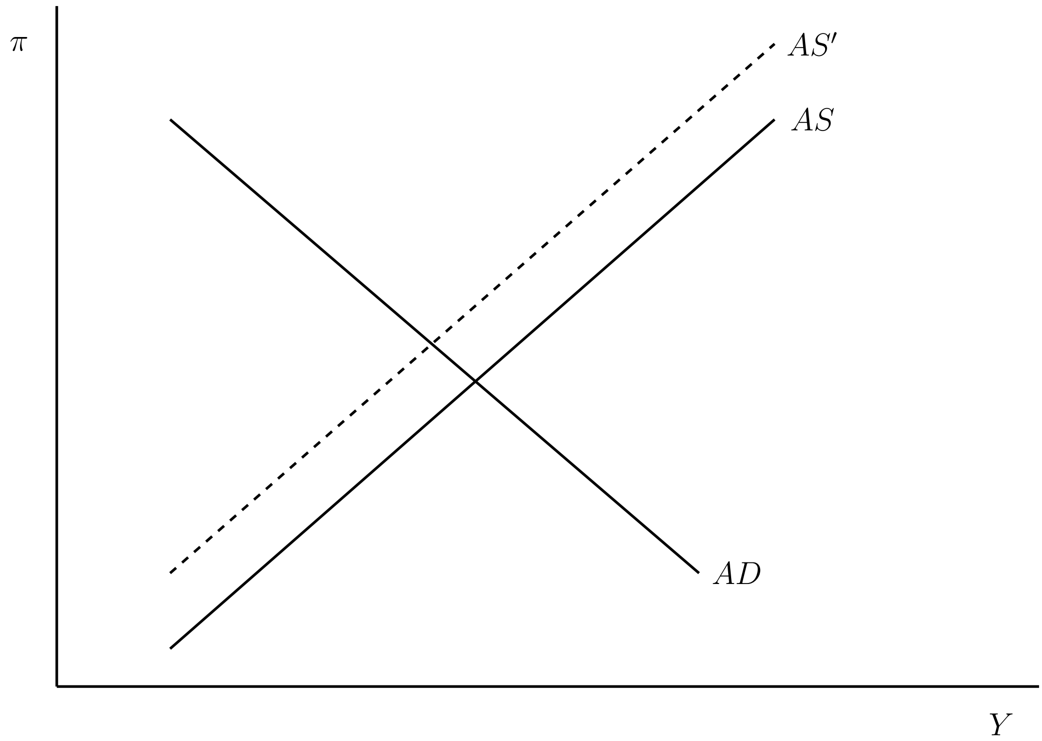

Figure 2: A negative supply shock in the standard *AD*-*AS* model

The breakdown of the standard Phillips curve relationship in the 1970s led to a period of *stagflation* in which both inflation and unemployment were increasing. Important factors contributing to inflation in this period were the two big hikes in OPEC oil prices in 1973 and 1979 as a result of conflicts in the Middle East. It is possible to explain simultaneous increases in inflation and unemployment in the traditional *AD*-*AS* framework as the result of negative shocks to aggregate supply as in Figure 2 which is drawn in $\pi - Y$ space.

However, Friedman (1968) and Phelps (1968) argued that the Phillips curve relationship was fundamentally flawed in ignoring inflationary expectations. In a situation in which agents expect price inflation to continue, they argued that inflationary expectations would be built into wage bargaining and price setting. In this world, the Phillips curve relationship (2.1) (rewritten in terms of price inflation π rather than wage inflation) would be augmented by inflation expectations π^e as in

$$\pi = \pi^e + \alpha(\overline{u} - u). \tag{3.1}$$

This is known as the *expectations-augmented Phillips curve*. The natural rate of unemployment $\overline{u}$ is also known as the *non-accelerating-inflation rate of unemployment* or *NAIRU* for short. Equation (3.1) says that as long as unemployment u is below $\overline{u}$, inflation is above its expected rate and so is accelerating. In long run equilibrium where

$$\pi = \pi^e, \tag{3.2}$$

u is constant at $\overline{u}$ which implies that the long run aggregate supply function is vertical. Thus in the *expectations-augmented Phillips curve* the upward-sloping aggregate supply function is only a short run phenomenon.

