A Kalman filter approach to estimating the UK NAIRU

Jennifer V Greenslade[*]

Richard G Pierse[**]

and

Jumana Saleheen[***]

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[*] External MPC Unit, Bank of England, Threadneedle Street, London, EC2R 8AH.
Email: jennifer.greenslade@bankofengland.co.uk

[**] Department of Economics, University of Surrey, Guildford, GU2 5XH and Bank of England, Threadneedle Street, London, EC2R 8AH.
Email: r.pierse@surrey.ac.uk

[***] Monetary Analysis, Bank of England, Threadneedle Street, London, EC2R 8AH.
Email: jumana.saleheen@bankofengland.co.uk

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Abstract

In this paper, the Kalman filter method is applied to UK Phillips-curve models and estimates are derived for the NAIRU from 1973 to 2000. The resulting profiles suggest that the NAIRU peaked around the mid-1980s and fell back thereafter. Structural changes in the labour market have reduced inflationary pressure from that source, and we suggest that temporary effects from real import prices and real oil prices were an important additional downward influence on inflation in the latter half of the 1990s. Some of the uncertainties around our NAIRU estimates are shown. But, even though there may be uncertainty about exactly where the NAIRU is, a variety of models suggest that unemployment was below the NAIRU for much of the second half of the 1990s.

Key words: Phillips curve, Kalman filter.
JEL classification: E24, E31.

Summary

In the second half of the 1990s, a period that was characterised generally by buoyant economic activity, unemployment in the United Kingdom fell continuously and reached its lowest level in over 20 years. In 2000, Labour Force Survey (LFS) unemployment stood at just over 5% of the labour force, which was nearly 2 percentage points below the lowest rate seen during the previous recovery. A key question then is at what level of unemployment will wage and price inflation begin to rise? This critical level of unemployment is usually referred to as the non-accelerating inflation rate of unemployment or NAIRU. If the unemployment rate falls below this level, it will put upward pressure on inflation and inflation will tend to rise (though effects from other variables may offset this pressure).

There are many possible methods that could be used to estimate the NAIRU. This paper adopts a statistical approach by applying Kalman filter techniques that allow the joint estimation of the Phillips curve and a time-varying measure of the NAIRU. We have used a variety of models (based on either price or wage inflation) and calculated time-varying NAIRU estimates from 1973 to 2000. According to these estimates, the NAIRU reached a peak in the mid-1980s and tended to decline thereafter. Such profiles are broadly in line with other UK estimates, often obtained from different approaches. Of course, the estimates presented in this paper should be regarded as illustrative and not interpreted as MPC estimates. In practice there are a range of labour market indicators that may be relevant for analysing inflationary pressures.

It is widely acknowledged that there is a great deal of uncertainty around NAIRU estimates, whichever approach is used. We illustrate this through the large standard error bands around our Kalman filter estimates. As a consequence, we would not place weight on any particular point estimate for the NAIRU. But even though there may be uncertainty about the level of the NAIRU, a range of specifications and assumptions tend to suggest that the NAIRU was falling through the 1990s (though we do not analyse the reasons for any fall in the NAIRU). Further, according to our models, it appears likely that unemployment at the end of the decade was below the NAIRU, suggesting some upward pressure on inflation from this source. Had the NAIRU estimates not fallen over this period, there would have been greater upward pressure on inflation from the labour market. So structural changes appear to have had a beneficial effect on UK inflation during this period.

However, the story does not end there. Our results suggest that temporary supply factors (captured by real import prices or real oil prices) are also likely to have played an important role in holding inflation down, especially in the 1997-99 period. Developments in import prices or oil prices, as well as movements in the unemployment gap, may therefore be important in assessing future inflationary pressures.

This paper has not touched on changes to the UK monetary policy regime, such as the move to inflation targeting at the end of 1992 or the granting of independence to the Bank of England in 1997, which may have had an impact on the formation of inflation expectations. It is possible that our NAIRU estimates are indirectly picking up any such changes, thus casting doubt on our estimates. But separate work including inflation expectations does not provide any strong evidence that this was a key factor for the United Kingdom.

1 Introduction

In the second half of the 1990s, a period that was characterised generally by buoyant economic activity, unemployment in the United Kingdom fell noticeably. In 2000, Labour Force Survey (LFS) unemployment was just over 5% of the labour force, which was nearly 2 percentage points below the lowest rate seen during the previous recovery. Such a fall in unemployment is not in itself surprising, but the levels to which unemployment has fallen without any sign of wage or price inflation is (see Charts 1 and 2). In the previous recovery, unemployment reached a low of under 7% in 1990. At that time annual wage inflation was around 10% and annual RPIX inflation about 8%. More recently, unemployment had fallen to 5.3% at the end of 2000, whereas price inflation had remained subdued at just over 2%, and wage inflation tended to remain at under 5%.

A key question then is at what level of unemployment will wage and price inflation begin to rise? This critical level of unemployment is usually referred to as the non-accelerating inflation rate of unemployment or NAIRU. If the unemployment rate falls below this level, it will put upward pressure on inflation and inflation will tend to rise (though effects from other variables may offset this pressure). This link between the labour market and the goods market means that knowledge about where the NAIRU is may help a central bank's understanding of inflationary pressure in the economy.

The UK was not alone in experiencing very low levels of unemployment without much wage and price inflation in the second half of the 1990s. Indeed the pick-up in US inflation was not substantial despite the fall in unemployment to 4%, below the commonly held value of a 6% NAIRU there.

To explain this UK and US experience, it has been argued that the NAIRU fell over this period (Katz and Krueger (1999) and Barwell (2000)). In practice, however, there is a great deal of uncertainty about the extent of this fall. This uncertainty is compounded by other developments in these economies. For example, there were movements in exchange rates that worked to dampen inflation (a factor that may be particularly important in the UK) — at least temporarily. And strong investment (in the US) or a fall in inflation expectations (in the UK) have raised the question as to whether price pressures were likely to have been as strong as they were in previous recoveries.

A number of explanations have been offered for a lower NAIRU in the UK. One is the much-discussed structural (or supply-side) reforms of the labour market. Those reforms of the 1980s and 1990s, for example, worked to reduce the collective bargaining power of workers: through de-unionisation and the promotion of flexible (part-time/temporary) but perhaps more insecure working arrangements. More recently the government's New Deal has also lowered unemployment by actively encouraging the young and long-term unemployed people back into work.

Chart 1: LFS unemployment rate

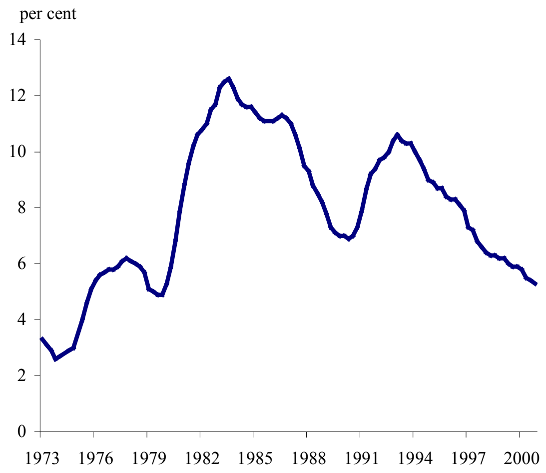
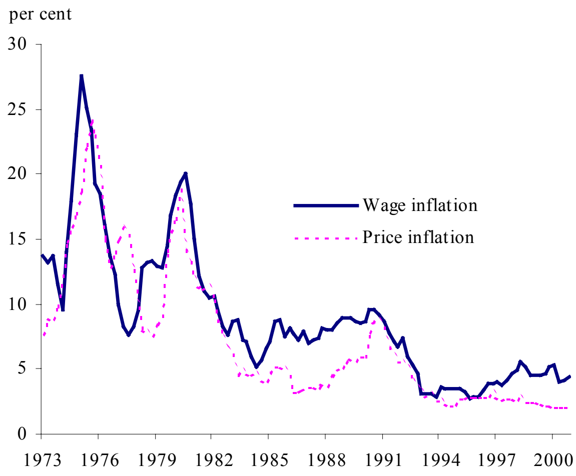


Chart 2: Wage and price inflation (annual % change)



We find empirical support for the hypothesis that the UK NAIRU rose till the mid-80s but fell back thereafter. We use Kalman filter techniques that allow the joint estimation of the Phillips curve and a time-varying measure of the NAIRU (see Bank of England (1999) for details of other work using this approach). The filtering process uses the rule that stable price inflation implies that unemployment must have been at the NAIRU (subject to effects from other variables). Rising (falling) inflation, however, implies that unemployment must have been below (above) the NAIRU. This intuitive simplicity is perhaps the main reason for the popularity of this statistical technique. Examples include work such as Gordon (1997 and 1998), Staiger, Stock and Watson (1997a), Boone (2000), Apel and Jansson (1999) and Rasi and Viikari (1998). Little work has used this approach for the UK. This paper aims to address this issue comprehensively.

