Chapter 3

WinSolve Tutorials

3.1 Tutorial on Klein Model 1

This tutorial looks at the famous Klein Model 1 in the version described and estimated by Theil (1971) on annual US data over the period 1921–1941. There are six endogenous variables: *consumption*, *investment*, *priv_wage_bill*, *production*, *profits* and *capital_stock*, and three exogenous variables not explained within the model: gov wage bill, gov expenditure and taxes.

3.1.1 Opening the model

Click on the (blue) open a model icon \mathbb{P} or select the *Open model* option from the *File* menu. A dialog box opens to allow you to select a model. Select the file *klein1.txt* and click *OK*. A new window will appear on the desktop representing the model object (Figure 3.1). The icon in this window is a



Figure 3.1: The Klein Model Object

link to the model equations. Double clicking this icon opens an editor to view these equations. The notation should be fairly self-explanatory. The WinSolve function time(1932) is a time trend taking the value one in 1932, two in 1933 etc.

{ Klein's Model I (1920-1941) Source: Theil (1971) data: p.456; parameter estimates: p.517 } @ Behavioural equations consumption = 16.44 + 0.1249*profits + 0.1631*profits(-1) +0.7901*(priv_wage_bill+gov_wage_bill); investment = 28.18 - 0.0131*profits + 0.7557*profits(-1) -0.1948*capital_stock(-1); priv_wage_bill = 1.8 + 0.4005*production + 0.1813*production(-1) + 0.1497*time(1932); @ Identities production = consumption + investment + gov_expenditure; profits = production - priv_wage_bill - taxes; capital_stock = capital_stock(-1) + investment;

Before the model can be solved we need to add a data set. This will define the maximum solution period for the model and provide observations on, at least, all exogenous variables, plus initial values for lagged endogenous variables. In this case, we have a data set, klein1.sdf, that contains historical data on all nine model variables. Click on the (green) open a data file icon or select the *Open data file* option from the *Data* menu. A dialog box opens to allow you to select a data file. Select the file klein1.sdf and click OK. Notice that several new icons on the toolbar are now ungreyed which indicates that they have become operational. This includes the (yellow) Solve model icon \Im that allows us to solve the model.

3.1.2 Single equation solution

Single equation solution mode solves a model equation by equation, using data values on the right hand side of each equation. The resulting solution should replicate the (static) prediction values of the estimated equations and the solution residuals should replicate the estimation residuals.

Click on the Solve model icon \Im or select the *Solve model* option from the *Solve* menu. This opens the Solution dialog box (Figure 3.2) that allows you to choose the select various solution options. Choose *Single equation solution* from the Solution mode list box and click *OK*. This opens the Solution status box (Figure 3.3) which displays information as the model

3.1. TUTORIAL ON KLEIN MODEL 1

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Figure 3.2: Model solution dialog box

solves. When model solution is complete, the status box can be closed by clicking on the OK button.

The results can now be viewed in the *Results* menu. Click on the graph icon \checkmark or select the *New table/graph* option in the *Results* menu. Choose the variable *consumption* and the variable type *Changes from base*. Click the *Add* button to add this item to the graph and then click *OK* to finish. The single equation residuals (Predicted minus actual) will be graphed (Figure 3.4). This graph can be displayed as a table by clicking on the system menu icon in the top right hand corner of the graph and selecting the *View as table* option.

3.1.3 Dynamic Forecasting

Dynamic model solution mode solves the model equations simultaneously. Past forecast values are used as lagged values in the solution so that forecast errors will tend to become magnified over time.

Click on the Solve model icon \Im or select the *Solve model* option from the *Solve* menu. This time choose the *Dynamic model solution* option from the Solution mode list box and click *OK*. When the model has solved, close the status box. Click on the graph icon \Join or select the *New table/graph* option in the *Results* menu. Select the variable *consumption* and choose the variable type *Changes from base*. Click the *Add* button to add this item to the graph and click *OK* to finish. The dynamic forecast residuals will be graphed (Figure 3.5). Comparing these residuals to the single equation residuals (Figure 3.4), it can be seen that the dynamic residuals are much larger since

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Figure 3.3: Solution status box

they incorporate the effect of errors in forecasting the other equations in the model, although, in this case, they are not being magnified but are actually decreasing over time.

However, the model seriously fails to forecast turning points as can be seen by graphing the forecast values (simulation values) against actuals (base values) as in Figure 3.6. The model forecasts (unbroken line) predicted a trough in 1929 which corresponded to a boom in actual consumption (ending in the Wall Street crash).