The *expectations-augmented Phillips curve* can explain the stability of the standard Phillips curve pre-1970 on the assumption that inflationary expectations were zero before this date and can also explain what has happened to the inflation-unemployment trade-off since 1970. It has become accepted by most macroeconomists as the standard model.

It can be rewritten as a relationship between inflation and output

$$\pi = \pi^e + \lambda(\log Y - \log \overline{Y}) \tag{3.3}$$

where $\log Y$ is the logarithm of output, $\log \overline{Y}$ is the logarithm of *normal output* and $\lambda > 0$ is a parameter. Normal output is the underlying long-run trend in output and is generally estimated by fitting a straight line through $\log Y$ or using the Hodrick-Prescott filter to fit a smooth non-linear trend. In the form of equation (3.3), the expectations-augmented Phillips curve is also known as the modern Keynesian aggregate supply function. It says that in periods where output is above normal output (peaks in the business cycle), inflation will be above expected values and in periods where output is below normal output (troughs in the business cycle) inflation will be below expected values. In long-run growth when $Y = \overline{Y}$ and $\pi = \pi^e$, output growth and inflation are independent and the long-run aggregate supply function is vertical.

4 Rational Expectations and the Lucas Critique

In discussion of the *expectations-augmented Phillips curve* in the previous section, we have made no assumption about how expectations are formed, other that in long-run equilibrium, expectations will be equal to realised values. The argument for this last assumption is simply that it would not be rational in long-run equilibrium for agents to hold expectations that are consistently proved wrong.

It is possible to extend this assumption that expectations are ‘rational’ to hold, not just in long-run equilibrium, but at all points in time. This is the *rational expectations hypothesis* first proposed by Muth (1961). It is easiest to explain this hypothesis in discrete time. Define

$$x^e_{t+1} = E(x_{t+1}|\mathscr{I}_t) \tag{4.1}$$

where x^e_{t+1} is the expected value of a variable x in period $t+1$ where the expectation is formed in period t. The right-hand side of the equation is the mathematical expectation (in rough language the 'average' value) conditional (|) on the set of information $\mathscr{I}_t$ available at time t. Different assumptions may be made about what is part of the agent's information set so that agents are not necessarily assumed to have complete information. There can also be asymmetries of information as, for example, between a buyer and seller at an auction. However, whatever is in the information set available to agents, the rational expectations hypothesis implies that the expectational error

$$\epsilon_{t+1} = x_{t+1} - x^e_{t+1} \tag{4.2}$$

could not have been predicted by agents when they formed their expectation or, formally, that

$$E(\epsilon_{t+1}|\mathscr{I}_t) = 0. \tag{4.3}$$

Thus the hypothesis says that agents do not make systematic errors when forming expectations or that, on average, their expectations are correct.

The rational expectations hypothesis is very powerful. Its widespread use in economics since Muth has been called the *rational expectations revolution*. In particular, Lucas (1976) made an important observation with respect to economic policy conducted by policymakers (governments or central banks) on agents who form rational expectations. Lucas pointed out that changes in policy are likely to affect agent's expectations and hence change their behaviour. This means that policymakers who try to take advantage of a statistical relationship to influence the behaviour of agents are likely to find that this statistical relationship breaks down because agents change their behaviour to thwart the policy. This criticism has come to be known as the *Lucas critique*.

One interpretation of the breakdown of the original Phillips curve relationship in the 1970s is that policymakers were beginning to use this statistical relationship to smooth out peaks and troughs in the business cycle. Suppose that in the 1960s, policmakers were trying to permanently raise inflation in order to reduce the real wage so that firms would increase employment. Once firms realised that this was happening, they began to expect higher inflation rates and so adjusted their expectations of inflation, so that the statistical relationship the policymakers were relying on began to break down.

The Lucas critique suggests that it is dangerous to rely on macroeconomic relationships that are not firmly founded on optimising behaviour at the microeconomic level based on preferences, technology and other 'deep parameters' that will not change with policy. This was a motivation for economists to search for the micro-founded theories of aggregate supply that are the basis for the *new Keynesian models.*

5 A New Keynesian Model of Sticky Prices

We saw in last week's lecture that an upward sloping aggregate supply function requires some form of inflexibility in reponse of nominal prices or wages to real shocks. The traditional Keynesian model is a static model, so in that model this had to take the form of a complete rigidity of nominal wages or nominal prices. In a dynamic model, all that is needed is that there is some inertia in response to changes in costs leading to price or wage 'stickiness'.