It is widely acknowledged that there is a great deal of uncertainty around NAIRU estimates, whichever approach is used. For example, Cross, Darby and Ireland (1997) found that this was the case when they used a variety of techniques (though not the Kalman filter approach) to estimate the NAIRU. We illustrate this through the large standard error bands around our Kalman filter estimates. But even though there may be uncertainty about the level of the NAIRU, a range of specifications and assumptions tend to suggest that the NAIRU was falling through the 1990s. However, this framework does not analyse the reasons for any fall in the NAIRU as it does not include structural variables (see Barrell, Pain and Young (1994) or Cassino and Thornton (2002) for work based on using structural variables).

The outline of the paper is as follows. The paper begins with a discussion of the theoretical ideas underlying the Phillips curve and the NAIRU. The intuition behind the Kalman filter method and estimates are described in Section 3 leaving the full technical details for Appendix B. Sections 4 and 5 present a range of Phillips curve models and the corresponding NAIRU profiles for 1973–2000. These models use different variables, assumptions and dynamics. Section 4 concentrates on models using RPIX or consumers' expenditure price inflation; and Section 5 uses wage inflation measured by the Average Earnings Index (AEI) or wages and salaries per employee. All models produce a broadly similar profile for the NAIRU. Section 7 shows how sensitive these results are to the assumptions made. It also demonstrates the very large uncertainty surrounding these NAIRU estimates. Section 8 concludes.

2 Phillips curve models

The apparent inverse relationship between UK money wage growth and unemployment, often called the Phillips curve, suggested an exploitable trade-off between inflation and unemployment (Phillips (1958)). The expectations augmented Phillips curve (Friedman (1968) and Phelps (1968)) developed this model further, by suggesting that such a trade-off could only be temporary and that the long-run Phillips curve is vertical. However, in the short-run the economy can be shifted away from its long-run equilibrium either because changes in aggregate demand create forecasting errors, or because of nominal inertia in the wage and/or price setting process.

A stylised version of the 'accelerationist Phillips curve model' may be written as:

$$\pi_t = \pi_{t-1} - \theta (u_t - u_t^*) \qquad \textbf{(1)}$$

where π_t is actual inflation, u_t is the unemployment rate, u_t^* is the natural rate of unemployment, and θ captures the impact of deviations in unemployment from its natural rate.

Various interpretations of Phillips curves have emerged.[1] One is the 'triangle' model of inflation, where the label indicates the dependence of the inflation rate on three factors: inertia, demand and supply (see Gordon (1997)). A general representation of the triangle Phillips model is of the following form:[2]

$$\pi_t = \alpha(L)\pi_{t-1} - \beta(L)(u_t - u_t^*) + \gamma(L)'\boldsymbol{z}_t + \varepsilon_t \qquad \textbf{(2)}$$

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[1] The expectations augmented Phillips curve may also be derived from a wide range of imperfectly competitive microeconomic models (see Roberts (1995, 1997)).

[2] The lag operator *L* allows the lagged specification to be written in short-hand form with $\alpha(L)$, $\beta(L)$, and $\gamma(L)$.

where inertia is represented by lags of inflation. Current and lagged values of the unemployment gap, $(u - u^*)$, are used as a proxy for excess demand and z_t represents supply-side factors, capturing inflationary pressure from the supply side – for example through a rise in the oil price.(3)

According to the equation, as soon as unemployment falls below the NAIRU, it will put upward pressure on inflation and inflation will tend to rise (though effects from other variables may offset this pressure). This framework has been used in many studies to provide time-varying estimates of the NAIRU or potential output, providing an indication of the level of excess demand or supply in the economy. The inclusion of supply-shock variables means that u_t^* is the NAIRU consistent with steady inflation in the absence of these temporary supply shocks. If these variables are excluded, the NAIRU can jump around in response to these temporary supply shocks. But this is not what one would commonly regard as a medium or longer-term concept of the NAIRU.(4)

Although the accelerationist Phillips curve usually relates labour market tightness (or the closeness of u to u^*) to price inflation, this relationship can also be represented in terms of wage inflation. The intuition is that the reduced supply of unemployed (or potential) workers puts upward pressures on wages and then on prices. Since the pressure is likely to be felt first in the labour market, it may be captured in wage inflation data before it is seen in price inflation data. For this reason, it is interesting to measure the NAIRU in both wage and price space. But real wages tend to grow in line with productivity. The wage inflation from this source will be independent of changes in labour market tightness, and — by leaving the firm's profitability unchanged — will not result in higher prices. So it makes sense when using wage inflation data to exclude these productivity-related effects. This can be done either by including productivity changes as a separate explanatory variable (as we do in Section 5), or by using a measure of wage inflation that has been adjusted for (trend) productivity growth (Gordon (1998)).

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(3) As in the case of most papers, we use a linear model (rather than a non-linear model, which would allow a different effect of unemployment on wages or prices when unemployment is low (eg a fall from 4% to 3%) than when it is high (eg a fall from 12% to 11%)). For examples of non-linear applications, see Debelle and Laxton (1997) or Gruen, Pagan and Thompson (1999).

(4) One may also be interested in knowing how the NAIRU moves in the short run. Short-run NAIRU estimates are affected by volatile temporary supply and are not discussed here.

3 Estimating a time-varying NAIRU

There are many possible methods that could be used to estimate the NAIRU and hence potential output.(5) The NAIRU may for example be modelled as a function of labour market or of demographic variables, or as a deterministic function of time, or as a stochastic process. Some approaches are characterised as economic approaches (such as the production function approach), others as statistical approaches (such as the Hodrick-Prescott filter(6) or multivariate filters), although the approaches are not mutually exclusive.(7)

In this paper, we treat the NAIRU as an unobserved stochastic process. We use the Kalman filter of Kalman (1960) and Kalman and Bucy (1961), since it has the major advantage of allowing a time-varying NAIRU to be estimated jointly with a Phillips curve. This *joint estimation* procedure ensures that the path of the estimated NAIRU is the one that performs best in a Phillips curve. This reduced-form approach has been widely used because of its intuitive simplicity.(8)

The main reason for preferring a multivariate approach is that it uses more information, including theory, to derive potential output or the NAIRU, rather than relying solely on the univariate properties of the unemployment rate.(9) These techniques also allow smooth, continuous updating of the estimate as new information becomes available. In addition, this method side-steps various modelling problems which are encountered when estimating a theoretical model of the NAIRU.(10) At a practical level this has included the difficulty in obtaining appropriate data to measure some of the key structural variables (eg union power and the replacement ratio, see Cassino and Thornton (2002) for a discussion of these issues).

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(5) For more details, see Giorno *et al* (1995), Barrell and Sefton (1995) or Cerra and Saxena (2000).

(6) Previous work at the Bank, using different values of the smoothing parameter, has found that an HP-filtered NAIRU (based solely on actual unemployment) has been highly correlated with movements in inflation.

(7) For an example of the SVAR approach for the UK, see Astley and Yates (1999). Other examples of this approach include Cerra and Saxena (2000), though this method produced a positive output gap for Sweden for most of the sample period, including the early 1990s, that the authors considered to be 'implausible'. They suggest that such an outturn may be related to difficulties relating the composite pure shocks to specific economic variables.

(8) For an example of the application of the Kalman filter technique to US output, see Kuttner (1994).

(9) Another example of a system approach for estimating the NAIRU and potential output is given in Adams and Coe (1990). This paper integrates wage and price data with 'real' and structural data.

(10) Manning (1993) argues that the commonly estimated wage price system is econometrically unidentified.

Further, unlike the HP filter method, the Kalman filter approach is not totally contingent on the choice of an arbitrary smoothing parameter – though, as we discuss below, some restrictions are typically made on the variability of the estimated NAIRU.

While some papers use the term natural rate of unemployment and NAIRU interchangeably (Gordon (1997) and Staiger, Stock and Watson (1997a)), a distinction is often drawn between these concepts. The natural rate concept captures the long-run real equilibrium determined by the structural characteristics of the labour and product markets, while the NAIRU is defined solely in relation to the level of unemployment that is consistent with a stable rate of inflation and so may be affected by the adjustment of the economy to past economic shocks.[(11)] This distinction is less important in the long run, as the effects of adjustment to shocks wash out and the NAIRU will tend towards the natural rate. In this paper, we use the NAIRU terminology because our models are reduced-form and do not explicitly incorporate information on the structural economic variables (eg union density or the replacement ratio) that determine the natural rate. But the resulting estimates are long run (as opposed to short run) in nature to the extent that we do allow for temporary supply-shock variables and 'smoothing' may proxy the fact that structural factors are likely to change slowly over time.

3.1 Kalman filter approach

The approach that we take to estimate a time-varying NAIRU is outlined below (for further details of the technique, see Appendix B). Our approach follows Harvey (1989,1993) or Hamilton (1994), but also includes exogenous variables (such as proxies for supply shocks).

