Exercise Sheet 3: Solutions

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1. (a) The levels regression produces the following results:

```
================================================================
Dependent Variable: FOOD
Method: Least Squares
Date: 10/24/02   Time: 13:45
Sample: 1959 1973
Included observations: 15
================================================================
Variable Coefficient Std. Error t-Statistic Prob.
================================================================
C         55.17662    3.624120   15.22483  0.0000
INCOME    0.094111    0.005517   17.05959  0.0000
================================================================
R-squared          0.957241     Mean dependent var     115.9600
Adjusted R-squared 0.953952     S.D. dependent var     11.96195
S.E. of regression 2.566894     Akaike info criterion  4.846836
Sum squared resid  85.65631     Schwarz criterion      4.941243
Log likelihood    -34.35127     F-statistic            291.0298
Durbin-Watson stat 0.947377     Prob(F-statistic)      0.000000
================================================================
```

The estimated intercept (55.17662) indicates that if *income* was zero, demand for *food* would be 55.18 units. The coefficient of *income* is the estimated marginal effect on *food* expenditure of a one unit increase in *income*.

Both coefficients have t-statistics in excess of 2.160 (the 0.05 critical value of t_{13}) are so are significantly different from zero at the 5% level of significance. Another way of seeing this is that the probability values (or p-values) of both statistics are zero (to 4 digits). This means that the probability of the true parameter values being equal to zero is 0.0000.

(b) The logs regression produces the following results:

```
===============================================================
Dependent Variable: LF
Method: Least Squares
Date: 10/24/02   Time: 13:53
Sample: 1959 1973
Included observations: 15
===============================================================
Variable Coefficient Std. Error t-Statistic Prob.
===============================================================
C        1.339974    0.161592    8.292322  0.0000
LY       0.528154    0.025030    21.10063  0.0000
===============================================================
R-squared          0.971630     Mean dependent var      4.748237
Adjusted R-squared 0.969448     S.D. dependent var      0.103815
S.E. of regression 0.018146     Akaike info criterion  -5.057172
Sum squared resid  0.004281     Schwarz criterion      -4.962765
Log likelihood     39.92879     F-statistic             445.2366
Durbin-Watson stat 1.006570     Prob(F-statistic)       0.000000
===============================================================
```

The coefficient of LY is the estimated income elasticity of the demand for food. For every 10% increase in income, it is estimated that food expenditure increases by 5.3% (i.e. an inelastic response).

Again, both coefficients have t-statistics in excess of 2.160 (the 0.05 critical value of t_{13}) are so are significantly different from zero at the 5% level of significance. Another way of seeing this is that the probability values (or p-values) of both statistics are zero (to 4 digits). This means that the probability of the true parameter values being equal to zero is 0.0000.

(c) There is no price variable in these equations. The data in the file *DATAB12.WF1* contains a nominal price index for food *PFOOD* and a total expenditure price deflator *PTPE*. From these we can construct a real price variable P defined by

$$P = PFOOD/PTPE$$

and its logarithm

$$LP = \log(P).$$

We can now estimate the regression

$$LF_t = \gamma + \delta LY_t + \lambda LP_t + u_t$$

The data in file *DATAB12.WF1* is available over a longer time span. Estimating the extended equation over the full data set we obtain:

```
================================================================
Dependent Variable: LF
Method: Least Squares
Date: 10/24/02   Time: 14:20
Sample: 1959 1983
Included observations: 25
================================================================
Variable Coefficient Std. Error t-Statistic Prob.

C          0.649353    0.137934    4.707702 0.0001
LY         0.644406    0.021504    29.96652 0.0000
LP        -0.477345    0.098878   -4.827614 0.0001
================================================================
R-squared           0.991063     Mean dependent var      4.842199
Adjusted R-squared 0.990250     S.D. dependent var      0.148444
S.E. of regression 0.014657     Akaike info criterion  -5.495571
Sum squared resid  0.004726     Schwarz criterion      -5.349306
Log likelihood     71.69464     F-statistic             1219.801
Durbin-Watson stat 1.070004     Prob(F-statistic)       0.000000
================================================================
```

Now we see that both income and price are significant and have the 'correct' signs. Demand for food is estimated to be both income and price inelastic. We note that we do have a 'low' *Durbin-Watson* statistic and this indicates that there is still something wrong with our specification.

2. (a) The regression yields the following results:

```
====================================================================
Dependent Variable: LC
Method: Least Squares
Date: 10/24/02   Time: 14:32
Sample(adjusted): 1949 1985
Included observations: 37 after adjusting endpoints
====================================================================
Variable Coefficient Std. Error t-Statistic Prob.
====================================================================
C         1.267844    0.078618   16.12674 0.0000
LY        0.877252    0.007317   119.8891 0.0000
INF      -0.126484    0.045418  -2.784851 0.0087
====================================================================
R-squared           0.998401     Mean dependent var      10.88271
Adjusted R-squared  0.998307     S.D. dependent var      0.258500
S.E. of regression  0.010636     Akaike info criterion  -6.171579
Sum squared resid   0.003846     Schwarz criterion      -6.040964
Log likelihood      117.1742     F-statistic             10615.94
Durbin-Watson stat  0.952491     Prob(F-statistic)       0.000000
====================================================================
```

(b) (i) The estimate of the income elasticity is 0.877252. A 95% confidence interval for the income elasticity is given by

$$0.877252 \pm 2.03 \times 0.007317 = (0.8624, 0.8921)$$

where 2.03 is the 5% critical value for a two-tail test of the T-distribution t_{34}.

(ii) A t-statistic for the hypothesis that $\beta_2 = 1$ is

$$\left|\frac{\beta_2 - 1}{\widehat{\sigma}_{\beta_2}}\right| = \left|\frac{0.877252 - 1}{0.007317}\right| = 16.7757.$$

The 5% critical value for a one-tail test of the T-distribution t_{34} is 1.69 so the hypothesis is strongly rejected. The hypothesis that $\beta_2 = 1$ is the hypothesis of a unit income elasticity of consumption.