One microeconomic reason for why prices might be slow to react to changes in costs is the existence of a fixed cost to changing prices, the so-called *menu cost.* The idea for this arises from the example of pricing in a restaurant. Each time a restaurant changes its prices, it must print a new version of its menu and there is a fixed cost involved in this. When the cost of menu-printing is high, the restaurant might postpone passing on small increases in costs until enough of these have cumulated to outweight the cost of re-printing the menu. Two key papers based on this idea are Mankiw (1985) and Akerlof and Yellen (1985).

Menu-cost models though appealing are technically difficult to analyse. An alternative approach which has proved very fruitful is to assume that prices (and wages) are not adjusted continuously but are set by multi-period contracts. In any period, some contracts will expire so those prices will be subject to change while other prices will be fixed. This assumption has the advantage of being realistic. In the real world, most wage contracts are reviewed annually and some biennially and many companies announce prices valid for six months or a year.

Three key papers using different versions of this idea are Fischer (1977), Taylor (1979) and Calvo (1983). In Fischer's and Taylor's models, prices are changed at fixed, determined intervals (in the simplest case every two periods). In Fischer's model the firm may set different prices for the different periods within the contract while Taylor assumes a single price for the duration of the contract. In the Calvo model, the opportunity for a firm to change its price arises randomly and is not within the control of the firm itself which makes the length of the contract unknown. As with Taylor, a single price is set for the duration of the contract. Though the assumptions underlying the Calvo (1983) pricing model may seem less realistic than the other two models, it turns out that it is technically simpler to analyse and easier to aggregate. As a consequence it has been widely used in the literature and for this reason, this is the model on which we will concentrate.

In the Calvo model, in each period only a random fraction $(1-\theta)$ of firms are able to reset their prices while all other firms keep their prices unchanged. Thus when firms do get a chance to reset their prices, they know that the price may then be fixed for an unknown number of periods. Calvo assumes that firms choose a log price $\overline{p}_t$ to minimise a quadratic loss function defined by

$$L(\overline{p}_t) = \sum_{k=0}^{\infty} (\theta\beta)^k E_t(\overline{p}_t - p^*_{t+k})^2 \tag{5.1}$$

where $0 < \beta < 1$ is a discount parameter, and p^*_{t+k} is the log of the optimal price that the firm would set in period $t + k$ if they were able to set prices in every period. The term $(\overline{p} - p^*_{t+k})^2$ is therefore a measure of the loss that the firm would bear in period $t + k$ from being unable to change its prices. Since the firm does not know at time t whether or not it will be able to set its price in period $t + k$, the expected value of its loss must be summed over all future periods, multiplied by the probability that its price will still be fixed in the $t + k$'th period which is θ^k. The firm is assumed to discount future losses at the discount rate β.

The optimal reset price can be derived from differentiating (5.1) with respect to $\overline{p}_t$ giving

$$\frac{\partial L}{\partial \overline{p}_t} = 2 \sum_{k=0}^{\infty} (\theta\beta)^k E_t(\overline{p}_t - p^*_{t+k}) = 0 \tag{5.2}$$

which implies that

$$\sum_{k=0}^{\infty} (\theta\beta)^k \overline{p}_t = \sum_{k=0}^{\infty} (\theta\beta)^k E_t p^*_{t+k}. \tag{5.3}$$

Now the infinite sum on the left-hand side of (5.3) can be written as

$$\sum_{k=0}^{\infty} (\theta\beta)^k = \frac{1}{1 - \theta\beta} \tag{5.4}$$

so, combining with (5.3), gives the solution

$$\overline{p}_t = (1 - \theta\beta) \sum_{k=0}^{\infty} (\theta\beta)^k E_t p^*_{t+k}. \tag{5.5}$$

Equation (5.5) says that the optimal solution for the firm is to set its price as a weighted average of the prices it would have expected to set in the future in the absence of any price rigidities. Consider the infinite sum on the right-hand side of (5.5)

$$\sum_{k=0}^{\infty} (\theta\beta)^k E_t p^*_{t+k} = p^*_t + (\theta\beta) E_t p^*_{t+1} + (\theta\beta)^2 E_t p^*_{t+2} + (\theta\beta)^3 E_t p^*_{t+3} + \cdots . \tag{5.6}$$