The basic model consists of two equations.

$$\pi_t = \alpha(L)\pi_{t-1} - \beta(L)(u_t - u_t^*) + \gamma(L)' z_t + \varepsilon_t \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \qquad \textbf{(2)}$$

$$u_t^* = u_{t-1}^* + \eta_t \qquad \eta_t \sim N(0, \sigma_\eta^2) \text{ and } \mathrm{cov}(\varepsilon_t, \eta_t) = 0 \qquad \textbf{(3)}$$

The first equation is the accelerationist Phillips curve discussed in Section 2 (equation **(2)**, repeated above). Since the Phillips curve is specified in terms of the unemployment gap, the coefficient on the unemployment term is constrained to be equal and opposite to that on the NAIRU term. Inflation in equation **(2)** could be specified in terms of wage inflation or price inflation. We follow a commonly used approach and estimate the model in first differences of inflation (see for example, Staiger, Stock and Watson

[(11)] For a discussion of the NAIRU and natural rate concepts, see King (1999).

(1997a)). This is a way of imposing dynamic homogeneity.[12] We follow the approach generally used in the Kalman filter literature and assume that inflation expectations are implicit in the inflation dynamics, rather than being explicitly identified. Theory suggests that where there is price stickiness, inflation expectations will play an important role as agents take account of such information in their decision-making process. For example, the New Keynesian Phillips curves model inflation as a forward-looking price mark-up equation (see for example, Galí and Gertler (1999)), though, in practise, backward-looking inflation dynamics also play an important empirical role in such models. The empirical importance of survey measures of inflation expectations are considered in a companion paper (see Driver, Greenslade and Pierse (2003)). To date, there is evidence to suggest that inflation expectations are playing some role in determining inflation in the UK, though the evidence is not yet conclusive. It is possible that our NAIRU estimates are indirectly picking up any such changes (which could be related to changes in the UK monetary policy regime).

Equation **(3)** specifies the time-series process generating the unobservable NAIRU (or u^*). It says that the NAIRU is an unobserved or stochastic process that follows a random walk.[13]

An important ratio in the above model is the ratio of the variances of the error terms in the two equations, the 'signal-to-noise ratio'. The signal-to-noise ratio measures the volatility or variance of the NAIRU relative to the variance of changes in inflation. In general one would expect the NAIRU to be less volatile than inflation and move little from quarter to quarter. But in practise, estimating the NAIRU without restricting this ratio leads to a NAIRU that is extremely volatile. So, in estimating the NAIRU it is typical to restrict the signal-to-noise ratio. The extreme case is for this ratio to be set to zero, $\sigma_\eta^2 = 0$, which would mean that the NAIRU would be a constant.

In this simple model, inflation changes for two reasons. First, a random exogenous event (or 'noise' measured in ε_t) might shock inflation. Second, the NAIRU itself might change. The model allows us to identify the source of the inflation change in each period. If normally distributed errors are assumed, the filter allows the computation of the log-likelihood function of the model that enables the parameters to be estimated using maximum likelihood methods. The estimation results include a

[12] Dynamic homogeneity is important as it ensures a meaningful NAIRU. Another way in which it can be imposed is to model inflation but impose the sum of lagged inflation terms to be equal to one. In terms of the RPIX models considered here, the NAIRU estimates do not appear to be very sensitive to such a choice of specification (see Driver, Greenslade and Pierse (2003) for details of models estimated using this latter approach).

[13] An ADF test for the stationarity of unemployment gave a statistic of -2.33 for the level and -3.21 for the change in unemployment (critical value of -2.89).

profile for the NAIRU (u_t^*) and an estimate of the accelerationist Phillips curve (these include the parameters $\alpha(L)$, $\beta(L)$, and $\gamma(L)$).

4 Empirical results: using price inflation

We estimate various models in this paper that are modified versions of the triangle Phillips curve model, of the following general form:

$$\Delta_1\pi_t = \alpha'(L)\Delta_1\pi_{t-1} - \beta(L)(u_t - u_t^*) + \gamma(L)\Delta z_t + \varepsilon_t \quad \textbf{(4)}$$

where Δ is the first difference operator, π_t is the annual inflation rate and u_t is LFS unemployment. We allow for proxies for supply shocks (z_t), so the NAIRU that we obtain is the unemployment rate that is compatible with stable inflation in the *absence of temporary supply shocks*. The main proxies for supply shocks (z_t) used in other studies are real import prices and real oil prices.(14) Our work also uses these variables. Chart 3 below shows annual inflation of real import prices and real import prices less oil and Chart 4 shows annual real oil price inflation for the period 1973–2000 (all deflated using RPIX inflation). Real import prices fell after around mid-1996 (as a result of the appreciation of sterling), though at a slowing rate. Real oil price inflation also fell from 1997 to early 1999, though oil prices strengthened after 1999 Q1 and real oil price inflation was positive at the end of our sample period.

Chart 3: Real import prices

(annual % change)

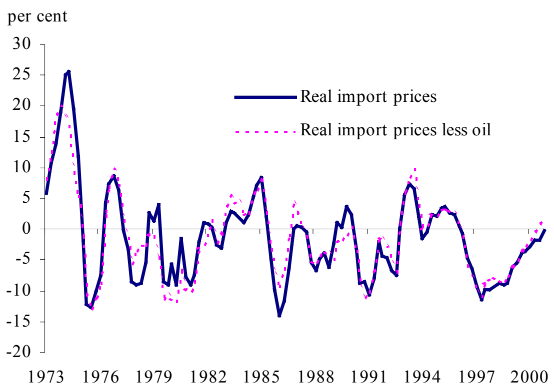
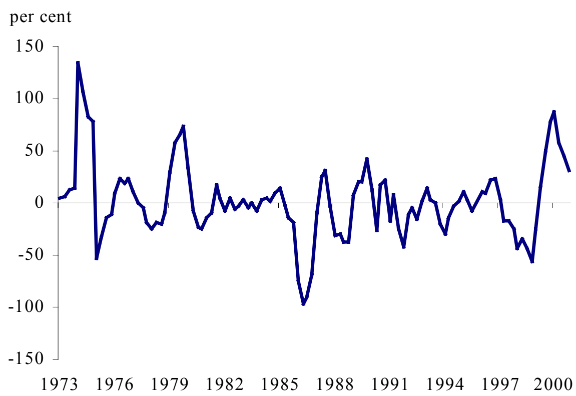


Chart 4: Real oil prices

(annual % change)



As outlined above, the extent to which the NAIRU can move around from quarter to quarter is usually restricted in the academic literature, reflecting the assumption that the NAIRU is determined by structural factors that evolve gradually over time. Obviously,

(14) Gordon also included the change in sensitive raw materials prices and the change in the real effective exchange rate, but he found these terms to be insignificant.

the choice of this restriction (the signal-to-noise ratio) is to some degree arbitrary and there is no universally accepted rule as to how to impose this restriction. We estimate a range of models using a general to specific estimation strategy based on a restriction for the signal-to-noise ratio of 0.16 (one of the higher values used in Gordon (1997)). The NAIRU profiles are influenced by this restriction and we do not suggest that this value is necessarily the United Kingdom's 'true' value. The sensitivity of the NAIRU profiles to other values for the signal-to-noise ratio is demonstrated in Section 7 below. All the estimates shown in this paper are *smoothed* NAIRU estimates (which use the full information set) rather than *filtered* estimates (which only use the information available at the time that the forecast was made).

Table A below shows the results of estimating models for RPIX inflation for the period from 1973 to 2000. We employ a general to specific estimation testing (omitting insignificant variables), where our regressors were the current value of the unemployment gap (the 'demand' effect in the triangle model), lagged annual RPIX inflation terms (capturing inertia) and lagged real import price inflation and real oil price inflation terms (the 'supply' component). The first row of Table A shows that changes in inflation are negatively correlated with the unemployment gap, suggesting that when unemployment is below the NAIRU, it will put upward pressure on inflation and inflation will tend to rise, though effects from the inertia and supply components in the model may offset this pressure.(15)

In other empirical papers, it is common to include at least one additional lag of the unemployment gap (since if there is at least one lag, the effect from the change in the demand variable on inflation is automatically captured (see Gordon (1997, page 16)). Gordon (1997, 1998) allows the unemployment rate to enter both contemporaneously and lagged, whereas Staiger, Stock and Watson (1997a) use specifications where only lagged values of the unemployment gaps enter. We allowed for both possibilities in our estimation strategy. When the lagged unemployment gap was added to our list of regressors, then the contemporanous value of the unemployment gap became insignificant and so this latter term was excluded from our model (this resulted in model 2). And when the general model was based on only lagged values of the unemployment gap (ie similar to the Staiger, Stock and Watson (1997a)), only, the lagged unemployment gap in period 1 was strongly significant (again delivering model 2). So, for our RPIX-based models, both estimation strategies deliver the same

(15) Preliminary work (based on an HP-filtered NAIRU) suggested that the equation diagnostics were improved by the inclusion of a dummy (=-1 in 1979 Q3, 1 in 1980 Q3, 0 at all other times). An oil price shock and VAT change occurred around this time, and this result suggests that normally distributed errors could only be achieved by including a dummy variable for this period.

model. Further, additional lags of the dependent variable were not statistically significant at conventional levels of testing and so were excluded.(16)

The final row in the table reports the log-likelihood (LL) for each model. A likelihood ratio (LR) test suggests that deleting any of the 'supply' variables leads to an inferior model.(17)

Since oil prices feed through into import prices, total import prices may to some extent contain the same information as the oil price term. To avoid the possibility of double-counting, an alternative is to use a real import price series stripping out oil together with real oil price series as our supply shock variables (models 3 and 4 below). Using a general to specific estimation approach based on the current unemployment gap delivers model 3. However, when lagged unemployment gaps were added to our general specification, once again, its lagged value in period 1 dominated either the contemporanous term or the additional lagged terms, resulting in model 4.