3.1.4 Simulating a government expenditure shock

We can explore the effect of shocks to the exogenous or endogenous variables on the Klein model . This is known as simulation. Simulations are computed relative to a base solution path.

Firstly, we need to create a simulation base. Click on the Solve model icon \checkmark or select the *Solve model* option from the *Solve* menu. Choose the *Create simulation base* option from the Solution mode list box and click *OK*. When the model has solved, close the status box. The original data values have now been replaced by the dynamic solution values. If we do a further dynamic forecast from this simulation base without making any adjustments to the variables, then the model will solve in a single iteration with zero forecast residuals. If we add a shock to one or more of the variables and then solve the model, the residuals will represent the difference between the shocked run



Figure 3.4: Consumption: single equation residuals 1921–1941



Figure 3.5: Consumption: Dynamic forecast residuals 1921–1941



Figure 3.6: Consumption: Dynamic Forecast and Actuals 1921–1941

and the base run, and hence show the effect of the shocks, relative to the base. We will look at the effect on the solution path of a 10% sustained increase in government expenditure starting in 1930. This can be represented as a *multiplicative adjustment* of 0.1 to the exogenous variable *gov-expenditure*. In *WinSolve*, adjustments to exogenous variables are by default assumed to be additive rather than multiplicative. Hence the first step is to override this default and declare that adjustments to *gov-expenditure* are multiplicative.

Choose the Change Adjustment type option from the Assumptions menu (Figure 3.7). Select the variable gov-expenditure and choose the Multiplicative option in the Override Type box. Click OK to leave the dialog box. Now click the edit dialog box icon \mathbf{M} or select the Edit data/adjustments option from the Data menu (Figure 3.8). Select type Adjustments and variable gov-expenditure. Then click on the Project level/growth rate button. This opens the dialog box in Figure 3.9. Select the projection period 1930–1941, click the Constant button and put the value 0.1 in the a: box. Then click OK to return to the edit dialog and click the Update button to update the adjustments. Finally click OK to leave the dialog box. This sets a 10% positive adjustment to the variable gov-expenditure over the period 1930–1941.

Now we can run the simulation. Click on the Solve model icon $\frac{1}{2}$ or



Figure 3.7: Adjustment Dialog Box

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Figure 3.8: Edit dialog box

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Figure 3.9: Projection dialog box

select the Solve model option from the Solve menu. Choose the Simulation run option from the Solution mode list box and click OK. When the model has solved, close the status box. To look at the results on production, click on the graph icon \swarrow or select the New table/graph option in the Results menu. Choose the variable production and the variable type % changes from base. Click the Add button to add this item to the graph and then click OK to finish. This graphs the percentage differences from the base solution (Figure 3.10). It can be seen that the changes from base are zero before the shock to government expenditure in 1930. They then rise to nearly 6% in 1932 before falling to 1% in 1938. After this they start to rise again, reaching 3.5% by the end of the simulation.

3.1.5 A target-instrument simulation

One interesting question is the following: what pattern of government expenditure would have ensured a steady growth rate in production of 5% throughout the whole period? This question can be answered by a target-instrument simulation. The target variable is *production* which will be fixed to grow at 5% per annum. The instrument is *gov-expenditure* and the model will determine a path for it to ensure that the target is met. This type of simulation is sometimes known as a Type 2 fix. It can be viewed as a simple form of optimal control. When the model is solved using Newton's method, *WinSolve* uses the model derivatives to determine how *gov-expenditure* needs to be changed. In a linear model such as Klein's Model 1, the derivatives are constant and the control problem will be solved in one iteration.



Figure 3.10: Production: % differences from base 1921–1941

Before we can start, the adjustments that were added to *gov-expenditure* in the previous simulation need to be removed. This can be done in one of two ways. Either the adjustments could be removed, in the same way that they were imposed, through the edit dialog box \mathbf{M} . However, in this case it is simpler to reset all adjustments back to zero. Select the *Reset data / adjustments* option from the *Data* menu and check the *Set to zero* box. Click *OK* to exit.

Now we are ready to set up the simulation. Select the Fix/Unfix variables option from the Assumptions menu. This opens the dialog box depicted in Figure (3.11). Select production from the Variable List and gov-expenditure from the Variable to instrument list. Then check the Fix indirectly box. The current status window will change to show the effect of the fix, as illustrated. Finally, click Done to exit the dialog.