Leading equation (5.5) by one period and taking expectations at time period t gives

$$E_t\overline{p}_{t+1} = (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k E_t p^*_{t+k+1} \tag{5.7}$$

where the infinite sum on the right-hand side of (5.7) is

$$\sum_{k=0}^{\infty}(\theta\beta)^k E_t p^*_{t+k+1} = E_t p^*_{t+1} + (\theta\beta)E_t p^*_{t+2} + (\theta\beta)^2 E_t p^*_{t+3} + \cdots . \tag{5.8}$$

Multiplying (5.7) by $\theta\beta$ and subtracting from (5.5) gives

$$\overline{p}_t - \theta\beta E_t\overline{p}_{t+1} = (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k E_t p^*_{t+k} - (1-\theta\beta)\theta\beta\sum_{k=0}^{\infty}(\theta\beta)^k E_t p^*_{t+k+1}. \tag{5.9}$$

However, comparing the terms in (5.6) and (5.8), it can be seen that

$$\sum_{k=0}^{\infty}(\theta\beta)^k E_t p^*_{t+k} - \theta\beta\sum_{k=0}^{\infty}(\theta\beta)^k E_t p^*_{t+k+1} = p^*_t \tag{5.10}$$

so that it is possible to rewrite (5.9) as

$$\overline{p}_t - \theta\beta E_t\overline{p}_{t+1} = (1-\theta\beta)p^*_t \tag{5.11}$$

or

$$\overline{p}_t = \theta\beta E_t\overline{p}_{t+1} + (1-\theta\beta)p^*_t. \tag{5.12}$$

We turn now from the pricing behaviour of individual firms to the aggregate price level in the economy as a whole. We assume that all firms are identical except for the fact that a fraction $1-\theta$ will be able to choose their prices in period t while the other θ will not (so that their prices will be the same as in the previous period). Thus the average (aggregate) log price in the Calvo economy will be a weighted average of last period's log price and the reset log price for this period as defined by

$$p_t = \theta p_{t-1} + (1-\theta)\overline{p}_t. \tag{5.13}$$

Note that when $\theta = 1$, this equation implies completely rigid aggregate prices $p_t = p_{t-1}$. Rearranging the equation gives

$$\overline{p}_t = \frac{1}{1-\theta}(p_t - \theta p_{t-1}) \tag{5.14}$$

and, leading by one period and taking expectations at period t,

$$E_t\overline{p}_{t+1} = \frac{1}{1-\theta}(E_t p_{t+1} - \theta p_t). \tag{5.15}$$

Combining (5.12) with (5.14) and (5.15) gives

$$\frac{\theta\beta}{1-\theta}(E_t p_{t+1} - \theta p_t) + (1-\theta\beta)p_t^* = \frac{1}{1-\theta}(p_t - \theta p_{t-1}) \tag{5.16}$$

or

$$\frac{\theta\beta}{1-\theta}E_t p_{t+1} = \frac{1+\theta^2\beta}{1-\theta}p_t - (1-\theta\beta)p_t^* - \frac{\theta}{1-\theta}p_{t-1}. \tag{5.17}$$

We want to rewrite this expression in terms of inflation $\pi_t = p_t - p_{t-1}$. Subtracting

$$\frac{\theta\beta}{1-\theta}p_t \tag{5.18}$$

from both sides gives

$$\frac{\theta\beta}{1-\theta}(E_t p_{t+1} - p_t) = \frac{1-\theta\beta+\theta^2\beta}{1-\theta}p_t - (1-\theta\beta)p_t^* - \frac{\theta}{1-\theta}p_{t-1} \tag{5.19}$$

and, adding and subtracting

$$\frac{\theta}{1-\theta}p_t \tag{5.20}$$

to the right-hand side gives

$$\frac{\theta\beta}{1-\theta}(E_t p_{t+1} - p_t) = \frac{1-\theta-\theta\beta+\theta^2\beta}{1-\theta}p_t - (1-\theta\beta)p_t^* + \frac{\theta}{1-\theta}(p_t - p_{t-1}) \tag{5.21}$$

