Exercise Sheet 6: Solutions

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1. (a) Regression yields:

```
==============================================================
Dependent Variable: LC
Method: Least Squares
Date: 10/29/02   Time: 18:37
Sample(adjusted): 1950 1985
Included observations: 36 after adjusting endpoints
==============================================================
Variable Coefficient Std. Error t-Statistic Prob.
==============================================================
C         0.244716    0.213958    1.143759 0.2618
LC(-1)    0.708128    0.150811    4.695459 0.0001
LY        0.660755    0.067809    9.744325 0.0000
LY(-1)   -0.392093    0.124026   -3.161394 0.0036
INF      -0.164111    0.052693   -3.114502 0.0040
INF(-1)   0.015959    0.052421    0.304433 0.7629
==============================================================
R-squared          0.999118     Mean dependent var      10.89442
Adjusted R-squared 0.998971     S.D. dependent var       0.252008
S.E. of regression 0.008085     Akaike info criterion   -6.646632
Sum squared resid  0.001961     Schwarz criterion       -6.382712
Log likelihood     125.6394     F-statistic              6795.131
Durbin-Watson stat 2.023213     Prob(F-statistic)        0.000000
==============================================================
```

(i) The estimated short run income elasticity is 0.660755.
The estimated long run income elasticity is

$$
\begin{aligned}
& \frac{\widehat{\beta}_3 + \widehat{\beta}_4}{1 - \widehat{\beta}_2} \\
= & \frac{0.660755 - 0.392093}{1 - 0.708128} \\
= & 0.920479
\end{aligned}
$$

Using the *EViews* Wald coefficient restrictions test facility, we can test the hypothesis that this long-run elasticity is zero:

Wald Test on Long Run Elasticity

```
==============================================================
Wald Test:
Equation: Untitled
==============================================================
Null Hypothesis: (C(3)+C(4))/(1-C(2))
==============================================================
F-statistic 733.3756  Probability 0.000000
Chi-square  733.3756  Probability 0.000000
==============================================================
```

(ii) The test for serial correlation yields:

```
Breusch-Godfrey Serial Correlation LM Test:
==============================================================
F-statistic   0.069339     Probability  0.794163
Obs*R-squared 0.085871     Probability  0.769494
==============================================================
```

On the basis of the Breusch-Godfrey serial correlation test, we would not reject the null of no autocorrelation.

(iii)

Wald Test of hypothesis: $\beta_2 + \beta_3 + \beta_4 = 1$

```
==============================================================
Wald Test:
Equation: Untitled
==============================================================
Null Hypothesis: C(2)+C(3)+C(4)=1
==============================================================
F-statistic 1.254592  Probability 0.271565
Chi-square  1.254592  Probability 0.262677
==============================================================
```

We do not reject H_0 at the 5% significance level.

(iv) Subtracting C_{t-1} from both sides of

$$\begin{aligned} LC_t &= \beta_1 + \beta_2 LC_{t-1} + \beta_3 LY_t + \beta_4 LY_{t-1} \\ &\quad + \beta_5 INF_t + \beta_6 INF_{t-1} + \varepsilon_t \end{aligned}$$

we get

$$\begin{aligned} \Delta LC_t &= \beta_1 + \beta_2 LC_{t-1} - LC_{t-1} + \beta_3 LY_t + \beta_4 LY_{t-1} \\ &\quad + \beta_5 INF_t + \beta_6 INF_{t-1} + \varepsilon_t, \end{aligned}$$

and, adding and subtracting $\beta_2 LY_{t-1}$ and $\beta_3 LY_{t-1}$, we obtain

$$\begin{aligned} \Delta LC_t &= \beta_1 + \beta_2 (LC_{t-1} - LY_{t-1}) - LC_{t-1} + \beta_3 \Delta LY_t \\ &\quad + (\beta_2 + \beta_3 + \beta_4) LY_{t-1} + \beta_5 INF_t + \beta_6 INF_{t-1} + \varepsilon_t. \end{aligned}$$

If the restriction

$$\beta_2 + \beta_3 + \beta_4 = 1 \quad \text{or} \quad \frac{\beta_3 + \beta_4}{1 - \beta_2} = 1$$

holds then we can rewrite this as

$$\begin{aligned} \Delta LC_t &= \beta_1 + (\beta_2 - 1)(LC_{t-1} - LY_{t-1}) + \beta_3 \Delta LY_t \\ &\quad + \beta_5 INF_t + \beta_6 INF_{t-1} + \varepsilon_t \end{aligned}$$

which is the same as the restricted model

$$\begin{aligned} \Delta LC_t &= \gamma_1 + \gamma_2 \Delta LY_t + \gamma_3 (LY_{t-1} - LC_{t-1}) \\ &\quad + \gamma_4 INF_t + \gamma_5 INF_{t-1} + \varepsilon_t. \end{aligned}$$

with the parameter correspondences: $\gamma_1 = \beta_1$, $\gamma_2 = \beta_3$, $\gamma_3 = 1 - \beta_2$, and $\gamma_4 = \beta_5$ and $\gamma_5 = \beta_6$.