There is also the question as to what is the appropriate price deflator that should be modelled. Some papers (such as Gordon (1997)) use a variety of measures. In addition to RPIX, we employed a general to specific estimation strategy for models based on the consumers' expenditure deflator. The results were broadly similar to those reported above. There was a statistically significant relationship between the unemployment gap and changes in inflation and once again, the results were dominated by the inclusion of the gap term in the previous period. Changes in real import prices were significant, though this was not the case for real oil prices. The results are given in the final column of Table A (Section 6 below compares the NAIRU profiles).(18)

To summarise the findings of Table A, the lagged unemployment gap appears to play an important role in the inflation process. Our results suggest that it is more influential than either the contemporaneous value of the unemployment gap or further lags (since when these terms are included, the lagged term in period 1 remains statistically significant at the expense of other terms). But in terms of the overall specification, there is little difference in the coefficients or the fit of these models. Excluding any of the supply-side variables would lead to an inferior model. Thus, model 2 will be considered as our 'preferred' RPIX-based model.

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(16) For example, when $\Delta\pi_{t-2}$ was added to model 2, the t-statistic was –1.58. For other models considered, it was even less significant.

(17) The likelihood ratio test is simply 2*(LL(unrestricted model)-LL(restricted model)) which is distributed as χ^2(no of restrictions). In the case where there is one restriction, a difference in the likelihoods reported in the tables of 1.92 or higher (=3.84/2) means that the restricted model can be rejected at the 95% level of confidence.

(18) Preliminary work (based on an HP-filtered NAIRU) suggested that the equation diagnostics were improved by the inclusion of the same dummy in 1979/1980, as well as a dummy for 1977 Q4.

Table A: Price inflation Phillips curve models estimated using the Kalman filter, 1973 Q1–2000 Q4

Dependent variable $\Delta\pi_t$ (rpix)	RPIX (1)	RPIX (2)	RPIX (3)	RPIX (4)	Dependent variable $\Delta\pi_t$ (personal consumption deflator)	PC (1)
$u_t - u_t^*$	-0.41 [-4.43]	-	-0.55 [-4.73]	-		-
$u_{t-1} - u_{t-1}^*$	-	-0.43 [-4.74]	-	-0.57 [-4.63]		-0.42 [-5.10]
$\Delta\pi_{t-2}$	-	-	-	-		-
$\Delta\pi_{t-4}$	-0.32 [-4.87]	-0.34 [-5.18]	-0.31 [-4.64]	-0.34 [-4.95]		-0.34 [-5.50]
ΔReal import price Inflation $_{t-1}$	0.34 [3.58]	0.37 [3.88]	-	-		0.34 [4.34]
ΔReal import price Inflation $_{t-4}$	0.25 [2.34]	0.25 [2.31]	-	-		0.33 [4.37]
ΔReal oil price Inflation $_{t-3}$	-	-	-	0.27 [3.03]		-
ΔReal oil price Inflation $_{t-4}$	0.22 [2.14]	0.19 [1.84]	0.35 [4.03]	-		-
ΔReal import price (Less oil) inflation $_{t-1}$	-	-	0.39 [3.75]	0.38 [3.58]		-
D79/80	-3.79 [-6.65]	-3.84 [-6.78]	-4.10 [-7.42]	-4.16 [-7.56]		-3.17 [-7.37]
D77Q4	-	-	-	-		-3.08 [-4.92]
LL	-139.5	-139.3	-141.4	-141.7		-112.7

LL is the log-likelihood, t-statistics are in parentheses.

Chart 5 shows the NAIRU profiles for models 1 and 2 in Table A above and Chart 6 shows the profiles for models 3 and 4 for the period from 1973 to 2000. The general shape of the profiles is similar, even though we are allowing the profiles to be quite volatile. All models show the NAIRU peaking in the mid-1980s and tending to fall back thereafter. This is consistent with a range of NAIRU estimates produced using a structural approach (see Coulton and Cromb (1994)). The models tend to show that actual unemployment was below the NAIRU in the second half of the 1990s, but the extent of this gap differs. The smoothness of the fall in the NAIRU in the 1990s also varies according to the model used. The models shown in Chart 5 are characterised by a steep decline in the estimated NAIRU during the second half of the 1980s and a slight rise in the early 1990s, declining again after around 1993.

Chart 5: Unemployment and *u profiles from models 1, 2 and 3a (per cent)**

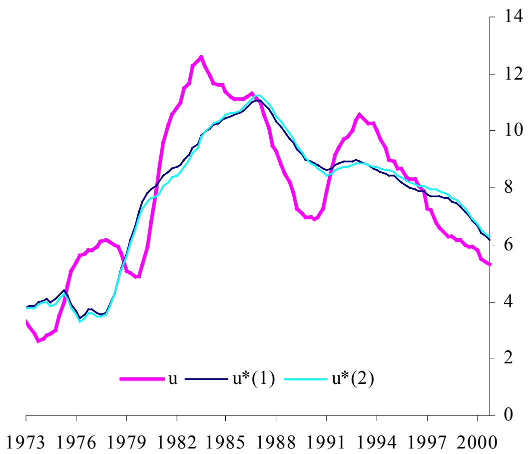
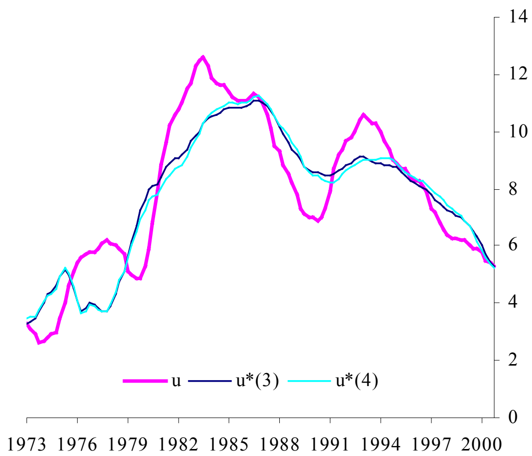


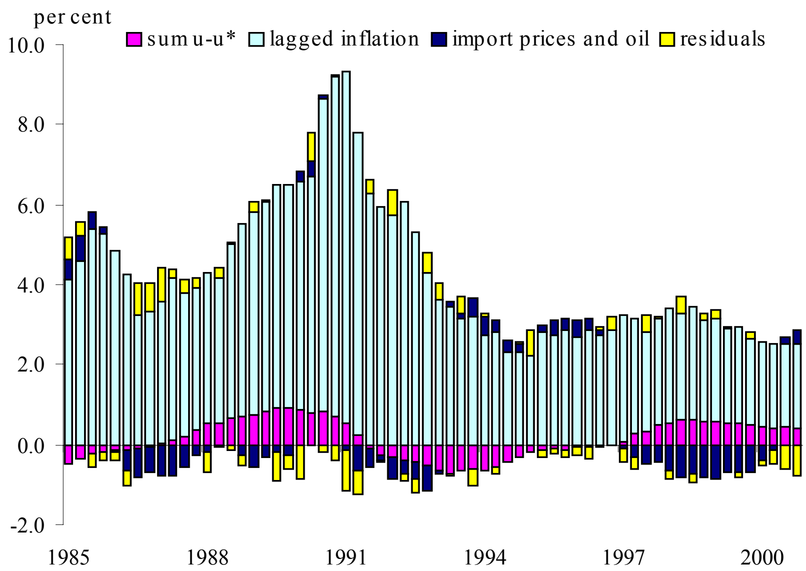
Chart 6: Unemployment and *u profiles from models 3 and 4 (per cent)**



If no supply-shock variables (such as import prices) were included in the model, then in an arithmetic sense, given the RPIX outturn, the NAIRU profile without supply-side variables would be lower for much of the period since 1997 (as real import price inflation has been negative during this period). The implication is that temporary supply shocks, such as the appreciation of sterling (which has led to lower import prices), have had a beneficial impact on RPIX inflation. In order to investigate this more fully, a static contributions exercise was undertaken for model 2. The results (which are expressed in terms of annual inflation) are shown in Chart 7 below.