$$= \frac{(1-\theta)(1-\theta\beta)}{1-\theta}p_t - (1-\theta\beta)p_t^* + \frac{\theta}{1-\theta}(p_t - p_{t-1}) \tag{5.22}$$

which can be rearranged for $p_t - p_{t-1}$ to give

$$p_t - p_{t-1} = \beta(E_t p_{t+1} - p_t) + \frac{(1-\theta)(1-\theta\beta)}{\theta}(p_t^* - p_t) \tag{5.23}$$

or

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta}(p_t^* - p_t) \tag{5.24}$$

where $\pi_t = p_t - p_{t-1} = \log(P_t/P_{t-1})$ is the standard definition for the rate of inflation in discrete time.

Finally, we need to make an assumption about how the optimal price p_t^* is determined. The standard assumption is that firms are imperfectly competitive and so set prices as a fixed mark-up over marginal costs in which case

$$p_t^* = \mu + x_t \tag{5.25}$$

where x_t is log marginal cost and $\mu > 0$ is a fixed mark-up. In this case $x_t - p_t$ is the log of real marginal cost. However, the real marginal cost though pro-cyclical is difficult to measure and so is generally proxied by the log output gap

$$y_t = \log Y_t - \log \overline{Y}_t \tag{5.26}$$

where Y_t is output and $\overline{Y}_t$ is normal output. Using this proxy we can write

$$p_t^* - p_t = \phi y_t \tag{5.27}$$

where y_t is the log output gap and $\phi > 0$, so that (5.24) becomes

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta} \phi y_t. \tag{5.28}$$

Equation (5.28) is known as the *new Keynesian Phillips Curve* and was first derived by Roberts (1995). It shows that inflation depends positively on expected future inflation and positively on the log output gap.

6 The Canonical New Keynesian Model

In this section we look briefly at a model by Clarida, Galí and Gertler (2000) that has come to be regarded as the canonical new Keynesian model and incorporates the new Keynesian Phillips curve (5.28) derived in the previous section. The complete model can be described by the three equations:

$$y_t = E_t y_{t+1} - \alpha r_t + u_{1t} \tag{6.1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + u_{2t} \tag{6.2}$$

and

$$r_t = \gamma E_t \pi_{t+1} + \delta E_t y_{t+1} + u_{3t} \tag{6.3}$$

where y_t is the log output gap, π_t is inflation, r_t the real interest rate and $\alpha > 0$, $0 < \beta < 1$, $\kappa > 0$, $\gamma > 0$ and $\delta > 0$. All the equations are linear. The first equation (6.1) is a forward-looking aggregate demand function where demand depends positively on expected demand in the next period and negatively on the interest rate. The second equation (6.2) is the new Keynesian Phillips curve or aggregate supply curve and relates inflation positively to expected future inflation and to output. The third equation (6.3) is a forward-looking Taylor rule for monetary policy and says that the central bank sets the interest rate on the basis of expectations of future inflation and the future output gap.

The terms u_{1t}, u_{2t} and u_{3t} are stochastic disturbances with zero mean that are assumed to be uncorrelated with each other. They represent stochastic shocks to respectively aggregate demand, aggregate supply and the interest rate rule. This model is an example of a *dynamic stochastic general equilbrium* model or *dsge* model for short. The deterministic steady state of the model is where $u_{1t} = u_{2t} = u_{3t} = 0$, $y_t = E_t y_{t+1} = y^*$ and $\pi_t = E_t \pi_{t+1} = \pi^*$. In this case $y^* = \pi^* = r^* = 0$.

Substituting (6.3) into (6.1) gives

$$y_t = (1 - \delta\alpha)E_t y_{t+1} - \gamma E_t \pi_{t+1} + u_{1t} - \alpha u_{3t} \tag{6.4}$$

and substituting this expression into (6.2) gives

$$\pi_t = (\beta - \kappa\gamma)E_t \pi_{t+1} + \kappa(1 - \delta\alpha)E_t y_{t+1} + \kappa u_{1t} + u_{2t} - \kappa\alpha u_{3t}. \tag{6.5}$$

Starting from deterministic steady state where $E_t y_{t+1} = 0$ and $E_t \pi_{t+1} = 0$, it can be seen that a positive shock to aggregate demand u_{1t} increases y_t and π_t but does not affect r_t. Conversely, a positive shock to aggregate supply u_{2t} increases π_t but leaves y_t and r_t unchanged. Finally, a positive (contractionary) shock to the interest rate rule u_{3t} increases r_t but decreases both y_t and π_t. Note that shocks have no permanence so that in period $t + 1$ the model returns to steady state.