The final step is to set the target path for production. Click the edit dialog box icon \checkmark or select the *Edit data/adjustments* option from the *Data* menu. Select variable *production* and then click on the *Project level/growth rate* button to open the Projection dialog box (Figure 3.9). Select the projection period 1921–1941, click the X(-1) button and put the value 1.05 in the c: box. Then click OK to return to the edit dialog and click the *Update* button to update the adjustments. Finally click OK to leave the dialog box. This

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Figure 3.11: Fix / unfix dialog box

sets a 5% growth rate on the variable *production* over the period 1921–1941. (Note that the simulation base has now been changed. It is important that you do not save this back to file unless you want to overwrite the original simulation base.)

The simulation can now be run. Click on the Solve model icon \Im or select the *Solve model* option from the Solve menu. Choose the *Dynamic model solution* option from the Solution mode list box and click *OK* to start the simulation. Figure (3.12) graphs the solution values for the instrument variable *gov-expenditure* (solid line) against actual values (dotted line). The solid line represents the level of government expenditure that would have kept production growing at 5% per annum over the whole solution period, despite the Great Crash. Note that this is less than actual expenditure for the first five years (over this period, production was growing faster than 5% anyway) but rises steadily as the Depression takes hold, with the difference reaching a maximum of \$16.5 billion in 1940.

3.2 Tutorial on a Stochastic Growth Model

In this tutorial we look at a simple real business cycle model set out in Taylor and Uhlig (1990). This paper, and the subsequent papers in the same issue of the Journal of Business and Economic Statistics, compare different numerical techniques for solving the stochastic growth model, including the



Figure 3.12: 5% target growth: required and actual government expenditure

parameterised expectations method of den Haan and Marcet (1990) which is implemented in *WinSolve*. We will use this method to solve the model and also apply the stacked Newton method.

3.2.1 The model

Agents live forever and maximise lifetime expected utility subject to a budget constraint. A constant relative risk aversion (CRRA) utility function is assumed, of the form

$$u(c_t) = (1 - \tau)^{-1} c_t^{1 - \tau}$$

where c_t is consumption and τ is the coefficient of relative risk aversion $0 < \tau < 1$.

Formally agents solve the problem:

$$\max E_0 \sum_{t=0}^{\infty} (1-\tau)^{-1} c_t^{1-\tau} \beta^t$$
(3.1)

subject to the resource constraint

$$c_t + k_t = \theta_t k_{t-1} \alpha + \mu k_{t-1} \tag{3.2}$$

where k_t is the end of period capital stock, and θ_t is technology. $1 - \mu$ is the rate of capital depreciation, $0 \le \mu \le 1$ and β is the rate of time discount, $0 < \beta < 1$.

Technology θ_t is assumed to be stochastic, following the process

$$\ln \theta_t = \rho \ln \theta_{t-1} + \varepsilon_t \tag{3.3}$$

where ε_t is a serially uncorrelated normally distributed random variable with zero mean and constant variance σ^2 . The first order Euler condition for capital in this model is given by

$$c_t^{-\tau} = \beta E_t [c_{t+1}^{-\tau} (\theta_{t+1} \alpha k_t^{\alpha - 1} + \mu)]$$
(3.4)

The solution to this problem is a decision rule for consumption and for the capital stock given by $c_t = f(k_{t-1}, \theta_t)$ and $k_t = g(k_{t-1}, \theta_t)$ respectively. However, the exact form of $f(\cdot)$ and $g(\cdot)$ is not known analytically as there is no closed-form solution to the model. Solutions must be found by numerical solution of the equations (3.4), (3.2) and (3.3).

Taylor and Uhlig (1990) and the subsequent papers in the same issue of the Journal of Business and Economic Statistics compare different numerical techniques for solving this model.

A special case: Brock-Mirman model

In the special case where the utility function is logarithmic ($\tau = 1$) and there is full depreciation ($\mu = 0$), then the model simplifies to

$$\max E_0 \sum_{t=0}^{\infty} \ln c_t \beta^t$$

subject to

$$c_t + k_t = \theta_t k_{t-1} \alpha$$

and the Euler condition becomes

$$c_t^{-1} = \beta E_t [c_{t+1}^{-1}(\theta_{t+1}\alpha k_t^{\alpha-1})].$$

This model is described in Brock and Mirman (1972) and is known as the *Brock-Mirman* economy. In this case there is a simple closed-form solution (see for example Sargent (1987) p122) given by

$$k_t = \alpha \beta k_{t-1}^a \theta_t$$

and

$$c_t = (1 - \alpha \beta) k_{t-1} \alpha \theta_t \; .$$

Parameterising expectations

The problem in solving the growth model is in finding the expectation

$$E_t[y_{t+1}] \tag{3.5}$$

in (3.4) where

$$y_t = c_t^{-\tau} (\theta_t \alpha k_{t-1}^{\alpha - 1} + \mu)$$

This expectation is a function of the state variables $x_t = \{k_{t-1}, \theta_t\}$ but its form is unknown. Note that on the assumption that expectations are model consistent,

$$E_t[y_{t+1}] = y_{t+1}$$
.