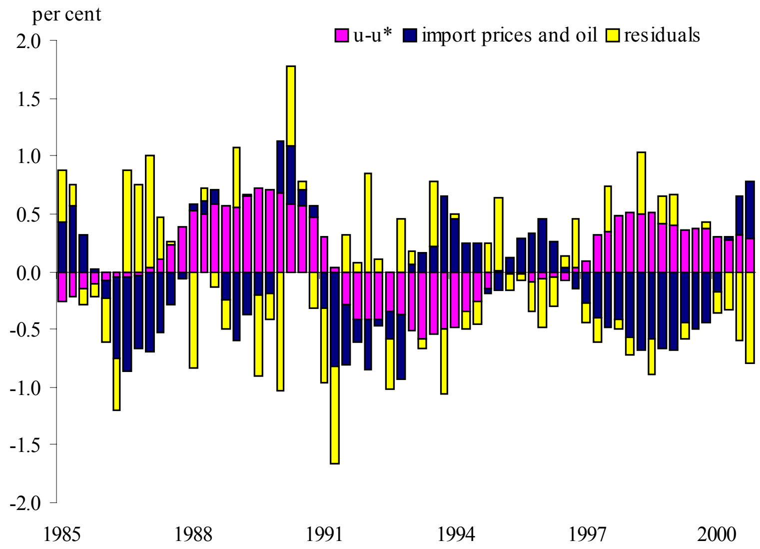
Unsurprisingly, inertia, or lagged inflation, is a key influence on current RPIX inflation. Following an upturn in economic activity, unemployment tends to fall below the estimated level of the NAIRU and this negative unemployment gap puts upward pressure on inflation. This can be seen for the expansion towards the end of the 1980s and for the period 1997-2000. Similarly, in the early 1990s, unemployment was above the NAIRU estimate suggested by this model and so put downward pressure on price inflation. This seems to accord with economic intuition. And we can also see that, according to this model, the fall in real import prices from 1997 to early 2000 provided a downward influence of almost 1 percentage point on price inflation, more than offsetting the positive impact from the unemployment gap effect. However, in the second half of 2000, the sharp rise in oil prices contributed upward pressure on prices, adding to the continued pressure from the unemployment gap effect. Had the NAIRU estimates not fallen in the second half of the 1990s, there would have been greater upward pressure on inflation from the labour market. So, both structural changes and favourable supply shocks appear to have had a beneficial effect on UK inflation over this period.

Chart 7: Contributions to annual RPIX inflation (based on model 2) – static exercise



Alternatively one could conduct a dynamic contributions exercise where the lagged inflation terms are decomposed into the contributions of lagged exogenous variables. Chart 8 below shows the results of a dynamic contributions exercise for model 2 (expressed in terms of the quarterly change). The story is similar to before – real import prices provided a downward influence on inflation for most periods after around 1997, offsetting the effect from the unemployment gap. At least in an accounting sense, a turnaround in real import price inflation could be accompanied by increased inflationary pressures. But the extent to which it might do so would of course depend on the future path of the other variables, which might themselves be affected by changes in real import price inflation.

Chart 8: Contributions to quarterly RPIX inflation (based on model 2) – dynamic exercise



5 Empirical results: using earnings inflation

Above we have considered a range of Phillips curve models specified in terms of price inflation, particularly RPIX. Now we express Kalman filter Phillips curve models in terms of earnings inflation. A general to specific estimation strategy was once again used where our regressors initially included a lagged inflation term, the current value of the unemployment gap and supply variables (real import prices and real oil prices). Real oil prices were not significant in the model at conventional levels of testing and so were excluded, as were additional lags of inflation or real import prices (model 1).[19] Model 1a also includes the lagged value of the unemployment gap. The coefficient on the current value of the unemployment gap becomes more negative as this additional term is added, though the coefficient on the lagged term is positive. But this model could be rewritten in terms of a level and a rate of change effect from the unemployment gap, both of which exert downward pressure on earnings inflation. Overall, the sum of the unemployment gap terms is negative and significant.[20] The NAIRU profiles for these models are shown in Chart 9.

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[19] The models shown in Table B use the RPIX deflator for the AEI-based models, but other models using the RPI or AEI index produced similar results.

[20] Inflation could depend on both the level and the change in the demand variable. The mixture of positive and negative coefficients in model a1 may reflect the change effect, whereas the significant (negative) sum of the coefficients reflects the importance of the effect of the level of the unemployment gap on the inflation process (Gordon (1997,1998)).

Table B: Earnings inflation Phillips curve estimated using the Kalman filter, 1973 Q1–2000 Q4

Dependent variable $\Delta\pi_t$ (average earnings index)	AEI (1)	AEI (1a)	AEI (2)	AEI (3)	Dependent variable $\Delta\pi_t$ (wages and salaries per employee)	WS (1)
$u_t - u_t^*$	-0.51	-0.90	-0.64	-		-0.61
	[-4.60]	[-4.39]	[-5.11]			[-3.65]
$u_{t-1} - u_{t-1}^*$	-	0.45	-	-1.33		-
		[2.22]		[-5.09]		
$u_{t-2} - u_{t-2}^*$	-	-	-	1.02		-
				[3.69]		
$\Delta\pi_{t-4}$	-0.43	-0.41	-0.43	-0.39		-0.48
	[-7.08]	[-6.59]	[-7.16]	[-5.87]		[-6.79]
ΔReal import price Inflation $_{t-4}$	0.40	0.43	0.47	0.38		0.60
	[3.93]	[4.26]	[4.33]	[3.88]		[4.24]
ΔReal oil price Inflation $_{t-1}$	-	-	-	-		0.25
						[1.71]
ΔProductivity			0.27	-		0.33
			[2.21]			[1.91]
D74	3.11	3.11	2.98	3.33		1.44
	[6.84]	[6.79]	[6.71]	[7.10]		[2.11]
LL	-155.7	-155.0	-153.2	-154.3		-187.8

Where the dependent variable is a measure of earnings inflation, as in the models estimated in Table B above, it may be argued that such a measure should be adjusted for productivity or trend productivity. Model 2 reports the results from our estimation whereby productivity growth is added to our list of regressors. Typically, lagged values of productivity appeared to be insignificant in the models tested, though this was not the case for the current value of productivity growth.(21) When this latter term was included in our model, the contemporaneous value of the unemployment gap remained statistically significant, though the lagged gap term became insignificant and so was

(21) Gordon (1998) includes a model where the dependent variable is specified in term of trend unit labour costs. This requires some assumptions regarding the calculation of trend productivity. Further, a model specified in terms of unit labour costs implies an immediate pass-through from productivity to earnings.

excluded. However, when we used the general to specific strategy based on a model that included only lagged values of the unemployment gap, it was productivity growth that was excluded from the model (model 3). It is interesting to note that NAIRU estimates for such a specification are broadly similar to those seen above, though tend to be somewhat noisier when only lagged unemployment gap terms are used (Chart 10).

There is no strong evidence to suggest any empirical superiority of a model where both the current and lagged gaps play a role. However, if productivity growth is also permitted to play a role in determining earnings growth, then model 2 may be considered as the preferred AEI-based model.

Chart 9: Comparison of NAIRU estimates from AEI Models 1 and 1a (per cent)

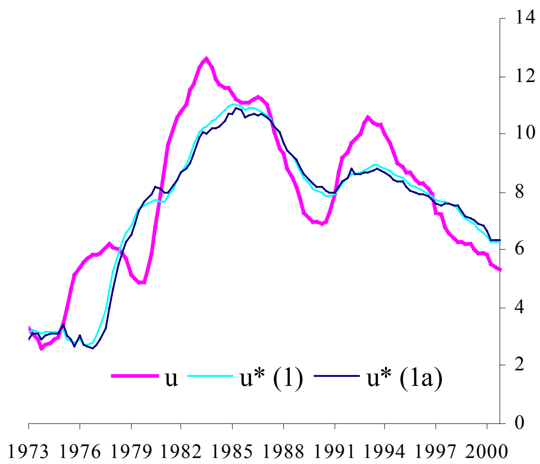
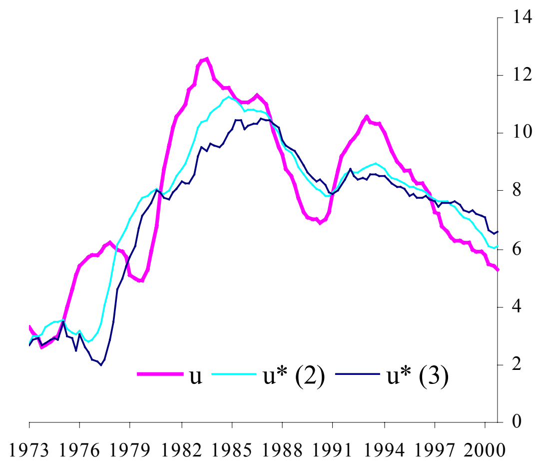


Chart 10: Comparison of NAIRU estimates from AEI Models 2 and 3 (per cent)



But other measures of earnings growth, in addition to the AEI may also be of interest. For example, the wages and salaries per employee measure is often used in earnings based Phillips curves. The final column in Table B reports the results of such a specification (Section 6 discusses differences in the profiles of the NAIRU estimates). Note that changes in wages and salaries per employee inflation also appear to depend on lagged inflation, the current value of the unemployment gap, real import price inflation and productivity growth, but in addition there is some role for real oil prices. Further, in such a model, there is no additional role for lagged values of the unemployment gap.

