Exercise Sheet 5: Solutions

R.G. Pierse

2. Estimation of **Model M1** yields the following results:

```
===============================================================
Dependent Variable: LRC
Method: Least Squares
Date: 10/24/02   Time: 18:06
Sample: 1955:1 1990:2
Included observations: 142
===============================================================
Variable Coefficient Std. Error t-Statistic Prob.

C         -1.448432   0.696587   -2.079327  0.0395
LPC       -0.306051   0.272836   -1.121740  0.2640
LPF       -0.326679   0.263148   -1.241427  0.2166
LPL        0.213974   0.095465    2.241382  0.0267
LPD        0.160577   0.088646    1.811439  0.0723
LNY        0.893699   0.062924    14.20274  0.0000
LP        -0.597475   0.272264   -2.194468  0.0300
D1        -0.371417   0.011886   -31.24701  0.0000
D2        -0.244535   0.010631   -23.00183  0.0000
D3        -0.261237   0.010494   -24.89505  0.0000
================================================================
R-squared          0.986629     Mean dependent var      7.776945
Adjusted R-squared 0.985718     S.D. dependent var      0.359173
S.E. of regression 0.042924     Akaike info criterion  -3.390938
Sum squared resid  0.243209     Schwarz criterion      -3.182781
Log likelihood     250.7566     F-statistic             1082.259
Durbin-Watson stat 1.089358     Prob(F-statistic)       0.000000
================================================================
```

(a) Noting the relationships between the variables:

$$\begin{aligned}
\log RPC_t &= \log(PC_t/P_t) = \log PC_t - \log P_t \\
\log RPF_t &= \log(PF_t/P_t) = \log PF_t - \log P_t \\
\log RPL_t &= \log(PL_t/P_t) = \log PL_t - \log P_t \\
\log RPD_t &= \log(PD_t/P_t) = \log PD_t - \log P_t \\
\log RY_t &= \log(Y_t/P_t) = \log Y_t - \log P_t
\end{aligned}$$

the model (M1) can be rewritten as

$$\begin{aligned}
\log RC_t = \quad & \beta_1 + \beta_2 \log RPC_t + \beta_2 \log P_t + \beta_3 \log RPF_t + \beta_3 \log P_t \\
& +\beta_4 \log RPL_t + \beta_4 \log P_t + \beta_5 \log RPD_t + \beta_5 \log P_t \qquad \text{(M1)} \\
& +\beta_6 \log RY_t + \beta_6 \log P_t + \beta_7 \log P_t \\
& +\beta_8 d1_t + \beta_9 d2_t + \beta_{10} d3_t + \varepsilon_t
\end{aligned}$$

so that the sum of the coefficients on P_t is $\beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7$. If the restriction $\beta_7 = \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6$ holds, then this is zero and the model collapses to (M2).

(b) The Wald test gives:

```
==============================================================
Wald Test:
Equation: Untitled
==============================================================
Null Hypothesis: C(2)+C(3)+C(4)+C(5)+C(6)+C(7)=0
==============================================================
F-statistic 0.392806  Probability 0.531910
Chi-square  0.392806  Probability 0.530828
==============================================================
```

The *p-value* for this test (in either form) is 0.53 so we fail to reject the null at the 5% level. Thus model (M2) is the preferred model.

3. Estimation of **Model M2** yields the following results:

```
==============================================================
Dependent Variable: LRC
Method: Least Squares
Date: 10/24/02   Time: 18:45
Sample: 1955:1 1990:2
Included observations: 142
==============================================================
Variable Coefficient Std. Error t-Statistic Prob.
==============================================================
C         -1.580734   0.662315   -2.386680  0.0184
LRPC      -0.472348   0.063384   -7.452214  0.0000
LRPF      -0.209961   0.185494   -1.131906  0.2597
LRPL       0.206184   0.094436    2.183312  0.0308
LRPD       0.180844   0.082349    2.196067  0.0298
LRY        0.905562   0.059873    15.12481  0.0000
D1        -0.372761   0.011665   -31.95594  0.0000
D2        -0.245944   0.010367   -23.72410  0.0000
D3        -0.261872   0.010421   -25.13045  0.0000
===============================================================
R-squared           0.986590     Mean dependent var     7.776945
Adjusted R-squared 0.985783     S.D. dependent var     0.359173
S.E. of regression 0.042826     Akaike info criterion  -3.402051
Sum squared resid  0.243933     Schwarz criterion     -3.214710
Log likelihood     250.5456     F-statistic            1223.076
Durbin-Watson stat 1.066690     Prob(F-statistic)      0.000000
==============================================================
```