den Haan and Marcet (1990) propose a general method for solving models by approximating expectations such as (3.5) using a functional form

$$\psi_t(\mathbf{x}_t; \boldsymbol{\delta})$$

where \mathbf{x}_t is a $p \times 1$ vector of state variables and $\boldsymbol{\delta}$ is a $k \times 1$ vector of parameters. These parameters are chosen such as to minimise the sum of squared residuals

$$\min_{\boldsymbol{\delta}} \sum_{t=1}^{T} (y_{t+1} - \psi_t(\mathbf{x}_t; \boldsymbol{\delta}))^2 .$$

This is simply a nonlinear least squares problem and can be solved iteratively.

The functional form of ψ should be chosen so as to be able to approximate the expectation as closely as possible. den Haan and Marcet suggest the class of *power functions*

$$\exp P_n(\ln(\mathbf{x}))$$

where P_n is a polynomial function of degree n. With large enough n, this class of functions can approximate any function $R_+^p \to R_+$ arbitrarily well. For the stochastic growth model they suggest the first order power function

$$\psi_t(k_{t-1}, \theta_t; \boldsymbol{\delta}) = \delta_1 k_{t-1}^{\delta_2} \theta_t^{\delta_3} = \exp P_1(\ln k_{t-1}, \ln \theta_t)$$

but also consider higher order power functions.

den Haan and Marcet (1994) proposes a test of solution accuracy that can be applied to the method of parameterised expectations. This is implemented by increasing the degree of the power function and testing the significance of the additional coefficients.

3.2.2 The stochastic growth model in WinSolve

In the *WinSolve* model definition language the stochastic growth model can be written as

$$\begin{aligned} ltheta &= rho * ltheta(-1) + norm(sigma ^2) ; theta = exp(ltheta) ; \\ c &= (beta * cexp(1)) ^(-1 / tau) ; \\ k &= theta * k(-1) ^alpha + mu * k(-1) - c ; \\ cexp &= C ^(-tau) * (alpha * theta * k(-1) ^(alpha - 1) + mu) ; \end{aligned}$$
(3.6)

where the model parameters ρ , α , β , μ , τ , and σ have been coded as WinSolve parameters *rho*, *alpha*, *beta*, *mu*, *tau* and *sigma* respectively. The equation on the second line corresponds to the Euler condition (3.4) where cexp(1) is the forward expectation of *cexp* which is defined by the fourth line.

This model has forward expectations and so requires a solution algorithm that can impose consistent expectations. *WinSolve* has three solution methods that can be applied to this model: the *Fair-Taylor* method (Fair and Taylor 1983), the *Stacked Newton* method (Newton 1687, Laffargue 1990, Boucekkine 1995) and the *parameterised expectations* method (den Haan and Marcet 1990, 1994). In practice, the *Fair-Taylor* method has problems with highly nonlinear models and will not solve this model.

The Stacked Newton method

The *Stacked Newton* method works by stacking all the equations over all time periods and solving the whole as a single simultaneous system using Newton's method with analytic derivatives. *WinSolve* makes use of the structure of the Jacobian matrix of the stacked system to reduce the computational burden following the suggestions of Laffargue (1990) and Boucekkine (1995). This method is very powerful.

Parameterising expectations in WinSolve

WinSolve implements the parameterised expectations algorithm of den Haan and Marcet (1990) through a function defined in the model definition language. For the case of the growth model, the expectation cexp(1) can be parameterised by replacing the second line in (3.6) with

$$c = (beta * parexp(cexp(1), k(-1), theta, 1, 2))^{(-1/tau)};$$
 (3.7)

The WinSolve function parexp() takes arguments defined by

$$parexp(y, x_1, \cdots, x_p [, \delta_1, \cdots, \delta_k], n, p)$$

where y is the expectation to be parameterised, x_1, \dots, x_p are the state variables, n is the order of the power function and p is the number of state variables. $\delta_1, \dots, \delta_k$ represent *optional* initial values for the parameters of the power function. Good initial values will improve the speed of convergence of the method. When a model has been solved once with parameterised expectations, *WinSolve* will save the solution values of the parameter vector $\boldsymbol{\delta}$ and will use these as starting values in subsequent solutions. This will speed up convergence in these subsequent runs.