These results show that the canonical new Keynesian model does not behave in a very Keynesian way. In particular, shocks to aggregate supply have no effect on real variables and shocks to aggregate demand do not affect real interest rates. Also, unless shocks are assumed to be autoregressive, the model fails to display the persistent dynamic behaviour that we observe in the real world. As a result, new Keynesian models have needed to complicate the canonical model to introduce more realistic features. Inflation inertia in the aggregate supply function can be introduced by means of adding indexation to the new Keynesian Phillips curve. Also, the Calvo pricing model can be extended to wage formation as well as price setting. Inertia can also be added to aggregate demand by including habit formation in the consumer's utility function. This is based on the idea that a consumer's utility might depend on relative consumption as well as the absolute level, either the consumer's own past consumption or that of other rival consumers ('*keeping up with the Joneses*').

All these ideas have been introduced into *dsge* models in an attempt to produce richer and more realistic behaviour. The credit crisis of 2008 has also served to remind economists that disruptions in credit markets can cause large swings in real economic activity and prompted research in introducing credit market inperfections into these models.

7 Conclusions

In this lecture we have seen how the Phillips curve, which initially seemed to provide empirical support for a Keynesian upward sloping aggregate supply function, later forced economists to construct new Keynesian models including the assumption of rational expectations and derived from optimising microeconomic behaviour. The Calvo pricing model gives a microeconomic rationale for a new Keynesian version of the upward sloping aggregate supply function. Combined with a new Keynesian version of aggregate demand and a forward looking interest rate rule, this can be used to construct simple new Keynesian *dsge* models.

Next week, in the final lecture of the module, we will look at optimal monetary policy in a (new Keynesian) macroeconomic model.

References

[1] Akerlof, G. A. and J. L. Yellen (1985), 'A near-rational model of the business cycle, with wage and price inertia', *Quarterly Journal of Economics*, 100, 823–838.

[2] Calvo, G. (1983), 'Staggered contracts in a utility maximizing framework', *Journal of Monetary Economics*, 12, 383–398.

[3] Clarida, R., J. Galí and M. Gertler (2000), 'Monetary policy rules and macroeconomic stability: evidence and some theory', *Quarterly Journal of Economics*, 115, 147–180.

[4] Fischer, S. (1977), 'Long-term contracts, rational expectations, and the optimal money supply rule', *Journal of Political Economy*, 85, 191–205.

[5] Friedman, M. (1968), 'The role of monetary policy', *American Economic Review*, 58, 1–17.

[6] Galí, J. and M. Gertler (1999), 'Inflation dynamics: a structural econometric analysis', *Journal of Monetary Economics*, 44, 195–222.

[7] Lucas, R. E. Jr. (1976), 'Econometric policy evaluation: a critique', *Carnegie-Rochester Conference Series on Public Policy*, 1, 19–46.

[8] Mankiw, N. G. (1985), 'Small menu costs and large business cycles: a macroeconomic model of monopoly', *Quarterly Journal of Economics*, 100, 529–539.

[9] Muth, J. F. (1961), 'Rational expectations and the theory of price movements', *Econometrica*, 29, 315–335.

[10] Phelps, E. S. (1968), 'Money-wage dynamics and labor market equilibrium', *Journal of Political Economy*, 76, 678–711.

[11] Phillips, A. W. (1958), 'The relationship between unemployment and the rate of change of money wages in the United Kingdom, 1861–1957', *Economica*, 25, 283–299.

[12] Roberts, J. M. (1995), 'New Keynesian economics and the Phillips curve', *Journal of Money, Credit and Banking*, 27, 975–984.

[13] Romer, D. (2012), *Advanced Macroeconomics*, Fourth edition, McGraw-Hill, Berkeley.

[14] Taylor, J. B. (1979), 'Staggered wage setting in a macro model', *American Economic Review*, 69, 108–113.