(a) The estimated coefficient of *D1* (-0.373) can be interpreted as an estimate of the change in the (logarithm of the) level of expenditure on clothing and footwear in the first quarter relative to the expenditure in the fourth quarter. The interpretation of *D2* and *D3* is similar.

(b) For first order autocorrelation, we can use the *Durbin-Watson* statistic. The value of the statistic is 1.067 which is below the critical value of d_L which is 1.622 ($T = 150$, $k = 9$) and so we reject the null of no autocorrelation. For fourth order autocorrelation, we use the *LM* test:

```
=================================================================
Breusch-Godfrey Serial Correlation LM Test:
=================================================================
F-statistic   43.92150     Probability  0.000000
Obs*R-squared 81.87909     Probability  0.000000
=================================================================
```

which rejects the null (both forms of the test have p-values of 0.0000). We would usually test for fourth order autocorrelation when we have quarterly data.

(c) (i) The own price elasticity is -0.472348 and the income elasticity is 0.905562.

(ii) Testing for unit price elasticity:

```
=================================================================
Wald Test:
Equation: Untitled
=================================================================
Null Hypothesis: C(2)=-1
=================================================================
F-statistic 69.30130  Probability 0.000000
Chi-square  69.30130  Probability 0.000000
=================================================================
```

the hypothesis of a unit price elasticity is strongly rejected

(iii) Testing for unit income elasticity:

```
=================================================================
Wald Test:
Equation: Untitled
=================================================================
Null Hypothesis: C(6)=1
=================================================================
F-statistic 2.487902  Probability 0.117100
Chi-square  2.487902  Probability 0.114725
=================================================================
```

we do not reject the null of a unit elasticity at the 5% level.

4. Creating a dummy variable $DUM1 = 1$ in 1973:1 but zero otherwise and a dummy variable $DUM2 = 1$ in 1973:2 but zero otherwise, we can add these dummies to the model and get the following results:

```
================================================================
Dependent Variable: LRC
Method: Least Squares
Date: 10/24/02   Time: 19:14
Sample: 1955:1 1990:2
Included observations: 142
================================================================
Variable Coefficient Std. Error t-Statistic Prob.
================================================================
C          -1.571912   0.653042   -2.407060 0.0175
LRPC       -0.484985   0.063084   -7.687981 0.0000
LRPF       -0.230243   0.182812   -1.259455 0.2101
LRPL        0.210778   0.092826    2.270672 0.0248
LRPD        0.202351   0.081701    2.476731 0.0145
LRY         0.904786   0.059034    15.32664 0.0000
D1         -0.375857   0.011526   -32.61012 0.0000
D2         -0.246012   0.010269   -23.95739 0.0000
D3         -0.262140   0.010241   -25.59621 0.0000
DUM1        0.111978   0.043352    2.583015 0.0109
DUM2       -0.004711   0.043793   -0.107562 0.9145
================================================================
R-squared          0.987243     Mean dependent var      7.776945
Adjusted R-squared 0.986269     S.D. dependent var      0.359173
S.E. of regression 0.042087     Akaike info criterion  -3.423861
Sum squared resid  0.232041     Schwarz criterion      -3.194888
Log likelihood     254.0941     F-statistic             1013.809
Durbin-Watson stat 1.003463     Prob(F-statistic)       0.000000
================================================================
```

We see that dummy *DUM1* (the dummy for the period *before* the tax change) is significant but dummy *DUM2* is not. We might want to test the hypothesis that the coefficients on the dummies are equal but opposite in sign, which is the hypothesis that agents perfectly anticipated the tax change. Conversely, we could just drop *DUM2*.

Note that all the models we have been looking at have very low *Durbin-Watson* statistics and so we might want to consider respecifying these models, possibly including lags of the model variables.