Exercise Sheet 4: Solutions

R.G. Pierse

Q1.

```
=================================================================
Dependent Variable: CX
Method: Least Squares
Date: 10/22/02   Time: 19:07
Sample: 1960 1986
Included observations: 27
=================================================================
Variable Coefficient Std. Error t-Statistic Prob.
=================================================================
C         0.155224    0.203471   0.762879    0.4527
Y         0.597069    0.060594   9.853648    0.0000
=================================================================
R-squared          0.795240     Mean dependent var     2.037449
Adjusted R-squared 0.787050     S.D. dependent var     0.789223
S.E. of regression 0.364199     Akaike info criterion  0.888954
Sum squared resid  3.316021     Schwarz criterion      0.984942
Log likelihood    -10.00088     F-statistic            97.09438
Durbin-Watson stat 0.462830     Prob(F-statistic)      0.000000
=================================================================
```

(a) Using the DW statistic to test for *AR(1)* errors we find that, (with 27 observations and 1 slope coefficient) $d_L = 1.32$. Since

$$DW = 0.462830 < d_L$$

we reject the null hypothesis of no autocorrelation against the alternative of *AR(1)* errors.

Using the Breusch-Godfrey LM test we get

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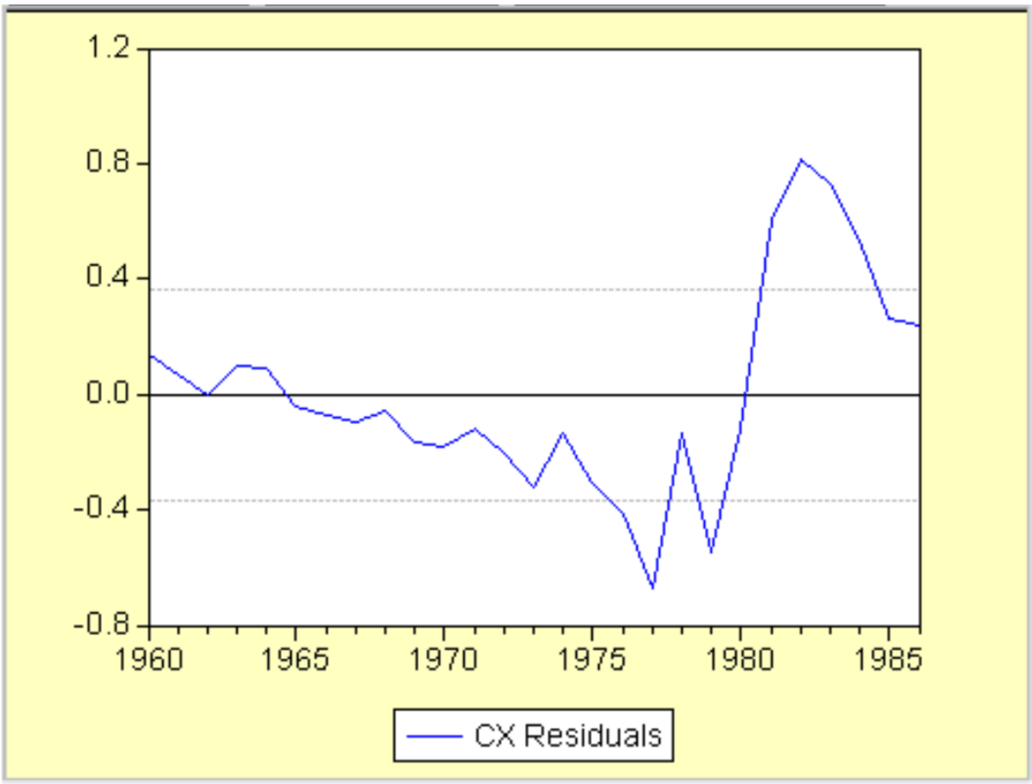
======================================================================
Breusch-Godfrey Serial Correlation LM Test:
======================================================================
F-statistic    34.31433      Probability  0.000005
Obs*R-squared 15.88781      Probability  0.000067
======================================================================
```

For both forms of the test we have p-values of 0.000 so that we reject the null hypothesis of no autocorrelation at e.g. the 1% or 5% levels. In this case (with no lagged dependent variables) we would prefer to use the DW test since it is an exact (rather than asymptotic) test.

(b) Plot of the residuals



Autocorrelations and Partial autocorrelations