Once again, we conduct contribution exercises, but this time in terms of an AEI-based model (2). Charts 11 and 12 below show similar results to above. The unemployment gap once again provided upward pressure on AEI inflation in the period from 1997 to 2000. Real import prices provided a downward impact on AEI-based inflation since around mid-1997, though to a slightly lower degree than suggested by the RPIX model.[22] [23]

Chart 11: Contributions to AEI inflation (based on model 2) – static exercise

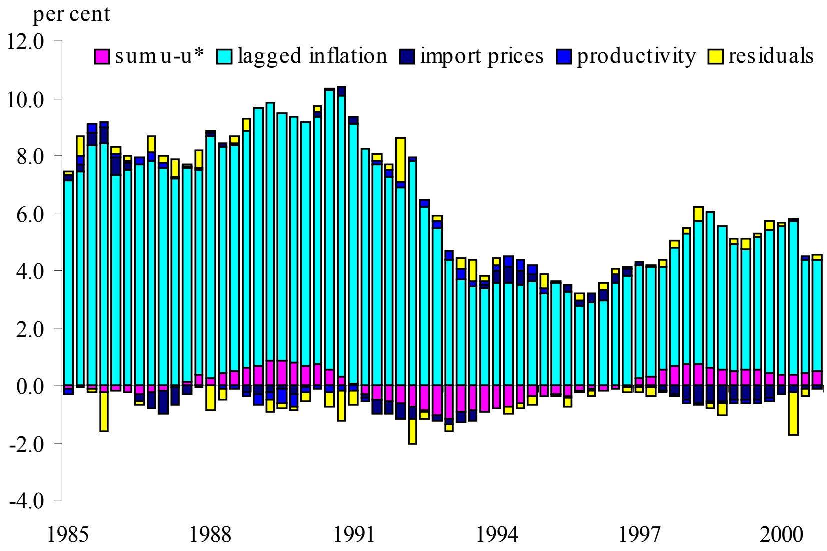
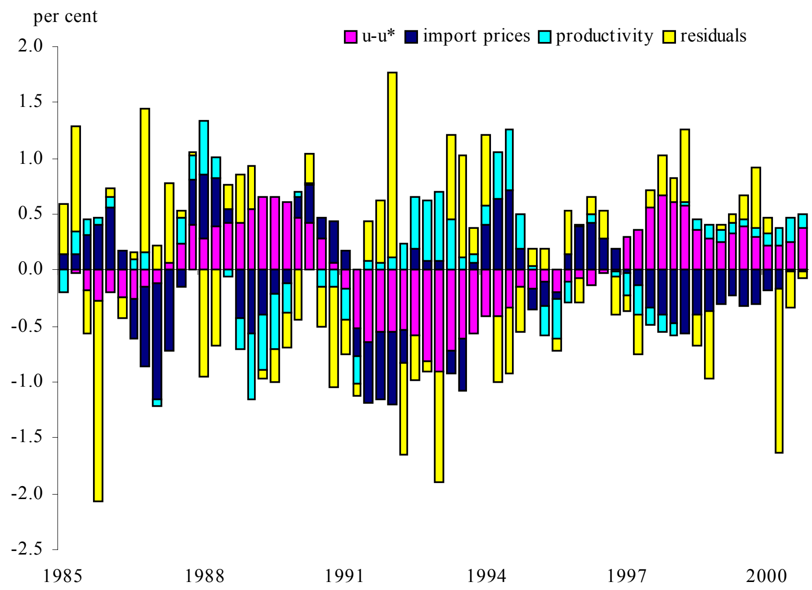


Chart 12: Contributions to AEI inflation (based on model 2) – dynamic exercise



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[22] This effect on AEI inflation is likely to be indirect in practice, working through the pricing channel.

[23] The large residual in 2000 Q2 is related to millenium effects. Annual wage inflation was boosted in early 2000 by factors such as additional overtime. Annual wage inflation fell back noticeably in 2000 Q2.

6 Comparing NAIRU estimates from price and earnings models

The estimates for the NAIRU that we have obtained – based on price inflation or earnings inflation – generally appear to show a similar profile over time. These estimates from a variety of models suggest that the NAIRU reached a peak in the mid-1980s and tended to decline through the 1990s. Average NAIRU/natural rate estimates based on structural models, rose in the second half of the 1970s and for most of the 1980s, before falling back slightly thereafter (Coulton and Cromb (1994)). Thus the estimates presented in this paper are broadly in line with other estimates, even though the approach taken is markedly different.

But closer examination of these results does reveal some differences. Chart 13 below shows the NAIRU estimates from RPIX model 2 and the consumers' expenditure deflator model. Both profiles tend to move together over time, but the latter model shows a greater decline since 1997. For the earnings-based models, the NAIRU estimates reach a similar peak, but the wages and salaries-based model has a smoother fall through the 1990s (Chart 14). The overall profile of the RPIX-based NAIRU estimates was broadly similar to the AEI-based models during the 1990s, though the RPIX-based estimates have been slightly higher than those from the AEI models.[24]

Chart 13: NAIRU estimates from price-based models (per cent)

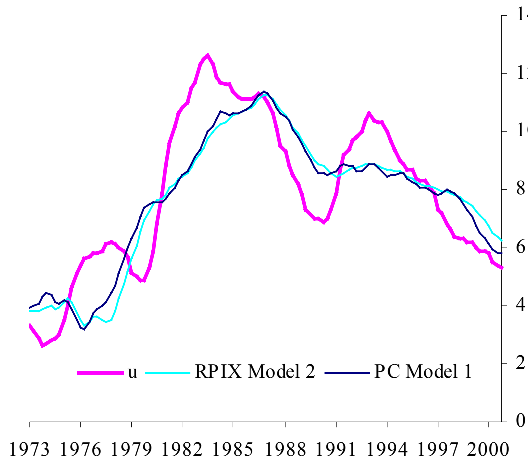
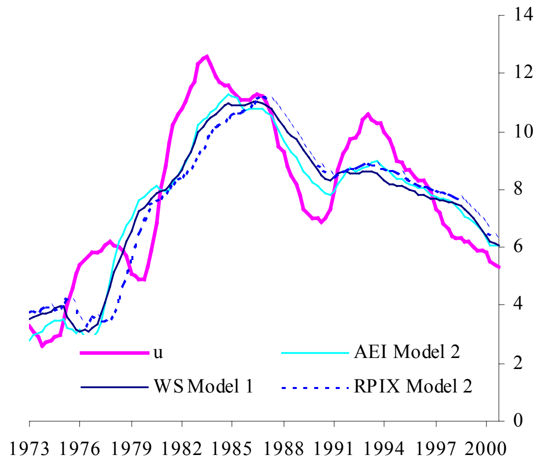


Chart 14: Comparison of NAIRU estimates from earnings and RPIX models (per cent)



[24] One possible reason for the minor divergence between the estimates could be the cyclical behaviour of firms' margins, though it is difficult to make precise inferences about this since these are reduced-form models.

Taking the four models above, we consider the broad patterns in the NAIRU estimates.[25] During the first half of the 1980s, all estimates rose by a similar amount, but the increase in these estimates was only around half of the increase that was recorded for actual unemployment. For the latter part of the 1980s, these estimates fell (the AEI-based measure saw the larger declines), as did unemployment. For the period 1990–2000 overall, there has been less change in either the NAIRU estimates or actual unemployment, though such a long period masks many developments. In the early 1990s, unemployment rose (reaching a peak in 1993), before falling back thereafter. Overall for the period 1990–95, actual unemployment rose by about 1.9 percentage points, whereas the estimated NAIRUs were broadly flat. The main difference during this period is that the AEI-based NAIRU estimates fell more during 1994 (around -0.5 percentage points) than did either of the price-based estimates (around -0.1 percentage points).

The final period, 1995–2000, is interesting. All NAIRU estimates fell during this period, though by much less than did unemployment, suggesting that these estimates do not just follow unemployment. This contrasts with results for a HP-filtered NAIRU that shows the NAIRU moving in line with unemployment during this period.[26] Overall, these results do not suggest any major difference between the wage-unemployment relationship and the price-unemployment relationship. Indeed, the estimates appear to be (perhaps surprisingly) robust to the variable choice.