2.

Unrestricted Distributed Lag

```
==============================================================
Dependent Variable: Y
Method: Least Squares
Date: 10/29/02   Time: 19:28
Sample(adjusted): 1953:08 1957:12
Included observations: 53 after adjusting endpoints
==============================================================
Variable Coefficient Std. Error t-Statistic Prob.
==============================================================
C          92.95484    56.37294    1.648927 0.1063
X          0.057709    0.052537    1.098451 0.2780
X(-1)      0.102538    0.078054    1.313676 0.1958
X(-2)      0.214550    0.077012    2.785941 0.0078
X(-3)      0.184946    0.076605    2.414266 0.0200
X(-4)      0.114580    0.077278    1.482704 0.1453
X(-5)      0.041156    0.076684    0.536696 0.5942
X(-6)      0.133245    0.079779    1.670182 0.1020
X(-7)      0.110570    0.058545    1.888639 0.0655
==============================================================
R-squared           0.989442     Mean dependent var     3229.849
Adjusted R-squared 0.987523     S.D. dependent var     1157.760
S.E. of regression 129.3240     Akaike info criterion  12.71604
Sum squared resid  735886.7     Schwarz criterion      13.05062
Log likelihood    -327.9750     F-statistic            515.4453
Durbin-Watson stat 0.425647     Prob(F-statistic)      0.000000
==============================================================
```

(a) The estimated impact (short-run) multiplier is 0.057709.
The estimated long-run multiplier is

$$\begin{aligned} & 0.057709 + 0.102538 + 0.214550 + 0.184946 \\ & +0.114580 + 0.041156 + 0.133245 + 0.110570 \\ = \; & 0.95930 \end{aligned}$$

Using the *EViews* Wald coefficient restrictions test facility, we can test the hypothesis that this long-run multiplier is zero:

Wald Test on Long Run Multiplier

```
=================================================================
Wald Test:
Equation: Untitled
=================================================================
Null Hypothesis: C(2)+C(3)+C(4)+C(5)+C(6)+C(7)+C(8)+C(9)=0
=================================================================
F-statistic 3209.189  Probability 0.000000
Chi-square  3209.189  Probability 0.000000
=================================================================
```

(b) Form the Almon variables:

$$\begin{aligned} Z0 &= X + X(-1) + X(-2) + X(-3) + X(-4) \\ &\quad + X(-5) + X(-6) + X(-7) \end{aligned}$$

$$\begin{aligned} Z1 &= X(-1) + 2 * X(-2) + 3 * X(-3) + 4 * X(-4) \\ &\quad + 5 * X(-5) + 6 * X(-6) + 7 * X(-7) \end{aligned}$$

$$\begin{aligned} Z2 &= X(-1) + 4 * X(-2) + 9 * X(-3) + 16 * X(-4) \\ &\quad + 25 * X(-5) + 36 * X(-6) + 49 * X(-7) \end{aligned}$$

$$\begin{aligned} Z3 &= X(-1) + 8 * X(-2) + 27 * X(-3) + 64 * X(-4) \\ &\quad + 125 * X(-5) + 6 * 36 * X(-6) + 7 * 49 * X(-7) \end{aligned}$$

$$\begin{aligned} Z4 &= X(-1) + 16 * X(-2) + 81 * X(-3) + 256 * X(-4) \\ &\quad + 625 * X(-5) + 36 * 36 * X(-6) + 49 * 49 * X(-7) \end{aligned}$$