```
=================================================================
Date: 10/22/02   Time: 19:45
Sample: 1960 1986
Included observations: 27
=================================================================
Autocorrelation Partial Correlation      AC     PAC  Q-Stat Prob
=================================================================
     .  |******  |      .  |******  | 1    0.757  0.757 17.249 0.000
     .  |****    |      . *|  .     | 2    0.518 -0.128 25.659 0.000
     .  |**.     |      .**|  .     | 3    0.266 -0.189 27.970 0.000
     .  |  .     |      . *|  .     | 4    0.020 -0.186 27.983 0.000
     .**|  .     |      . *|  .     | 5   -0.190 -0.147 29.266 0.000
     .**|  .     |      .  |* .     | 6   -0.248  0.135 31.558 0.000
     .**|  .     |      .  |  .     | 7   -0.237  0.037 33.765 0.000
     .**|  .     |      . *|  .     | 8   -0.213 -0.087 35.630 0.000
     .**|  .     |      .**|  .     | 9   -0.230 -0.235 37.934 0.000
     . *|  .     |      .  |  .     | 10 -0.175  0.064 39.343 0.000
     . *|  .     |      . *|  .     | 11 -0.175 -0.088 40.844 0.000
     . *|  .     |      .  |  .     | 12 -0.177 -0.024 42.470 0.000
=================================================================
```

Note that the residuals are not randomly scattered about zero and *at least* the first three autocorrelation function values are significantly different from zero - both figures indicate the presence of autocorrelation.

(c) Denote the saved residuals as RES and obtain

```
================================================================
Dependent Variable: RES
Method: Least Squares
Date: 10/22/02   Time: 19:59
Sample(adjusted): 1961 1986
Included observations: 26 after adjusting endpoints
================================================================
Variable Coefficient Std. Error t-Statistic Prob.
================================================================
RES(-1)   0.770406    0.129372    5.954944 0.0000
================================================================
R-squared 0.586418     Mean dependent var  -0.005381
Adjusted R-squared 0.586418     S.D. dependent var      0.363081
S.E. of regression 0.233499     Akaike info criterion  -0.033580
Sum squared resid  1.363039     Schwarz criterion       0.014809
Log likelihood     1.436536     Durbin-Watson stat      1.801323
================================================================
```

Generating the transformed variables:

$$CC = CX - 0.770406 * CX(-1)$$

and

$$YY = Y - 0.770406 * Y(-1)$$

we can run a regression to obtain the (2-step) Cochrane-Orcutt estimates of α and β.

```
===============================================================
Dependent Variable: CC
Method: Least Squares
Date: 10/22/02   Time: 20:07
Sample(adjusted): 1961 1986
Included observations: 26 after adjusting endpoints
Variable Coefficient Std. Error t-Statistic Prob.
===============================================================
C         0.143885    0.123017   1.169637 0.2536
YY        0.465313    0.141247   3.294325 0.0031
===============================================================
R-squared          0.311385     Mean dependent var     0.519862
Adjusted R-squared 0.282693     S.D. dependent var     0.276407
S.E. of regression 0.234100     Akaike info criterion  0.007669
Sum squared resid  1.315271     Schwarz criterion      0.104446
Log likelihood     1.900302     F-statistic            10.85257
Durbin-Watson stat 1.721340     Prob(F-statistic)      0.003054
===============================================================
Breusch-Godfrey Serial Correlation LM Test:
===============================================================
F-statistic   0.427383     Probability  0.519759
Obs*R-squared 0.474314     Probability  0.491009
===============================================================
```

Neither the *Durbin-Watson* nor the *LM* tests show any evidence of serial correlation.

Q2.

```
=================================================================
Dependent Variable: CX
Method: Least Squares
Date: 10/24/02   Time: 15:48
Sample(adjusted): 1961 1986
Included observations: 26 after adjusting endpoints
=================================================================
Variable Coefficient Std. Error t-Statistic Prob.
=================================================================
C         0.003962    0.122430    0.032362  0.9745
Y         0.207495    0.064345    3.224707  0.0038
CX(-1)    0.695319    0.094013    7.396025  0.0000
=================================================================
R-squared          0.937124     Mean dependent var      2.068732
Adjusted R-squared 0.931656     S.D. dependent var      0.787597
S.E. of regression 0.205899     Akaike info criterion  -0.214697
Sum squared resid  0.975069     Schwarz criterion      -0.069532
Log likelihood     5.791066     F-statistic             171.3986
Durbin-Watson stat 1.919159     Prob(F-statistic)       0.000000
=================================================================
```

There is a lagged dependent variable in this model so that the *Durbin-Watson* statistic is invalid. Instead we use the *Breuch-Pagan LM* test statistic.

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=================================================================
Breusch-Godfrey Serial Correlation LM Test:
=================================================================
F-statistic   0.045571     Probability  0.832923
Obs*R-squared 0.053745     Probability  0.816670
=================================================================
```

The p-value for the *LM* test is 0.833 so that we do not reject the null that there is no autocorrelation.

One possible explanation for why we found autocorrelation in the original model, but not here, is that original model was misspecified by the incorrect exclusion of the lagged dependent variable C_{t-1}.