Table C: Comparison of NAIRU estimates, 1980–2000

Sample/percentage points change	Actual change in unemployment	Change in u^* AEI (2)	Change in u^* WS (1)	Change in u^* RPIX (2)	Change in u^* PC (1)
1980–85	6.1	3.4	3.4	3.3	3.2
1985–90	-4.4	-3.1	-2.2	-1.7	-2.1
1990–2000	-1.7	-2.0	-2.6	-2.6	-2.7
of which 1990–95	1.9	0.3	-0.6	-0.3	0.0
1995–2000	-3.6	-2.3	-2.0	-2.3	-2.7

[25] This exercise could be repeated for any model. Obviously, the results will depend on the exact specification and variables used, as well as the restriction of the signal-to-noise ratio, though the broad pattern for the models that we have considered from the tables above is the same.

[26] We use the value for lambda normally used for quarterly data, 1600, for this exercise.

7 Sensitivity analysis

The NAIRU estimates above were all determined on the basis of a restriction of the signal-to-noise ratio, consistent with the idea that the NAIRU should not jump around from quarter to quarter, but instead evolves more gradually over time. Below we show the effect of using a wide range of restrictions for the ratio of the variances of the error terms before providing some standard error bands. For conciseness, we conduct these exercises using only an AEI-based model 1a above.

7.1 Restricting the signal-to-noise ratio

Though the variability of the NAIRU can in principle be estimated from the data, a restriction is usually adopted in the literature (this involves restricting the ratio of the variances of the error terms (σ_η^2 and σ_ε^2 above)). For a given variation in ΔΠ, the signal-to-noise ratio measures the volatility or variance of the NAIRU relative to the variance of ΔΠ. In modelling US price inflation, Gordon (1997) experiments with different restrictions and chooses the restriction that produces a time-varying NAIRU that is less volatile than unemployment. He uses values of 0.2 and 0.4 for the standard deviation of the NAIRU for the US (or 0.04 and 0.16 for its variance), suggesting that this value allows movements in the NAIRU, but not sharp quarter-to-quarter changes. Gordon (1998) uses similar values to his earlier paper: the imposed standard deviation ranges from 0.045 to 0.271. Staiger, Stock and Watson (1997a) use values of 0.05 and 0.15 for the variance when modelling US price inflation, whereas Laubach (1997) uses a value of 0.09 (for the variance). We have conducted similar exercises; Table D below gives some results for a model based on earnings inflation (model 1a from Table B).

Table D: Estimating earnings inflation using the Kalman filter, 1973 Q1–2000 Q4

	$\Delta\Pi_t$	$\Delta\Pi_t$	$\Delta\Pi_t$	$\Delta\Pi_t$
	Variance =0.04	Variance =0.09	Variance =0.16	Variance =0.25
$u_t - u_t^*$	-0.94	-0.92	-0.90	-0.83
	[-3.21]	[-2.98]	[-4.39]	[- 4.39]
$u_{t-1} - u_{t-1}^*$	0.86	0.60	0.45	0.38
	[2.90]	[1.78]	[2.22]	[1.87]
$\Delta\pi_{t-4}$	-0.33	-0.39	-0.41	-0.42
	[-4.71]	[-5.53]	[-6.59]	[-6.73]
ΔReal import price	0.35	0.41	0.43	0.42
Inflation $_{t-4}$	[3.40]	[4.01]	[4.26]	[4.06]

The sensitivity of the estimated NAIRU to different restrictions is shown in Chart 15. In the case where the variance is 0.04, most of the new data on inflation and unemployment are treated as noise rather than signal, and so the NAIRU is constrained to be extremely smooth (the inflation residuals would be larger). As more of the information is treated as signal, corresponding to an increase in variance in Chart 15, the NAIRU becomes more volatile (though the extent of the volatility depends on the model used). But the basic profile of the NAIRU is broadly the same with similar turning points within a plausible range of signal-to-noise restrictions. Note that when the signal-to-noise ratio is freely estimated, the point estimate is numerically larger than the restriction used throughout this paper (0.16) but lies within the confidence interval. The resulting NAIRU estimates show implausibly large variation (perhaps because the model insufficiently captures supply side or expectations shocks, particularly in the 1970s), so we maintain the lower figure.

The sum of the coefficients becomes more negative as the signal-to-noise ratio rises. This does not necessarily imply in itself that these terms make a larger contribution to inflation as the ratio increases, since the associated unemployment gap is smaller (Chart 16 shows the different unemployment gaps associated with the various estimates of the NAIRU).

Chart 15: Different NAIRU estimates based on alternative variances

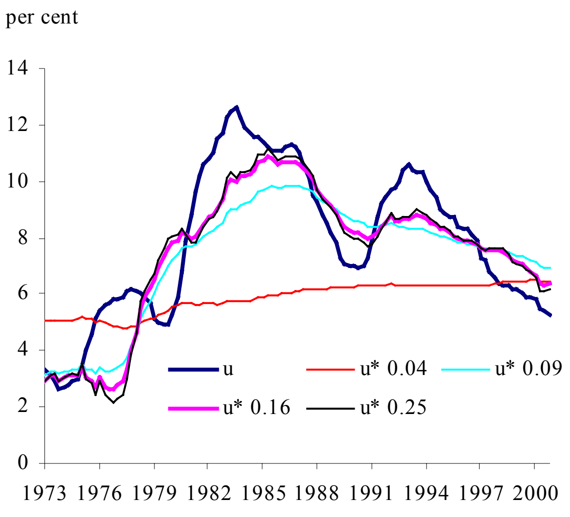
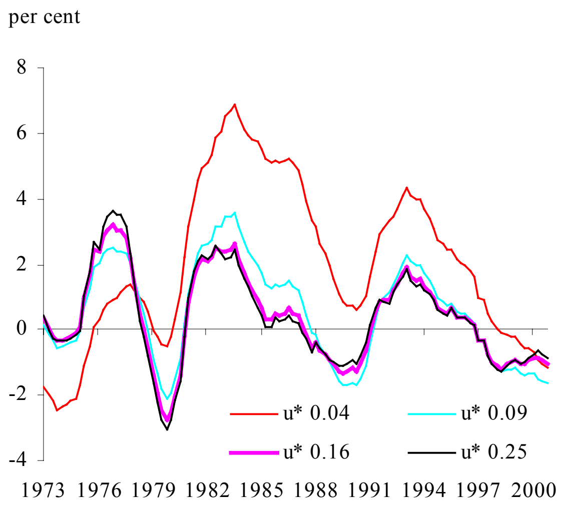


Chart 16: Different unemployment gaps based on alternative NAIRU estimates



7.2 Standard error bands for the NAIRU

Restricting the signal-to-noise ratio is one way in which the sensitivity of the NAIRU estimates can be considered, as demonstrated above. But another way of considering the sensitivity issue is in terms of the standard error bands around the NAIRU. The Kalman filter approach allows us to do this, as do some other approaches. For an excellent example of different techniques to derive estimates of the US NAIRU, see Staiger, Stock and Watson (1997a). Using various methods, they found that the confidence intervals around the US NAIRU were large.[27]

We calculate confidence intervals for the NAIRU using AEI-based model 1a above.[28] We consider only the *filter uncertainty* that is calculated from the smoothing iteration. This represents uncertainty that would be present even if true values of the parameters were known.[29] Chart 17 below shows an example of the standard error bands, confirming that both the 90% and 95% confidence intervals around the NAIRU are wide. This is in line with the findings of Cross, Darby and Ireland (1997) who used a variety of specifications, but not the Kalman filter approach, to derive estimates of the confidence intervals for the NAIRU and found a large degree of uncertainty around these estimates. Using their preferred specification for the UK (which allowed a mean shift in unemployment), the 95% confidence intervals for the NAIRU were in the range of 7.41 to 10.29 for the period 1980 Q1–1995 Q3. However, for an alternative approach, based on a 2 knot cubic spline specification, the 95% confidence intervals were between –9.98 and 14.55 for 1994 Q1. Our findings confirm the general belief that estimates of the NAIRU tend to be imprecisely measured, suggesting that such estimates should be treated with caution. For example, at times, we may be 90% certain that actual unemployment has not been above (or below) the NAIRU. However, this information may still be useful, though it does suggest not placing too much emphasis on a particular point estimate.

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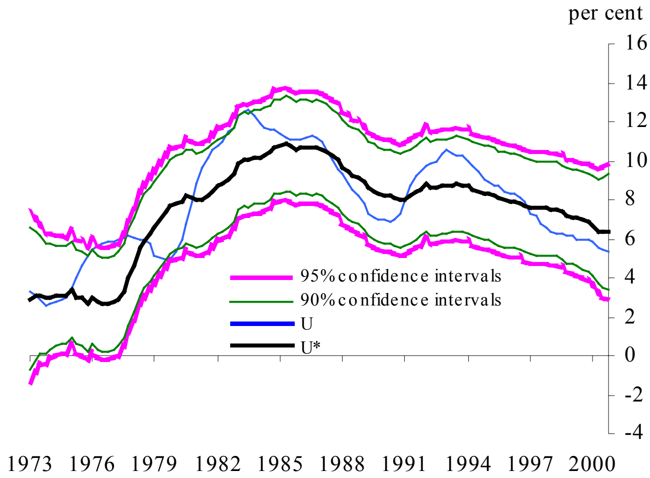
(27) For their Kalman filter based time-varying models, the standard errors reported for the NAIRU are the square root of the sum of the Kalman smoothed estimates of the variance of the state and the delta method estimate of the variance of the estimate of the state (Ansley and Kohn (1986)).