Note that parameterising expectations does not require a separate solution algorithm in *WinSolve*. The *Fair-Taylor* method should be selected but the parameterised expectations algorithm will be doing all the work since, apart from the function parexp(), the model is completely backward looking.

3.2.3 Opening the model

Click on the open a model icon \square or select the *Open model* option from the *File* menu. Select the file *rbc.txt* in the dialog box and click *OK*. A new window will appear on the desktop (Figure 3.13) representing the stochastic



Figure 3.13: Stochastic growth model object

growth model. The model parameters have been set to the values $\rho = .95$, $\alpha = .33$, $\beta = .95$, $\mu = 0.7$, $\tau = 1$, and $\sigma = .1$ corresponding to one of the high variance cases reported in den Haan and Marcet (1990). We now open a data set. Click on the (green) open a data file icon from the *Data* menu. Select the file *rbc.sdf* and click *OK*. This data set allows the model to be solved for 2500 periods.

3.2.4 Solving the model

We will first solve the model using the *Stacked Newton* algorithm. This is the default so no changes need to be made. Click on the Solve model icon \Im or select the *Solve model* option from the *Solve* menu. Make sure that the *Dynamic model solution* option in the Solution mode list box is selected and

then click OK. This opens the status box and initiates solution. When the model has solved, close the status box by clicking on the OK button. The results can be viewed in the *Results* menu.

We will now solve the model again, this time using the parameterised expectations algorithm. To do this, we must replace the equation for c in (3.6) with the parameterised equation (3.7). Since (3.7) has already been coded up in the model as an alternative equation, all we need to do is to switch it in. Select the *Switch alternative equations* option in the *Assumptions* menu which brings up the dialog box in Figure (3.14). Choose the equation with



Figure 3.14: Switch alternative equations dialog box

description '1st order parameterised equation' from the Description list box, and click *Done*.

Now we are ready to solve the model again. WinSolve will automatically select an appropriate solution method for the parameterised expectation equation. Simply click on the Solve model icon \Im or select the Solve model option from the Solve menu, and then click on OK to commence model solution.

When this run has solved, the two model solutions can be compared in the *Results* menu. Figure (3.15) shows solution values for c from the two runs, graphed over the full 2500 observations. As can be seen, the two solutions are nearly identical. The final values for the power function parameters in the parameterised expectation are 1.44, -.48 and -.74 which are very close to the results reported by den Haan and Marcet for this case (1.44, -.49 and -.72).

3.2.5 Stochastic simulation

The dynamic solutions considered in the previous section are based on a single drawing of the random shock to technology. A new drawing will generate a different solution path. Consider taking repeated dawings of ε_t in (3.3),



Figure 3.15: Dynamic simulation of consumption in stochastic growth model

assuming a normal distribution with zero mean and variance σ^2 . The average of these drawings will be a consistent estimate of the expected value of the solution path. This procedure is known as *stochastic simulation* and increasing the number of drawings or *replications* increases its precision.

We will now do a stochastic simulation with the growth model, solving it using the parameterised expectations algorithm. The model is solved once without shocks to estimate the parameters of the expectations function. Then, in the stochastic replications, the expectations function is treated as fixed and not re-estimated. This is equivalent to the assumption that agents do not anticipate future shocks, so that the simulations are based on *rational* rather than *consistent* expectations. Computationally, it means that the simulations are much cheaper to perform.

Click on the Stochastic simulation icon \bigotimes or select the *Stochastic simulation* option from the *Solve* menu. This opens the Stochastic simulation dialog box (Figure 3.16). Select the *Diagonal variance matrix* option from the Shock generation method box. This option allows you to enter a variance for the shock to be applied to each equation, which is the correct option here since only one of our equations, that for *ltheta*, is stochastic. After clicking *OK*, another dialog box will open in which you enter values for the variance elements. Select the variable *ltheta* and enter the value 0.01. Then click the

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Figure 3.16: Stochastic simulation dialog box

OK button which opens the status box and starts the simulation.

The average of the stochastic simulations is plotted as the solid line in Figure (3.17) while the broken line represents a single draw dynamic solution.

3.3 Further reading

Whitley (1994) is a simple introduction to macroeconomic modelling and forecasting. Pierse (1997) is a concise review of solution methods for nonlinear models, or see Fisher (1992) for a very detailed treatment. Taylor and Uhlig (and the following 10 papers in the same volume) compare different ways of solving the stochastic growth model set out here.



Figure 3.17: Consumption: stochastic simulation average versus one draw

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