Fourth Degree Almon Lag Polynomial

```
================================================================
Dependent Variable: Y
Method: Least Squares
Date: 10/29/02   Time: 19:45
Sample(adjusted): 1953:08 1957:12
Included observations: 53 after adjusting endpoints
================================================================
Variable Coefficient Std. Error t-Statistic Prob.
================================================================
C         91.15685    55.03268   1.656413 0.1043
Z0        0.036566    0.045387   0.805649 0.4245
Z1        0.162404    0.172128   0.943511 0.3502
Z2       -0.051813    0.115507  -0.448572 0.6558
Z3        0.003884    0.025901   0.149944 0.8815
Z4        7.05E-05    0.001845   0.038189 0.9697
================================================================
R-squared          0.989220     Mean dependent var     3229.849
Adjusted R-squared 0.988073     S.D. dependent var     1157.760
S.E. of regression 126.4396     Akaike info criterion  12.62368
Sum squared resid  751387.9     Schwarz criterion      12.84673
Log likelihood    -328.5275     F-statistic            862.5750
Durbin-Watson stat 0.490582     Prob(F-statistic)      0.000000
================================================================
```

We can see from the p-value that the coefficient on the fourth order term, $Z4$, is not significant so we drop it and estimate the third degree polynomial Almon lag.

Third Degree Almon Lag Polynomial

```
================================================================
Dependent Variable: Y
Method: Least Squares
Date: 10/29/02   Time: 19:49
Sample(adjusted): 1953:08 1957:12
Included observations: 53 after adjusting endpoints
================================================================
Variable Coefficient Std. Error t-Statistic Prob.
================================================================
C          91.31976   54.29340   1.681968  0.0991
Z0         0.035553   0.036435   0.975776  0.3341
Z1         0.168311   0.074723   2.252456  0.0289
Z2        -0.056093   0.027670  -2.027187  0.0482
Z3         0.004867   0.002645   1.840519  0.0719
================================================================
R-squared          0.989220     Mean dependent var     3229.849
Adjusted R-squared 0.988321     S.D. dependent var     1157.760
S.E. of regression 125.1175     Akaike info criterion  12.58597
Sum squared resid  751411.2     Schwarz criterion      12.77185
Log likelihood    -328.5283     F-statistic            1101.125
Durbin-Watson stat 0.489410     Prob(F-statistic)      0.000000
================================================================
```

We can see from the p-value that the coefficient on the third order term, *Z3*, is not significant so we drop it and estimate the second degree polynomial Almon lag.

Second Degree Almon Lag Polynomial

```
======================================================================
Dependent Variable: Y
Method: Least Squares
Date: 10/29/02   Time: 19:56
Sample(adjusted): 1953:08 1957:12
Included observations: 53 after adjusting endpoints
======================================================================
Variable Coefficient Std. Error t-Statistic Prob.
======================================================================
C         98.01741    55.47535   1.766864  0.0835
Z0         0.092330    0.019855   4.650281  0.0000
Z1         0.034612    0.017935   1.929833  0.0594
Z2       -0.005375    0.002569  -2.092453  0.0416
======================================================================
R-squared           0.988459     Mean dependent var     3229.849
Adjusted R-squared 0.987752     S.D. dependent var     1157.760
S.E. of regression 128.1294     Akaike info criterion  12.61643
Sum squared resid  804440.6     Schwarz criterion      12.76513
Log likelihood    -330.3354     F-statistic            1398.878
Durbin-Watson stat 0.493917     Prob(F-statistic)      0.000000
======================================================================
```

This time the p-value of the coefficient on the highest order term, *Z2*, shows that it is significant and so we stop searching and choose a second degree polynomial.

(c) One way to obtain an estimate of the long-run (equilibrium) multiplier is to note that *in equilibrium*

$$X_t = X_{t-1} = X_{t-1} = \cdots = X_{t-7}$$

so that

$$\begin{aligned} Y_t &= c + 7 * \alpha_0 * X_t + (1+2+3+\cdots+7) * \alpha_1 * X_t \\ &\quad + (1+4+9+\cdots+49) * \alpha_2 * X_t. \end{aligned}$$

The long-run multiplier is therefore given by

$$\begin{aligned} & 7 * \widehat{\alpha}_0 + (1+2+3+\cdots+7) * \widehat{\alpha}_1 + (1+4+9+\cdots+49) * \widehat{\alpha}_2 \\ = \;& 0.86298. \end{aligned}$$

A *Wald* test for the hypothesis that the long-run multiplier is zero is given by:

Wald Test on Long Run Multiplier

```
=================================================================
Wald Test:
Equation: Untitled
=================================================================
Null Hypothesis: 7*C(2)+7*4*C(3)+(1+4+9+16+25+36+49)*C(4)=0
=================================================================
F-statistic 1742.687  Probability 0.000000
Chi-square  1742.687  Probability 0.000000
=================================================================
```