(28) We use a signal-to-noise restriction of 0.16. The confidence intervals will depend on the amount of variation allowed in the NAIRU to some extent. Laubach (1997) notes a positive correlation between the width of the confidence intervals and the volatility in the NAIRU. We find the opposite – as the NAIRU is allowed to move more, the estimated standard errors are smaller. This matches the result in Boone (2000).

(29) One could in principle also calculate the parameter uncertainty by Monte Carlo simulations. We leave this as a possible future extension.

Chart 17: Standard error bands



8 Conclusions

We have used a variety of models, based on either price or wage inflation, and calculated time-varying NAIRU estimates using Kalman filter techniques. According to these estimates, the NAIRU reached a peak in the mid-1980s and tended to decline through the 1990s. We find these profiles plausible, though they vary to some degree, depending on the variables included and the extent to which the signal-to-noise ratio has been restricted. Such profiles are broadly in line with other UK estimates, often obtained from different approaches. We demonstrate that there is a huge amount of uncertainty surrounding NAIRU estimates, as the confidence bands are likely to be wide. As a consequence, we would not place weight on any particular point estimate for the NAIRU. But the models tend to suggest that unemployment was below the NAIRU for much of the second half of the 1990s, suggesting some upward pressure on inflation from this source. Had the NAIRU estimates not fallen over this period, there would have been greater upward pressure on inflation from the labour market. So structural changes appear to have had a beneficial effect on UK inflation during this period.

However, the story does not end there. Our results suggest that temporary supply factors (captured by real import prices or real oil prices) are also likely to have played an important role in holding inflation down, especially in the 1997-99 period. Developments in import prices or oil prices, as well as movements in the unemployment gap, may therefore be important in assessing future inflationary pressures.

This paper has not touched on changes to the UK monetary policy regime, such as the move to inflation targeting at the end of 1992 or the granting of independence to the Bank of England in 1997, which may have had an impact on the formation of inflation expectations. It is possible that our NAIRU estimates are indirectly picking up any such changes, thus casting doubt on our estimates. But separate work including inflation expectations does not provide any strong evidence that this was a key factor for the United Kingdom (see Driver, Greenslade and Pierse (2003)).

Appendix A: Data definitions

We use ONS data where available (ONS codes in parentheses).

Prices: Retail Price Index excluding mortgage interest payments (RPIX) since 1974 [code CHMK]. Prior to 1974, we obtain a series for RPIX by applying the growth rates on the changes in the RPI index [code CHAW] to the level of RPIX in 1974.

Prices: Total Final Consumers' Expenditure deflator (PC) [code (ABJK+HAYE) / (ABJR+HAYO)].

Earnings: Average Earnings Index (AEI) [code LNMQ].

Earnings: Wages and Salaries per employee (WS). This is wages and salaries from the national accounts [code DTWL] divided by the sum of employees in employment plus HM Forces.

Unemployment: LFS unemployment from 1984 [code MGSX] and OECD measure prior to 1984.

Real import prices: Nominal total import prices are given by the implicit import price deflator [code = IKBI/IKBL] and import prices less oil are total trade in goods less oil [code BQKL]. In both cases import prices are deflated using RPIX or relevant price deflator.

Real oil prices: Brent oil prices in US dollars [code IFS.UK.IFS.11276AAZZF] converted into pounds sterling [code AJFA]. This series is also deflated using RPIX or relevant price deflator.

Productivity: Output per head [code LNNN].

Appendix B: Kalman filter technique

The Kalman filter of Kalman (1960) and Kalman and Bucy (1961) is an algorithm for generating minimum mean square error forecasts in a state space model. If Gaussian errors are assumed, the filter allows the computation of the log-likelihood function of the model. This enables the parameters to be estimated easily using maximum likelihood methods.[1]

The state space form comprises two equations: a measurement equation and a transition equation. The *measurement equation* specifies how the vector of *n* observed variables, y_t, is related to a vector of *m* unobserved state variables, α_t (the *state vector*), and is given by:

$$y_t = Z_t \alpha_t + X_t d + \varepsilon_t \qquad t = 1,\ldots\ldots,T \qquad \textbf{(1)}$$

where Z_t is an *n* x *m* matrix and X_t is an *n* x *k* matrix of exogenous variables and where ε_t is an observational error with

$$\text{var}(\varepsilon_t) = \sigma^2 H_t$$

The *transition equation* specifies the time-series process generating the unobservable state variables and is given by:

$$\alpha_t = T_t \alpha_{t-1} + c_t + R_t \eta_t \qquad t = 1,\ldots\ldots,T \qquad \textbf{(2)}$$

where T_t is an *m* x *m* matrix, c_t is an *m* x 1 vector, R_t is an *m* x *g* matrix and η_t is a *g* x *1* vector of serially uncorrelated disturbances with

$$\text{var}(\eta_t) = \sigma^2 Q_t$$

The matrices Z_t, H_t, T_t, R_t, Q_t and G_t are known as the *system matrices*. Most of the elements of these matrices will be fixed values, mainly ones and zeros. However, they will also contain elements corresponding to the underlying parameters of the system, known as the system *hyper-parameters*. The vectors c_t and *d* may also contain parameters but these do not affect the stochastic properties of the model.

Let a_{t-1} be the minimum mean square linear estimator (*MMSLE*) of the state vector α_{t-1} based on information available at time *t*-1 and let P_{t-1} be the *m* x *m*, covariance matrix of the estimation error defined by

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[1] This appendix treats the general case. In the application in this paper, we have $n = m = 1$ so that the variance matrices H_t and Q_t are scalars and by assumption are time-invariant.

$$P_{t-1} = E(a_{t-1} - \alpha_{t-1})(a_{t-1} - \alpha_{t-1})'$$

Then the Kalman filter comprises two sets of recursive equations: the prediction equations and the updating equations.

The prediction equations give the optimal predictors of the state vector α_t and its covariance matrix based on information available at time t-1.

$$a_{t|t-1} = T_t a_{t-1} + c_t \tag{3}$$

$$P_{t|t-1} = T_t P_{t-1} T_t \ + R_t Q_t R_t \tag{4}$$

The updating equations update this predictor using new information available at time t embodied in the prediction error

$$v_t = y_t - Z_t a_{t\ |t-1} - X_t d \tag{5}$$

The updating equations are given by:

$$a_t = a_{t\ |t-1} + P_{t\ |t-1} Z_t \ F_t^{-1} v_t \tag{6}$$

and

$$P_t \ = P_{t\ |t-1} - P_{t\ |t-1} Z_t \ F_t^{-1} Z_t P_{t\ |t-1} \tag{7}$$

where

$$F_t \ = Z_t P_{t\ |t-1} Z_t \ + H_t$$

Thus, the Kalman Filter is a recursive process for calculating the optimal estimator of the state vector given the information set at that time. The repeated process of optimal prediction, getting the prediction errors and updating the predictions are the essence of the Kalman filter algorithm.

Assuming that the disturbances are normally distributed, the log-likelihood function for the model can be computed from the prediction errors v_t and their associated covariance matrix F_t and is defined by:

$$L = -\frac{nT}{2}\log(2\pi\sigma^2) - \frac{1}{2}\sum_{t=1}^{T}\log|F_t| - \frac{1}{2\sigma^2}\sum_{t=1}^{T} v_t{}' F_t^{-1} v_t \tag{8}$$

The Kalman filter predictors $a_{t|t-1}$ and $P_{t|t-1}$ give the optimal predictors of the state vector α_t and its covariance matrix based on information available at time *t*-1. So this procedure of obtaining *filtered* estimates of unobserved state variable (in this case the NAIRU) does not use all the available information. The Kalman filter allows a backward recursion known as *smoothing.* The *smoothed* estimators $a_{t|T}$ and $P_{t|T}$ give the optimal predictors of α_t and $\mathrm{var}(\alpha_t)$ based on *all* the information in the sample. These smoothed estimators can be generated from the backward recursions

$$a_{t|T} = a_t + P_t^*(a_{t+1|T} - T_{t+1}a_t - c_t) \qquad \textbf{(9)}$$

and

$$P_{t|T} = P_t + P_t^*(P_{t+1|T} - P_{t+1|t})P_t^{*'} \qquad \textbf{(10)}$$

where

$$P_t^* = P_t T_{t+1} P_{t+1|t}^{-1}$$

If $P_{t+1|t}$ is singular, its inverse can be replaced by a generalised inverse $P_{t+1|t}^-$.

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